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Hicks versus Slutsky: Analytical Insights from the Cobb–Douglas Utility Framework

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Abstract
This paper examines the Hicks and Slutsky effects that arise when the price of a good changes. Starting from the distinction between Marshallian and Hicksian demand functions, the analysis highlights the decomposition of total demand variation into substitution and income effects. The methodological approach employs the Cobb–Douglas utility function, which allows explicit derivations and a transparent comparison of the two decompositions. Analytical results show that the relative importance of substitution versus income effects depends on the magnitude and direction of price changes. In particular, the Hicks and Slutsky decompositions provide different shares of the total demand adjustment, with Hicks assigning a larger weight to the income effect when prices decrease, and Slutsky emphasizing the substitution effect. The findings demonstrate that the prevalence of one decomposition over the other is determined by the price change ratio and the utility parameters, offering theoretical insights with relevance for demand analysis, welfare evaluation, and consumer behavior modeling.

Keywords: utility, Hicks effect, Slutsky effect, Cobb–Douglas, income

1 Introduction

The study of consumer behavior in the face of price changes is one of the most fundamental aspects of microeconomic theory. Prices convey information, guide allocation decisions, and ultimately shape both individual welfare and market equilibria. When the price of a good varies, the consumer's demand adjusts not only because relative prices change, but also because real purchasing power is affected. This adjustment process is classically decomposed into two components: the substitution effect and the income effect (Azim et al., 2024).

Two major frameworks dominate this analysis. The Hicksian decomposition isolates the substitution effect by keeping utility constant, allowing economists to study how consumers reallocate consumption across goods purely due to relative price changes (Nosheen et al., 2025). By contrast, the Slutsky decomposition holds purchasing power constant, making it more directly linked to observable behavior in real markets. Although both decompositions aim to separate the same total demand effect, the methodological differences lead to distinct interpretations and sometimes different policy implications (Coto-Millán, 2012).

Understanding these differences is not merely of theoretical interest. In applied economics, welfare evaluations, tax policy design, and demand forecasting all rely on assumptions about how substitution and income effects operate. For example, the choice between Hicksian and Slutsky approaches can alter conclusions about consumer welfare after a tax reform or a subsidy scheme. Hence, clarifying the relative weight of substitution and income effects under each decomposition remains a relevant question.

In this paper, we contribute to this discussion by conducting a comparative analysis of Hicks and Slutsky effects using a Cobb–Douglas utility function. This functional form is chosen because of its analytical tractability, well-known properties, and widespread application in both theoretical and empirical studies. By explicitly deriving the effects, we are able to determine the conditions under which one decomposition assigns greater importance to substitution relative to income, and vice versa (Madden, 1991; Wang, J., Yu, S., & Liu, T. (2021).

The main objective of this study is to show how the relative dominance of Hicks or Slutsky effects depends on the ratio of price change and the parameters of consumer preferences. In doing so, we aim to provide a clear analytical framework that highlights the differences between the two decompositions, while also offering insights that can be extended to more general utility specifications (Lenfant, 2018).

The novelty of this paper lies in the explicit comparative analysis of Hicks and Slutsky decompositions using the Cobb–Douglas utility function as a tractable framework (Ioan & Ioan, 2015). While the distinction between compensated and uncompensated demand is well established in microeconomic theory, most treatments focus either on theoretical definitions or on numerical illustrations (Sasakura, 2016). This study extends the discussion by deriving analytical expressions for both decompositions, computing the substitution and income effects, and determining the thresholds at which one decomposition attributes more weight to the income effect relative to the substitution effect. In particular, the paper highlights how the ratio of price change and the preference parameters (α, β) influence the relative dominance of Hicks or Slutsky effects. These results provide not only a clearer understanding of the theoretical differences but also an original perspective on their quantitative implications.

2. Literature review

The decomposition of demand responses into substitution and income effects has been a central theme in microeconomic theory since the early formulations of consumer demand analysis. Classical consumer theory initially relied on the Marshallian demand framework, in which changes in consumption are explained solely by adjustments to relative prices and income, without distinguishing between utility compensation and purchasing power compensation (Goodwin et al., 2018). However, this approach provided limited insight into the underlying mechanisms of consumer choice when prices vary (Zambelli, 2024).

The Hicksian formulation advanced the analysis by introducing the concept of compensated demand, which holds utility constant (Brown, 2018). This allowed for a more precise isolation of the substitution effect, separating it from the income effect generated by a change in real purchasing power. In parallel, the Slutsky decomposition offered an alternative view, in which the substitution effect is defined by keeping real income constant rather than utility (Aguiar & Serrano 2025). This made the Slutsky approach particularly appealing for empirical applications, since it is directly linked to observable behavior and measurable changes in expenditure (Araar & Verme, 2016).

The distinction between Hicks and Slutsky decompositions has generated extensive debate in the literature, as the two frameworks, although mathematically related, emphasize different economic interpretations (Ghosh et al., 2016). Hicks' approach is often viewed as more theoretically rigorous, since it preserves consumer welfare at the initial level of utility. Slutsky's decomposition, on the other hand, is more intuitive in applied contexts, since it reflects how a consumer could maintain the same consumption bundle after a price change, before reallocating resources to maximize utility (Mohajan, 2017).

Over time, these theoretical constructs have been applied to a wide range of utility functions and market settings (Durán, 2024). The Cobb–Douglas utility function has frequently been used in such studies due to its tractability, homotheticity, and ability to capture constant expenditure shares. Its analytical properties make it particularly suitable for deriving explicit substitution and income effects and for comparing the two decompositions in a transparent manner (Jensen, 2024).

While the Hicks and Slutsky effects are well established in the microeconomic literature, relatively few studies have undertaken a systematic comparison of the two approaches within the same functional framework (Zhang, 2020). Most contributions either focus on the properties of compensated versus uncompensated demand in general or illustrate one decomposition in isolation. A direct comparison, especially in the case of Cobb–Douglas preferences, offers a valuable perspective by highlighting not only the differences in theoretical formulation but also the quantitative implications of each approach (Castro, 2024; Brown, 2018).

The gap in the literature lies in providing explicit analytical results that show under which conditions Hicks or Slutsky assigns greater weight to the substitution or income effect. By filling this gap, the present study contributes to a deeper understanding of how theoretical decompositions translate into measurable outcomes when prices change. This comparison is relevant not only from a theoretical standpoint but also for applications in welfare analysis, policy design, and empirical demand estimation, where the choice between Hicksian and Slutsky effects can affect conclusions about consumer behavior.

3. Theoretical Framework

3.1. Marshallian (or Walrasian) demand

Let a consumer be faced with choosing a certain number of quantities from an ordered set of goods $B_1,...,B_n$, SC – their consumption space and selling prices: $p_1,...,p_n$. We will assume that the entire income V available to the consumer can be allocated to the act of purchase, his preferences not being affected by the size of V. We will say, in this case, that the demand for goods is uncompensated. Also, let $U:SC\rightarrow\mathbf{R}_+$ be a utility function. Considering the budget area $ZB=\{(x_1,...,x_n)\in SC \mid \sum_{i=1}^{n} p_i x_i \le V\}$ we pose the problem of determining the consumption basket so that the utility is maximum.

The problem becomes:

$$\begin{cases} max \;\; U(x_1,\ldots,x_n) \\ \displaystyle\sum_{i=1}^{n} p_i x_i \le V \\ x_1,\ldots,x_n \in SC \end{cases}$$

It is shown that under the conditions where U is a concave function and SC is a convex set, then the optimal solution of the problem is located on the border of the budget area, that is, it satisfies the conditions:

$$\begin{cases} max \;\; U(x_1,\ldots,x_n) \\ \sum_{i=1}^{n} p_i x_i = V \\ x_1,\ldots,x_n \in SC \end{cases}$$

Applying the method of Lagrange multipliers, we obtain: $\frac{U_{m,1}}{p_1} = \ldots = \frac{U_{m,n}}{p_n}$ - Gossen's second law and characteristic system:

$$\begin{cases} \frac{U_{m,1}}{p_1} = \ldots = \frac{U_{m,n}}{p_n} \\ \sum_{i=1}^{n} p_i x_i = V \end{cases}$$

The solution to the problem is::

$$\begin{cases} \bar{x}_1 = f_1(p_1,\ldots,p_n,V) \\ \ldots \\ \bar{x}_n = f_n(p_1,\ldots,p_n,V) \end{cases}$$

It can be shown that the restriction of the function U to the hyperplane $\sum_{i=1}^{n} p_i x_i = V$ has the same nature as U, so it is concave. As a result of this fact, the point $(\bar{x}_1,\ldots,\bar{x}_n)$ is a local maximum. We will say in this case that the demand is of the Marshall type.

3.2. Hicks demand

Now let the same consumer who wants to satisfy a given level of utility under the conditions in which he is willing to allocate the lowest income to achieve his goals. We will say, in this case, that the demand for goods is compensated. Considering the utility function U:SC→$\mathbf{R}_+$ and $\overline{u}$ the desired utility, the problem of determining the consumption basket so that the allocated income is minimal becomes:

$$\begin{cases} min \sum_{i=1}^{n} p_i x_i \\ U(x_1,\ldots,x_n) \geq \overline{u} \\ x_1,\ldots,x_n \in SC \end{cases}$$

As in the previous section, we obtain that, under the conditions in which the objective function is linear, it is also convex, in particular, the optimal solution of the problem is located on the border of the area $U(x_1,...,x_n) \geq \overline{u}$.

The problem becomes:

$$\begin{cases} min \sum_{i=1}^{n} p_i x_i \\ U(x_1, \dots, x_n) = \overline{u} \\ x_1, \dots, x_n \in SC \end{cases}$$

Applying the method of Lagrange multipliers, we obtain again Gossen's second law and characteristic system:

$$\begin{cases} \frac{U_{m,1}}{p_1} = \dots = \frac{U_{m,n}}{p_n} \\ U(x_1, \dots, x_n) = \overline{u} \end{cases}$$

and problem's solution:

$$\begin{cases} \tilde{x}_1 = g_1(p_1, \dots, p_n, \overline{u}) \\ \dots \\ \tilde{x}_n = g_n(p_1, \dots, p_n, \overline{u}) \end{cases}$$

We will say in this case that the demand is of the Hicks type.

4. Methodology

The methodological framework of this paper relies on the decomposition of demand variation into substitution and income effects using two classical approaches: the Hicks and the Slutsky methods. The analysis proceeds in several steps:

a. **Specification of demand functions**

The Marshallian demand function is derived from the maximization of consumer utility subject to a budget constraint.

The Hicksian (compensated) demand function is derived from the minimization of expenditure subject to a given level of utility.

b. **Definition of effects**

The **Hicks decomposition** isolates the substitution effect by holding utility constant and adjusting income, followed by an income effect that restores the consumer to the actual budget.

The **Slutsky decomposition** isolates the substitution effect by holding real purchasing power constant, followed by an income effect that adjusts to the initial level of income.

c. **Analytical setting**

The Cobb–Douglas utility function is employed as a tractable functional form:

$$U=AX^{\alpha}Y^{\beta}, \text{ unde } \alpha ,\beta>0$$

Where:

X – the quantity consumed of good A (or good 1);

Y – the quantity consumed of good B (or good 2);

α– a preference parameter that indicates the relative importance of good X;

β – a preference parameter that indicates the relative importance of good Y;

In equilibrium, the consumer spends α/(α+β) of income on X and β/(α+β) on Y.

A – a scaling constant (normalization factor). It does not affect consumption choices, only the "scale" of utility.

Initial equilibrium is derived from the first-order conditions of utility maximization under the budget constraint.

d. **Comparative analysis**

For a change in the price of one good, both Hicksian and Slutsky decompositions are applied.

Substitution and income effects are computed explicitly for the Cobb–Douglas case.

Ratios are defined to evaluate the relative importance of the substitution and income effects under each decomposition.

e. **Evaluation criteria**

The prevalence of Hicks or Slutsky effects is assessed based on the **price change ratio** and the **parameter structure** of the utility function (α, β).

Analytical expressions are used to identify thresholds where the dominance of one effect

over the other reverses.

Building on the Cobb–Douglas specification, the analysis proceeds by applying both the Hicksian and Slutsky decompositions to a price change scenario. The structure of the utility function allows explicit derivation of Marshallian and Hicksian demand functions, which are then used to separate total demand variation into substitution and income components. The role of the preference parameters α and β becomes central, since they determine the expenditure shares and, consequently, the sensitivity of each good to changes in relative prices. By comparing the two decompositions under the same functional framework, the methodology enables a direct evaluation of how the Hicks and Slutsky effects differ in magnitude and relative importance, depending on both the price ratio and the distribution of preferences.

This methodology ensures a rigorous comparison between the two decompositions, while maintaining analytical clarity through the use of a standard utility function widely applied in microeconomic theory.

5. Results and discussions.

5.1 Hicks and Slutsky effects

In the previous section we studied the case of Marshall and Hicks demand.

Naturally, the question of the evolution of demand arises when the prices of goods vary. In principle, whether it is uncompensated or compensated demand, the respective system of conditions is solved again, obtaining the new solution. The interesting aspect, however, is that of the transition from the old vector of purchased goods to the new one, given that this process is, as a rule, one that takes place over time and therefore has a series of implications and transitory effects.

5.1.1. Hicks effect

Let the new prices of goods $B_1,...,B_n$ be: $p'_1,...,p'_n$.

We will consider in this analysis that, to begin with, the consumer will change his demand so as to preserve his original level of utility. The compensated demand will therefore satisfy the problem:

$$\begin{cases} min \sum_{i=1}^{n} p'_i x_i \\ U(x_1,\dots,x_n) = \overline{u} \\ x_1,\dots,x_n \in SC \end{cases}$$

where $\overline{u}$ is the initial level of utility.

Let the solution:

$$\begin{cases} \tilde{x}_1 = f_1(p'_1,\dots,p'_n,\overline{u}) \\ \dots \\ \tilde{x}_n = f_n(p'_1,\dots,p'_n,\overline{u}) \end{cases}$$

$V'=\sum_{i=1}^{n} p'_i\tilde{x}_i$ – the income necessary to purchase the respective basket of goods, and $U(\tilde{x}_1,...,\tilde{x}_n)=\overline{u}$.

We will call the transition from the initial basket of goods $(x_1,...,x_n)$ to $(\tilde{x}_1,...,\tilde{x}_n)$ the **Hicks substitution effect** (abbreviated Hs) and we have: $\Delta_{Hs,i}=\tilde{x}_i-x_i$, $i=\overline{1,n}$.

The second phase derives from the fact that if $V'\neq V$ (V – the initial income), the consumer will again modify his demand vector (corresponding to his real income), proportional to the previous one that preserved his utility. In this case, the problem of uncompensated demand arises, namely:

$$\begin{cases} max\ U(x_1,\dots,x_n) \\ \sum_{i=1}^{n} p'_i x_i = V \\ x_1,\dots,x_n \in SC \end{cases}$$

with the solution:

$$\begin{cases} \tilde{\tilde{x}}_1 = g_1(p'_1,\dots,p'_n,V) \\ \dots \\ \tilde{\tilde{x}}_n = g_n(p'_1,\dots,p'_n,V) \end{cases}$$

In this case, we have: V=$\sum_{i=1}^{n} p'_i\tilde{\tilde{x}}_i$, U=U($\tilde{\tilde{x}}_1$,...,$\tilde{\tilde{x}}_n$) – the utility obtained. We will call the transition from the intermediate basket of goods ($\tilde{x}_1$,...,$\tilde{x}_n$) to ($\tilde{\tilde{x}}_1$,...,$\tilde{\tilde{x}}_n$) the **Hicks income effect** (abbreviated Hv) and we have: $\Delta_{Hv,i}$=$\tilde{\tilde{x}}_i$-$\tilde{x}_i$, i=$\overline{1,n}$.

The total effect of these two stages is:$\Delta_{H,i}$=$\Delta_{Hs,i}$+$\Delta_{Hv,i}$=$\tilde{x}_i$-x_i+$\tilde{\tilde{x}}_i$-$\tilde{x}_i$=$\tilde{\tilde{x}}_i$-x_i, i=$\overline{1,n}$.

From a geometric point of view, the Hicks effect consists in a first displacement of the income hyperplane determined by the new prices so that it remains tangent to the utility hypersurface U=$\overline{u}$, its intersections with the coordinate axes determining, relative to the old intersections, the effect of Hicks substitution for each individual product. The second stage consists in its parallel displacement until the initial income is reached and the determination of the utility hypersurface tangent to it. The new shift of the intersection points will determine the Hicks income effect for each product.

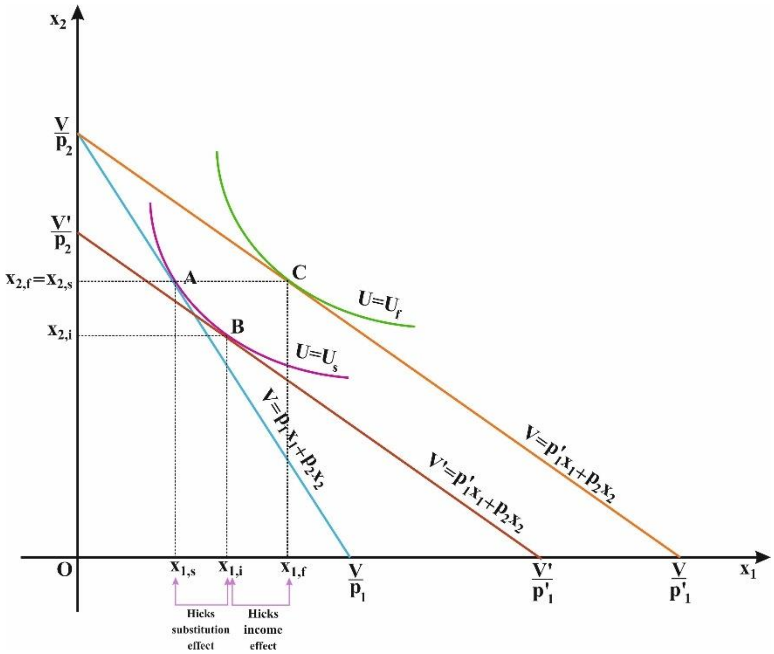
In particular, for two goods B_1 and B_2 whose initial prices are p_1 and p_2, respectively, and the consumer's income is V, we will assume that the good x_1 undergoes a price reduction from p_1 to p'_1<p_1.

The straight line of income, relative to initial prices, is: V=p_1x_1+p_2x_2. Considering the utility function U=U(x_1,x_2), the income line becomes tangent to one of the isoutility curves (U=constant) at point A($x_{1,s}$,$x_{2,s}$) and the utility will be: U=U($x_{1,s}$,$x_{2,s}$)=U_s (we noted with the index s from start).

As a result of the change in the price of good B_1, in the first phase, the consumer will change his consumption vector so that he keeps the same level of maximum utility as he had before the change. Therefore, the new revenue line: V'=p'_1x_1+p_2x_2 (where V' is currently unknown) will move parallel to itself until it becomes tangent to the isoutility curve U=U_s at point B($x_{1,i}$,$x_{2,i}$) and the quantities consumed will be: $x_{1,i}$>$x_{1,s}$ (naturally, as a result of the decrease in the price of B_1) and $x_{2,i}$<$x_{2,s}$ (the consumer moving to the good B_1) (we noted with the index i from intermediate). It is observed that, since the new income line cuts the Ox_2 (*figure 1*) axis at a point closer to the origin, it follows that V'<V. The difference $x_{1,i}$-$x_{1,s}$ is precisely the Hicks substitution effect.

The second stage consists in reallocating the additional income difference (V-V') to a new consumption vector. In this case, the consumer increases his maximum utility to U=U_f (we noted with the index f from final) obtaining a new consumption: $x_{1,f}$>$x_{1,i}$ (as a result of the additional allocated income), respectively $x_{2,f}$=$x_{2,s}$. The difference $x_{1,f}$-$x_{1,i}$ is the Hicks income effect.

Figure 1. Hicks effect



5.1.2. Slutsky effect

Let, again, the new prices of goods B_1,...,B_n be: p'_1,...,p'_n. We will consider, this time, the situation in which, first, the consumer will conserve his purchasing power. The income necessary for this fact becomes: V'= $\sum_{i=1}^{n} p'_i x_i$, the income hyperplane rotating around the initial point (x_1,...,x_n) which will determine a new maximal utility, determined by the tangent of the utility hypersurface to it.

So there is a problem of uncompensated demand, the consumer wanting to maximize his utility under the conditions of the newcomer. So we have:

$$\begin{cases} max \ \ U(x_1, \dots, x_n) \\ \sum_{i=1}^{n} p'_i x_i = V' \\ x_1, \dots, x_n \in SC \end{cases}$$

with solution:

$$\begin{cases} \bar{x}_1 = f_1(p'_1, \dots, p'_n, V') \\ \dots \\ \bar{x}_n = f_n(p'_1, \dots, p'_n, V') \end{cases}$$

the utility being: U=U($\bar{x}_1$,...,$\bar{x}_n$).

We will call the transition from the initial basket of goods (x_1,...,x_n) to ($\bar{x}_1$,...,$\bar{x}_n$) **Slutsky-type substitution effect** (abbreviated Ss) and we have: $\Delta_{Ss,i}=\bar{x}_i - x_i$, $i=\overline{1,n}$.

In the second stage, due to the fact that V'≠V (V – the initial income), the consumer will again modify his demand vector (corresponding to his real income), proportional to the previous one that had maximized his utility. The new problem that arises is again uncompensated demand:

$$\begin{cases} max \;\; U(x_1,\dots,x_n) \\ \sum_{i=1}^{n} p'_i x_i = V \\ x_1,\dots,x_n \in SC \end{cases}$$

with solution:

$$\begin{cases} \bar{\bar{x}}_1 = g_1(p'_1,\dots,p'_n,V) \\ \dots \\ \bar{\bar{x}}_n = g_n(p'_1,\dots,p'_n,V) \end{cases}$$

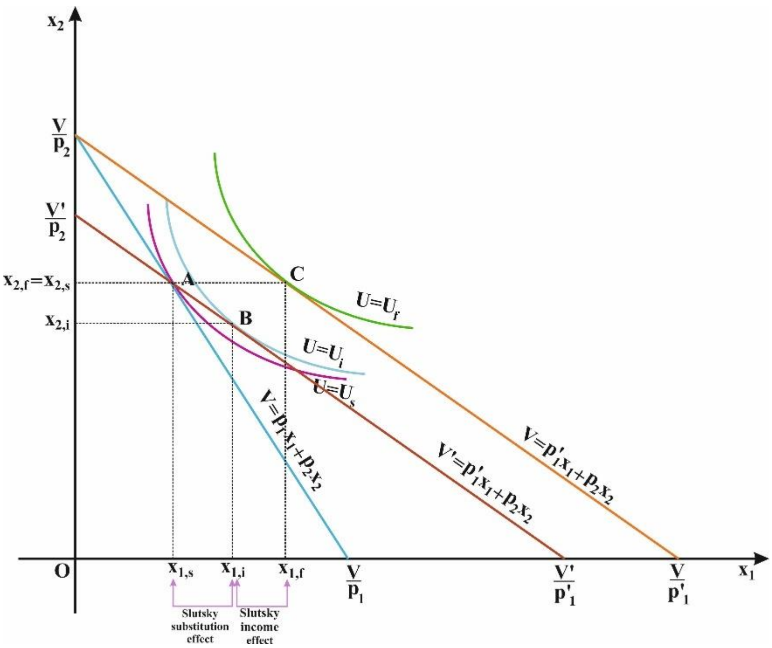
In this case, we have: $V=\sum_{i=1}^{n} p'_i \bar{\bar{x}}_i$, $U=U(\bar{\bar{x}}_1,\dots,\bar{\bar{x}}_n)$ – the utility obtained. We will call the transition from the intermediate basket of goods $(\bar{x}_1,\dots,\bar{x}_n)$ to $(\bar{\bar{x}}_1,\dots,\bar{\bar{x}}_n)$ the **Slutsky income effect** (abbreviated Sv) and we have: $\Delta_{Sv,i}=\bar{\bar{x}}_i-\bar{x}_i$, $i=\overline{1,n}$. Geometrically, the income hyperplane: $\sum_{i=1}^{n} p'_i x_i = V'$ is translated parallel to the old income V, then the tangent utility hypersurface is determined. The total effect of these two stages is: $\Delta_{S,i}=\Delta_{Ss,i}+\Delta_{Sv,i}=\bar{x}_i-x_i+\bar{\bar{x}}_i-\bar{x}_i=\bar{\bar{x}}_i-x_i$, $i=\overline{1,n}$.

In particular, for two goods B_1 and B_2 whose initial prices are p_1 and p_2, respectively, and the consumer's income is V, we will assume that the good B_1 undergoes a price reduction from p_1 to $p'_1<p_1$. In this case, the sequential transition from initial to final consumption is carried out in several stages. The line of income, relative to initial prices, is: $V=p_1x_1+p_2x_2$. Considering the utility function $U=U(x_1,x_2)$, the income line becomes tangent to one of the isoutility curves (U=constant) at the point $A(x_{1,s},x_{2,s})$, and the utility will be: $U=U(x_{1,s},x_{2,s})=U_s$ (we denoted with the index s from start).

As a result of the change in the price of good B_1, in the first phase, the consumer will keep his initial purchasing power, so he will opt for the same consumption vector $(x_{1,s},x_{2,s})$. In this case, however, the income line, having a lower slope (as a result of the decrease in price p_1) will no longer be tangent to the isoutility curve $U=U_s$. Therefore, the consumer will modify his consumption vector, to obtain maximum utility, moving to point $B(x_{1,i},x_{2,i})$, and the quantities consumed will be: $x_{1,i}>x_{1,s}$ (naturally, as a result of the decrease in the price of B_1) and $x_{2,i}<x_{2,s}$ (the consumer moving to the good B_1) (we noted with the index i from intermediate). It is observed that since the new income line cuts the Ox_2 (*figure 2*) axis closer to the origin, it follows that V'<V. The difference $x_{1,i}-x_{1,s}$ is the Slutsky substitution effect. The second stage consists in reallocating the additional income difference (V-V') to a new consumption vector. In this case, the consumer increases his maximum utility to $U=U_f$ (we noted with the index f from final) obtaining a new consumption: $x_{1,f}>x_{1,i}$

(as a result of the additional allocated income), respectively $x_{2,f}=x_{2,s}$. The difference $x_{1,f}-x_{1,i}$ is the Slutsky income effect.

Figure 2. Slutsky effect



5.2. Substitution and Income Effect Analysis for a Cobb-Douglas Utility Function

Let there be two goods A and B whose initial prices are p_A and p_B, the utility function being of the Cobb-Douglas type $U=TX^{\alpha}Y^{\beta}$, $\alpha,\beta>0$, where X and Y are the quantities of product A and B, respectively. Also let V be the consumer's income.

The utility maximization condition for the allocated income V is:

$$\begin{cases} \frac{U_{m,A}}{U_{m,B}} = \frac{p_A}{p_B} \\ V = p_A X + p_B Y \end{cases}$$

where $U_{m,A}=\alpha TX^{\alpha-1}Y^{\beta}$ and $U_{m,B}=\beta TX^{\alpha}Y^{\beta-1}$ are the marginal utilities corresponding to the two goods.

The solution of the system of equations is:

$$\begin{cases} X_1 = \frac{\alpha}{(\alpha+\beta)p_A} V \\ Y_1 = \frac{\beta}{(\alpha+\beta)p_B} V \end{cases}$$

and the corresponding utility is:

$$U_1 = TV^{\alpha+\beta} \frac{\alpha^{\alpha}\beta^{\beta}}{(\alpha+\beta)^{\alpha+\beta} p_A^{\alpha} p_B^{\beta}}$$

Now suppose that the price of good B changes from p_B to p'_B with income remaining constant. From the above relations, we obtain:

$$\begin{cases} X_3 = \frac{\alpha}{(\alpha+\beta)p_A} V \\ Y_3 = \frac{\beta}{(\alpha+\beta)p'_B} V \end{cases}$$

and the corresponding utility: $U_3 = TV^{\alpha+\beta} \frac{\alpha^{\alpha}\beta^{\beta}}{(\alpha+\beta)^{\alpha+\beta} p_A^{\alpha} p'^{\beta}_B}$.

Let us first study the Hicks effect.

On changing the price of B, for the same utility $U_1 = TV^{\alpha+\beta} \frac{\alpha^{\alpha}\beta^{\beta}}{(\alpha+\beta)^{\alpha+\beta} p_A^{\alpha} p_B^{\beta}}$ we will have: $U_1 = TV'^{\alpha+\beta} \frac{\alpha^{\alpha}\beta^{\beta}}{(\alpha+\beta)^{\alpha+\beta} p_A^{\alpha} p'^{\beta}_B}$ from where:

$$TV^{\alpha+\beta} \frac{\alpha^{\alpha}\beta^{\beta}}{(\alpha+\beta)^{\alpha+\beta} p_A^{\alpha} p_B^{\beta}} = TV'^{\alpha+\beta} \frac{\alpha^{\alpha}\beta^{\beta}}{(\alpha+\beta)^{\alpha+\beta} p_A^{\alpha} p'^{\beta}_B}$$

therefore:

$$V'^{\alpha+\beta} = V^{\alpha+\beta} \frac{p'^{\beta}_B}{p_B^{\beta}}$$

or, much simpler:

$$V' = V \left(\frac{p'_B}{p_B}\right)^{\frac{\beta}{\alpha+\beta}}$$

With the new income:

$$\begin{cases} X_{2H} = \frac{\alpha}{(\alpha+\beta)p_A} V' \\ Y_{2H} = \frac{\beta}{(\alpha+\beta)p'_B} V' \end{cases}$$

or, in terms of prices:

$$\begin{cases} X_{2H} = \frac{\alpha}{(\alpha+\beta)p_A} \left(\frac{p'_B}{p_B}\right)^{\frac{\beta}{\alpha+\beta}} V \\ Y_{2H} = \frac{\beta}{(\alpha+\beta)p_B} \left(\frac{p'_B}{p_B}\right)^{-\frac{\alpha}{\alpha+\beta}} V \end{cases}$$

The Hicks substitution effect is therefore:

$$\Delta_{1H}X = X_{2H} - X_1 = \frac{\alpha}{(\alpha+\beta)p_A}\left(\left(\frac{p'_B}{p_B}\right)^{\frac{\beta}{\alpha+\beta}} - 1\right) V$$

$$\Delta_{1H}Y = Y_{2H} - Y_1 = \frac{\beta}{(\alpha+\beta)p_B}\left(\left(\frac{p'_B}{p_B}\right)^{-\frac{\alpha}{\alpha+\beta}} - 1\right) V$$

Considering now the initial income V instead of V' we get:

$$\Delta_{2H}X = X_3 - X_{2H} = \frac{\alpha}{(\alpha+\beta)p_A}\left(1 - \left(\frac{p'_B}{p_B}\right)^{\frac{\beta}{\alpha+\beta}}\right) V$$

$$\Delta_{2H}Y = Y_3 - Y_{2H} = \frac{\beta}{(\alpha+\beta)p'_B}\left(1 - \left(\frac{p'_B}{p_B}\right)^{\frac{\beta}{\alpha+\beta}}\right) V$$

which represents the Hicks income effect.

We will now apply Slutsky's method to the analysis of the two effects. When the price of B changes, the revenue corresponding to the same optimal combination is:

$$V' = p_A X_1 + p'_B Y_1 = p_A \frac{\alpha}{(\alpha+\beta)p_A} V + p'_B \frac{\beta}{(\alpha+\beta)p_B} V = \frac{V}{\alpha+\beta}\left(\alpha + \beta \frac{p'_B}{p_B}\right)$$

so:

$$\begin{cases} X_{2S} = \frac{\alpha}{(\alpha+\beta)^2 p_A}\left(\alpha + \beta \frac{p'_B}{p_B}\right) V \\ Y_{2S} = \frac{\beta}{(\alpha+\beta)^2 p'_B}\left(\alpha + \beta \frac{p'_B}{p_B}\right) V \end{cases}$$

and the corresponding utility: $U_2 = T X_{2S}^{\alpha} Y_{2S}^{\beta} = T \frac{\alpha^{\alpha}\beta^{\beta}}{(\alpha+\beta)^{2(\alpha+\beta)} p_A^{\alpha} {p'}_B^{\beta}} V^{\alpha+\beta} \left(\alpha + \beta \frac{p'_B}{p_B}\right)^{\alpha+\beta}$.

The Slutsky substitution effect (which does not preserve utility) is therefore:

$$\Delta_{1S}X=X_{2S}-X_1=\frac{\alpha\beta\left(\frac{p'_B}{p_B}-1\right)}{(\alpha+\beta)^2 p_A}V$$

$$\Delta_{1S}Y=Y_{2S}-Y_1=\frac{\alpha\beta\left(1-\frac{p'_B}{p_B}\right)}{(\alpha+\beta)^2 p'_B}V$$

and the Slutsky income effect:

$$\Delta_{2S}X=X_3-X_{2S}=\frac{\alpha\beta\left(1-\frac{p'_B}{p_B}\right)}{(\alpha+\beta)^2 p_A}V$$

$$\Delta_{2S}Y=Y_3-Y_{2S}=\frac{\beta^2\left(1-\frac{p'_B}{p_B}\right)}{(\alpha+\beta)^2 p'_B}V$$

We define, in the following, the ratios:

- $\alpha_Y=\frac{Y_2-Y_1}{Y_3-Y_1}$ - the share of the total change in consumption due to the substitution effect;
- $\beta_Y=\frac{Y_3-Y_2}{Y_3-Y_1}$ - the share of the total change in consumption due to the income effect;
- $r_Y=\frac{\beta_Y}{\alpha_Y}=\frac{Y_3-Y_2}{Y_2-Y_1}$ - the ratio between the income effect and the substitution effect.

We have: $\alpha_Y+\beta_Y=1$ and $r_Y=\frac{1}{\alpha_Y}-1=\frac{1}{\frac{1}{\beta_Y}-1}$.

In the case of the Hicks effect:

- $\alpha_{YH}=\frac{\Delta_{1H}Y}{\Delta_{1H}Y+\Delta_{2H}Y}=\frac{\left(\frac{p'_B}{p_B}\right)^{\frac{\beta}{\alpha+\beta}}-\frac{p'_B}{p_B}}{1-\frac{p'_B}{p_B}}$
- $\beta_{YH}=\frac{\Delta_{2H}Y}{\Delta_{1H}Y+\Delta_{2H}Y}=\frac{1-\left(\frac{p'_B}{p_B}\right)^{\frac{\beta}{\alpha+\beta}}}{1-\frac{p'_B}{p_B}}$
- $r_{YH}=\frac{\beta_{YH}}{\alpha_{YH}}=\frac{1-\left(\frac{p'_B}{p_B}\right)^{\frac{\beta}{\alpha+\beta}}}{\left(\frac{p'_B}{p_B}\right)^{\frac{\beta}{\alpha+\beta}}-\frac{p'_B}{p_B}}$

In the case of the Slutsky effect:

- $\alpha_{YS}=\frac{\Delta_{1S}Y}{\Delta_{1S}Y+\Delta_{2S}Y}=\frac{\alpha}{\alpha+\beta}$
- $\beta_{YS}=\frac{\Delta_{2S}Y}{\Delta_{1S}Y+\Delta_{2S}Y}=\frac{\beta}{\alpha+\beta}$
- $r_{YS}=\frac{\beta_{YS}}{\alpha_{YS}}=\frac{\beta}{\alpha}$

Let us now write $x=\frac{p'_B}{p_B}$ – the ratio of the new to the old price of B, assuming for non-triviality that $x\neq 1$.

In the case of Hicks, we have: $\alpha_{YH}=\frac{x^{\frac{\beta}{\alpha+\beta}}-x}{1-x}$, $\beta_{YH}=\frac{1-x^{\frac{\beta}{\alpha+\beta}}}{1-x}$, $r_{YH}=\frac{1-x^{\frac{\beta}{\alpha+\beta}}}{x^{\frac{\beta}{\alpha+\beta}}-x}$.

Let's consider the function f:$(0,1)\cup(1,\infty)\to\mathbf{R}$, f(x)=$\frac{x^{\frac{\beta}{\alpha+\beta}}-x}{1-x}$.

By calculating its derivative:

$$f'(x)=\frac{1}{(1-x)^2x^{\frac{\alpha}{\alpha+\beta}}}\left(\frac{\beta}{\alpha+\beta}+\frac{\alpha}{\alpha+\beta}x-x^{\frac{\alpha}{\alpha+\beta}}\right)$$

On the other hand, the function g(x)= $\frac{\beta}{\alpha+\beta}+\frac{\alpha}{\alpha+\beta}x-x^{\frac{\alpha}{\alpha+\beta}}$ has $g'(x)$ $=\frac{\alpha}{\alpha+\beta}\left(1-x^{-\frac{\beta}{\alpha+\beta}}\right)$ so for $x\in(0,1)$ g decreases and for $x\in(1,\infty)$ g will increase. Because g(1)=0 follows that g(x)>0 $\forall x\in(0,1)\cup(1,\infty)$.

From the expression of f' we obtain that f'(x)>0 which means that f is a strictly increasing function.

On the other hand, because $\lim_{x\to 0} f(x)=0$, $\lim_{x\to 1} f(x)=\frac{\alpha}{\alpha+\beta}$, $\lim_{x\to\infty} f(x)=1$ we have that for $x\in(0,1)$: $\alpha_{YH}\in\left(0,\frac{\alpha}{\alpha+\beta}\right)$, $x\in(1,\infty)$: $\alpha_{YH}\in\left(\frac{\alpha}{\alpha+\beta},1\right)$. Reversing the relations, we obtain for $x\in(0,1)$: $\frac{1}{\alpha_{YH}}\in\left(\frac{\alpha+\beta}{\alpha},\infty\right)$, $x\in(1,\infty)$: $\frac{1}{\alpha_{YH}}\in\left(1,\frac{\alpha+\beta}{\alpha}\right)$.

Also, because $\beta_{YH}=1-\alpha_{YH}$, we have for $x\in(0,1)$: $\beta_{YH}\in\left(\frac{\beta}{\alpha+\beta},1\right)$ and for $x\in(1,\infty)$: $\beta_{YH}\in\left(0,\frac{\beta}{\alpha+\beta}\right)$.

Finally, for $x\in(0,1)$: $r_{YH}\in\left(\frac{\beta}{\alpha},\infty\right)$ and for $x\in(1,\infty)$: $r_{YH}\in\left(0,\frac{\beta}{\alpha}\right)$.

Also, equality $r_{YH}=1$ is equivalent to: $\alpha_{YH}=\frac{1}{2}$ so: $\frac{x^{\frac{\beta}{\alpha+\beta}}-x}{1-x}=\frac{1}{2}$ or: $2x^{\frac{\beta}{\alpha+\beta}}-x-1=0$.

Let now the function h:$(0,1)\cup(1,\infty)\to\mathbf{R}$, h(x)=$2x^{\frac{\beta}{\alpha+\beta}}-x-1$.

We have: $h'(x)=\frac{2\beta}{\alpha+\beta}x^{-\frac{\alpha}{\alpha+\beta}}-1$ so the stationary point of h (root of derivative h') is $x_d=\left(\frac{2\beta}{\alpha+\beta}\right)^{\frac{\alpha+\beta}{\alpha}}$. The fact that $\lim_{x\to 0} h'(x)=\infty$, $\lim_{x\to 1} h'(x)=\frac{\beta-\alpha}{\alpha+\beta}$ and $\lim_{x\to\infty} h'(x)=-1$ involves the following cases:

- if $\alpha<\beta \Rightarrow x_d>1$, $h'(x)>0$ for $x\in(0,1)$, $x\in(1,x_d)\Rightarrow h'(x)>0$ and $x\in(x_d,\infty)\Rightarrow h'(x)<0$;
- if $\alpha>\beta \Rightarrow x_d<1$, $h'(x)>0$ for $x\in(0,x_d)$, $x\in(x_d,1)\Rightarrow h'(x)<0$ and $x\in(1,\infty)\Rightarrow h'(x)<0$.

Because $\lim_{x\to 0} h(x)=-1$, $\lim_{x\to 1} h(x)=0$, $\lim_{x\to\infty} h(x)=-\infty$ we obtain that:

- if $\alpha<\beta$ then the only real root of h is in the interval (x_d,∞);
- if $\alpha>\beta$ then the only real root of h is in the interval $(0,x_d)$.

We have now $h''(x) = -\frac{2\alpha\beta}{(\alpha+\beta)^2} x^{-\frac{2\alpha+\beta}{\alpha+\beta}} < 0$ therefore h is concave.

To determine the real root $\overline{x}$ of h, we will apply Newton's method of approximation for functions of a real variable. Because the starting point x_0 of the method for the function h:[a,b]→**R**, which preserves its monotony and concavity, is the one for which $h(x_0)h''(x_0)>0$ and in the present case $h''(x_0)<0$ for any x_0, we will choose x_0 such that $h(x_0)<0$. According to Newton's method, we have:

$$x_{n+1} = x_n - \frac{h(x_n)}{h'(x_n)} = \frac{1 - \frac{2\alpha}{\alpha+\beta} x_n^{\frac{\beta}{\alpha+\beta}}}{\frac{2\beta}{\alpha+\beta} x_n^{-\frac{\alpha}{\alpha+\beta}} - 1},\ n \geq 0$$

where for α<β: x_0 is big enough and for α>β: x_0 is small enough.

Therefore, since r_{YH} is a decreasing function, we have that:

- for α<β: if $x \in (0,1) \cup (1,\overline{x})$ then $r_{YH}>1$ and if $x \in (\overline{x},\infty)$ then $r_{YH}<1$;
- for α>β: if $x \in (0,\overline{x})$ then $r_{YH}>1$ and if $x \in (\overline{x},1) \cup (1,\infty)$ then $r_{YH}<1$.

In conclusion, for $x=\frac{p'_B}{p_B}<1$ we have that $\alpha_{YH}<\alpha_{YS}$ so the part of the total change in demand for good B due to the substitution effect is greater in the case of the Slutsky effect than in the case of Hicks and, of course, $\beta_{YH}>\beta_{YS}$ represents the fact that the portion of the total change in demand for good B due to the income effect is smaller under the Slutsky effect than under the Hicks effect. Since $r_{YH}>r_{YS}$ we have that the ratio between the income effect and the substitution effect is higher in Hicks' case than in Slutsky's.

For $x=\frac{p'_B}{p_B}>1$ we have that $\alpha_{YH}>\alpha_{YS}$ so the part of the total change in demand for good B due to the substitution effect is smaller in the case of the Slutsky effect than in the case of Hicks and $\beta_{YH}>\beta_{YS}$ implies that the part of the total change of demand for good B due to the income effect is greater under the Slutsky effect than under the Hicks effect. Because $r_{YH}<r_{YS}$ it follows that the ratio between the income effect and the substitution effect is smaller in Hicks' case than in Slutsky's.

Also:

- for α<β: if $x=\frac{p'_B}{p_B}>\overline{x}>1$ then the income effect is lower than the substitution effect and vice versa for the other case;
- for α>β: if $x=\frac{p'_B}{p_B}<\overline{x}<1$ then the substitution effect is smaller than the income effect, the conclusion being reversed for the other inequality.

6. Conclusions

This paper has examined the decomposition of demand changes into substitution and income effects under the Hicksian and Slutsky frameworks. Starting from the theoretical foundations of Marshallian and Hicksian demand, the analysis applied both decompositions to the Cobb–Douglas utility function, which provided a tractable setting for explicit derivations.

The results demonstrate that the relative importance of substitution and income effects depends not only on the direction of the price change but also on the structure of consumer preferences. When the price of a good decreases, the Hicksian decomposition assigns a larger share to the income effect, whereas the Slutsky decomposition emphasizes the substitution effect. In contrast, when the price increases, Hicks highlights substitution, while Slutsky attributes greater weight to the income effect. The preference parameters α\alphaα and β\betaβ further shape these outcomes by determining the expenditure shares and the sensitivity of each good to relative price changes.

The novelty of this study lies in providing explicit analytical expressions and a comparative assessment of the two decompositions within the same functional framework. By deriving clear conditions under which one effect dominates the other, the paper contributes to a more nuanced understanding of consumer behavior under price changes.

Beyond the theoretical contribution, the findings have implications for applied economics. The choice between Hicksian and Slutsky decompositions may affect welfare analysis, policy evaluation, and demand forecasting, especially in contexts where the direction and magnitude of price changes play a crucial role. Future research could extend this approach to more general utility specifications or empirical estimations, providing further insights into the robustness of the comparative results.

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