NONLINEAR DISCRETE MODEL FOR THE CONTROL OF A SEWER NETWORK

Viorel MINZU and Iulian ARAMA

Department of Automation and Electrical Engineering,
"Dunarea de Jos" University of Galati, Romania,

Abstract: This paper proposes a nonlinear discrete model for a sewer network aiming to minimize the total overflow volume generated by the influent collected in its area, in the horizon of the control problem. The sewer network is modeled as a nonlinear discrete dynamic system. The control objective leads to an optimization problem with constraints on control and state variables.

Keywords: nonlinear discrete dynamic systems, optimal control; sewer network.

1. INTRODUCTION

The large cities in the world have a growing development that led to the production of significant quantities of wastewater which are discharged into rivers, seas, lakes, etc. If appropriate actions are not taken, the quality of these natural receptors is strongly affected. These actions concern mainly the building of sufficient number of wastewater treatment plants and the construction of sewer networks allowing the passage of a larger percentage of wastewater through a treatment plant. These issues have been addressed in many publications in the filed (Rojas, *et all*, 2012; O'Brien, *et all*, 2011; Aulinas, *et all*, 2010.).

It is necessary to construct more sewer networks allowing the passage of a larger quantity of wastewater through a treatment plant, before being discharged into natural receivers. But this imperative has a big impediment: the limited areas that can be used for this purpose in the large cities. The main problem is related to the conjoint effects of the rainfall waters with wastewater discharged by human and industrial consumers. A realistic solution is to have an optimal automatic control that allows an efficient usage of the existing sewer networks, without the need to build new retention facilities. There are two categories of control techniques that were used for this purpose. The first category contains the intelligent control techniques: fuzzy logic control (Carp, *et all*, 2013; Carp, 2014), expert systems (Aulinas, 2010) or even multi-agent systems (Murillo, *et all*, 2011; Minzu, *et all*, 2014.). The second one contains optimal control techniques: discrete dynamic programming (Carp, 2014), nonlinear optimal control (Marinaki,and Papageorgiou, 2005), model predictive control for hybrid systems (Ocampo-Martinez, 2010) or neural-optimal control algorithm (Dorsano and Labadie, 2007).

In section II a model of the Sewer Network (SN) is proposed. The SN is regarded as a nonlinear discrete dynamic system. The wastewater volumes contained in the tanks represent the state variables and the wastewater volumes evacuated by the water columns are regarded as control inputs. The rainfall water

This paper was recommended for publication by Adrian Filipescu

together with wastewater discharged by human and industrial consumers is called in this paper *influent*. This one is an exogenous input that generates the disturbance variables. In section III, the control objective is described by an optimization problem whose objective function is the total overflow volume. The discharge control of the SN means to minimize the total overflow volume on the control horizon. However the optimization problem is solved, in Section 4 is presented a way of using an optimal solution to the implementation of a realistic optimal controller.

2. SEWER NETWORK MODELLING

In order to obtain a general sewer network model, we examine firstly a particular structure of a SN illustrated in Fig.1. Here a triangular element represents a retention tank for the wastewater and a circular element is a collector for all the flow capacities that are inputs of the tanks. The influent that affects the SN is represented by the flow capacities $d_i(t)$, i=1,…,5, from the five catchment areas. The discharge of the tank i caused by its evacuation pump and it is denoted by $u_i(t)$, i=1,…,5. A characteristic of the SN is the fact that any tank has only one downstream tank. When the tanks reach their maximum capacity V_i, i=1,…,5, expressed by volume units, there are overflows denoted by $q^{over}{}_i(t)$.

Let $q_i(t)$ be the total flow capacity of the wastewater entering in the retention tank i. For our example, we have

$$(1)\quad \begin{array}{l} q_1(t) = d_1(t);\ q_2(t) = d_2(t) + u_1(t);\ q_3(t) = d_3(t) \\ q_4(t) = d_4(t);\ q_5(t) = d_5(t) + u_2(t) + u_3(t) + u_4(t) \end{array}$$

The next equation expresses the growth of the wastewater volume in a time interval dt:

$$(2)\quad dV_i(t) = [q_i(t) - u_i(t)] \cdot dt$$

If T is the sample time, the integration of equation (2) using the rectangle's method leads to the following equation:

$$(3)\quad V_i(t+T) - V_i(t) = [q_i(t) - u_i(t)] \cdot T\ ,\ t \in \boldsymbol{R}$$

One can consider the water volume of the tank i as a state variable associated with this one.

$$(4)\quad x_i(t) = V_i(t), \quad i = 1,...,5$$

For our example, the state equations are:

$$(5)\quad \begin{cases} x_1(t+T) = x_1(t) + [q_1(t) - u_1(t)] \cdot T \\ x_2(t+T) = x_2(t) + [q_2(t) - u_2(t)] \cdot T \\ x_3(t+T) = x_3(t) + [q_3(t) - u_3(t)] \cdot T \\ x_4(t+T) = x_4(t) + [q_4(t) - u_4(t)] \cdot T \\ x_5(t+T) = x_5(t) + [q_5(t) - u_5(t)] \cdot T \end{cases}$$

Here after, a discrete model of the SN is presented ([1]). Let's keep the same notation, t, for the discrete time $t \in I$, t= 0,…, N, with the sampling period T. Hence, the horizon of the control problem is $N \cdot T$. For the sake of simplicity, the model uses some hypothesis. The first hypothesis stats all the water columns have the same evacuation power. This may be expressed by the wastewater volume evacuated in a sampling period T. Let ΔV be this volume. Because the balance of the wastewater is done for every sampling period, the model has a simpler form if it uses the next new defined variables:

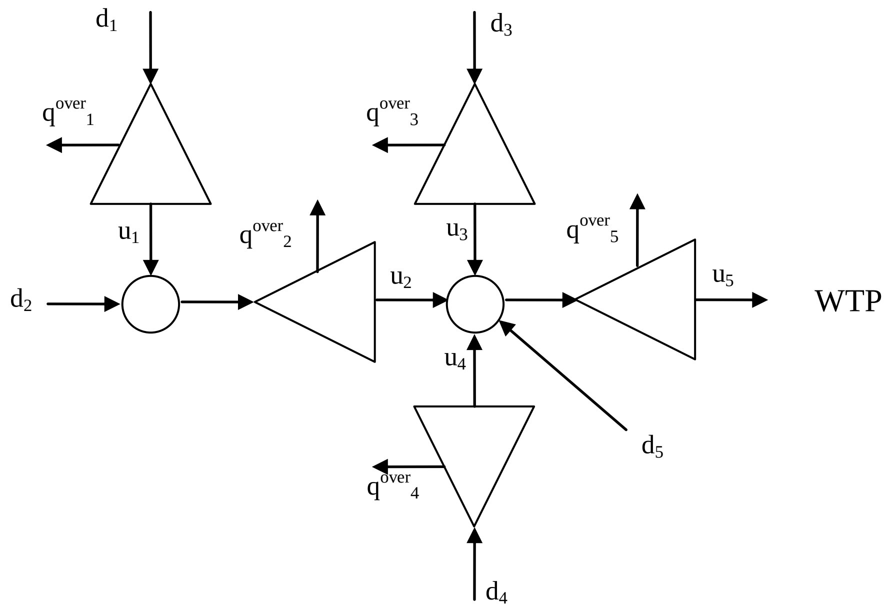

Fig.1. – Sewer network structure

$$(6)\quad D_i(t) \overset{D}{=} d_i(t)\cdot T/\Delta V \qquad U_i(t) \overset{D}{=} u_i(t)\cdot T/\Delta V$$
$$Q_i(t) \overset{D}{=} q_i(t)\cdot T/\Delta V \qquad M_i \overset{D}{=} V_i/\Delta V$$

The time horizon for the evolution of our sewer network has N sampling periods:

$$(7)\ t=0,\dots,N$$

The second hypothesis stats that $D_i(t)$, $U_i(t)$, $Q_i(t)$ and M_i, i=1,…,5, have integer values representing volumes that are multiples of ΔV.

The state variables represent the remaining wastewater volumes in the tanks of the SN. For example, in the tank i, at the moment t, it remains a wastewater volume of (x_i(t)·ΔV) m^3:

$$(8)\quad x(t) = [x_1(t) \quad x_2(t) \quad \cdots \quad x_5(t)]^T,$$

$$(9)\quad x_i(t) \in \{0, 1, \dots, M_i\},\ i = 1,\dots,5$$

From the equations (5) and using the notations (6), we deduce the new equations for the discrete time model:

$$(10)\quad \begin{cases} x_1(t+1) = x_1(t) + Q_1(t) - U_1(t) \\ x_2(t+1) = x_2(t) + Q_2(t) - U_2(t) \\ x_3(t+1) = x_3(t) + Q_3(t) - U_3(t) \\ x_4(t+1) = x_4(t) + Q_4(t) - U_4(t) \\ x_5(t+1) = x_5(t) + Q_5(t) - U_5(t) \end{cases}$$

where the input flows are:

$$(11)\quad \begin{cases} Q_1(t) = D_1(t) \\ Q_2(t) = D_2(t) + U_1(t) \\ Q_3(t) = D_3(t) \\ Q_4(t) = D_4(t) \\ Q_5(t) = D_5(t) + U_2(t) + U_3(t) + U_4(t) \end{cases}$$

Aiming to find the state equation of the SN and to express the wastewater overflow let's take the example of tank 2. For instance, one can strike the balance of the wastewater for the second tank at the moment t+1:

$$(12)\quad x_2(t+1) = x_2(t) + D_2(t) + U_1(t) - U_2(t).$$

But the wastewater volume in the tank is less or equals the tank volume M_2. So, the correct value is

$$(13)\quad x_2(t+1) = \min(x_2(t) + D_2(t) + U_1(t) - U_2(t), M_2)$$
.

If the value $x_2(t) + D_2(t) + U_1(t) - U_2(t) - M_2$ is positive, then it represents the overflow volume. Hence, for this sampling period and for the second tank the overflow volume can be expressed by the following value

$$(14)\quad \max(0,\ x_2(t) + D_2(t) + U_1(t) - U_2(t) - M_2).$$

If we want to memorize the total overflow volume corresponding to each tank, we introduce 5 new state variables, x_i, i= 6, …, 10. For our example, the additional state equation is:

$$(15)\quad \begin{aligned} & x_7(t+1) = x_7(t) + \\ & + \max(0, x_2(t) + D_2(t) + U_1(t) - U_2(t) - M_2) \end{aligned}$$

Here after, we give the extended set of state equations:

$$(16)\quad \begin{cases} x_1(t+1) = \min(M_1,\ x_1(t) + D_1(t) - U_1(t)) \\ x_2(t+1) = \min(M_2,\ x_2(t) + D_2(t) + U_1(t) - U_2(t)) \\ x_3(t+1) = \min(M_3,\ x_3(t) + D_3(t) - U_3(t)) \\ x_4(t+1) = \min(M_4,\ x_4(t) + D_4(t) - U_4(t)) \\ x_5(t+1) = \min(M_5,\ x_5(t) + D_5(t) + U_2(t) + U_3(t) + U_4(t) - U_5(t)) \\ x_6(t+1) = x_6(t) + \max(0,\ x_1(t) + D_1(t) - U_1(t) - M_1) \\ x_7(t+1) = x_7(t) + \max(0,\ x_2(t) + D_2(t) + U_1(t) - U_2(t) - M_2) \\ x_8(t+1) = x_8(t) + \max(0,\ x_3(t) + D_3(t) - U_3(t) - M_3) \\ x_9(t+1) = x_9(t) + \max(0,\ x_4(t) + D_4(t) - U_4(t) - M_4) \\ x_{10}(t+1) = x_{10}(t) + \max(0,\ x_5(t) + D_5(t) + U_2(t) + U_3(t) + U_4(t) - U_5(t) - M_5) \end{cases}.$$

In the sequence, we shall see that we have introduced the extended state vector only to calculate the total overflow volume. In the final dynamic model, we shall use only the state variables associated with the tanks.

One can consider the output variable, $y(t)$, as being the wastewater volume that exits from the last tank and goes to the wastewater treatment process (WTP, see Fig. 1). In our example, we have:

(17) $y(t)=U_5(t)$

In order to complete the role of each variable of the proposed model, we can add that the external influent generates the disturbance variables, $D_i(t)$, i=1,….5. The wastewater volumes evacuated by the water columns at the moment t, $U_i(t)$, are regarded as control inputs.

The water volumes inside each tank at the moment $t=0$ are supposed to be known, that is:

(18) $x_i(0)=x_{i0}$, i=1,…, 5

A rational supposition is that there is no overflow for the moment $t=0$. Hence, we have:

(19) $x_i(0)=0$, i=6,…, 10

The volumes

(20) $D_i(t)$, i=1,….5, t=0,…, N,

are the combined contribution of the wastewater discharged by human and industrial consumers with the rainfall waters. These values are known for all the tanks and for the entire time horizon, thanks to a forecasting service that delivers this information.

The model of the SN is entirely described by the equations (20), (11), (16), (17) and the initial values of the state variables (18) and (19).

Under these conditions, the system evolution for the entire time horizon is completely known, if the control inputs $U_i(t)$ are specified for each tank and for each time moment.

3. OPTIMAL CONTROL OF THE SEWER NETWORK

The main control objective for our system is to minimize the total overflow volume, which is the sum of overflow volumes over all time moments and all retention tanks. Let's observe that the total overflow volume, denoted by I, can be expressed by the sum

(21) $$I = x_6(N) + x_7(N) + x_8(N) + x_9(N) + x_{10}(N)$$

We write the state equation for x_6 and for t=1,…, N. So, we obtain the following equations:

(22) $$\begin{cases} x_6(1) = x_6(0) + \\ \quad + \max\left(0,\, x_1(0) + Q_1(0) - U_1(0) - M_1\right) \\ x_6(2) = x_6(1) + \\ \quad + \max\left(0,\, x_1(1) + Q_1(1) - U_1(1) - M_1\right) \\ \ldots \\ x_6(N) = x_6(N-1) + \max(0,\, x_1(N-1) + \\ \quad + Q_1(N-1) - U_1(N-1) - M_1) \end{cases}$$

Making the sum of all equations from (22) , we can express $x_6(N)$ only depending on $x_6(0)$ (that is null), $Q_1(t)$ and $U_1(t)$, with t=0,…,N-1.

We can repeat the same calculus for the state variables x_7, x_8, x_9 and x_{10}. It holds:

(23) $$\begin{cases} x_6(N) = x_6(0) + \sum_{t=0}^{N-1} \max(0,\, x_1(t) + D_1(t) - U_1(t) - M_1) \\ x_7(N) = x_7(0) + \sum_{t=0}^{N-1} \max(0,\, x_2(t) + D_2(t) + U_1(t) - U_2(t) - M_2) \\ x_8(N) = x_8(0) + \sum_{t=0}^{N-1} \max(0,\, x_3(t) + D_3(t) - U_3(t) - M_3) \\ x_9(N) = x_9(0) + \sum_{t=0}^{N-1} \max(0,\, x_4(t) + D_4(t) - U_4(t) - M_4) \\ x_{10}(N) = x_{10}(0) + \sum_{t=0}^{N-1} \max(0,\, x_5(t) + D_5(t) + U_2(t) + U_3(t) + U_4(t) - U_5(t) - M_5) \end{cases}$$

$$
\begin{aligned}
I = \sum_{t=0}^{N-1} & [\max(0, x_1(t)+D_1(t)-U_1(t)-M_1)+ \\
&+\max(0, x_2(t)+D_2(t)+U_1(t)-U_2(t)-M_2)+ \\
&+\max(0, x_3(t)+D_3(t)-U_3(t)-M_3)+ \\
&+\max(0, x_4(t)+D_4(t)-U_4(t)-M_4)+ \\
&+\max(0, x_5(t)+D_5(t)+U_2(t)+U_3(t)+U_4(t)-U_5(t)-M_5)]
\end{aligned} \quad (24) \quad .
$$

From equations (21) and (23), the total overflow volume is given by equation (24):

If we consider the system evolution beginning with the time moment *t*, the cost function for our optimization problem denoted by *I*(*t*) would be:

$$
\begin{aligned}
I(t) = \sum_{s=t}^{N-1} & [\max(0, x_1(s)+Q_1(s)-U_1(s)-M_1)+ \\
&+\max(0, x_2(s)+Q_2(s)-U_2(s)-M_2)+ \\
&+\max(0, x_3(s)+Q_3(s)-U_3(s)-M_3)+ \\
&+\max(0, x_4(s)+Q_4(s)-U_4(s)-M_4)+ \\
&+\max(0, x_5(s)+Q_5(s)-U_5(s)-M_5)]
\end{aligned} \quad (25) ,
$$

where $Q_i(s)$ are defined by the equations (11)

Because we found out an expression of the objective function depending on the first five state variables $x_i(t), \quad i=1,\dots,5$, the next 5 state variables $x_i(t), \quad i=6,\dots,10$ would be eliminated from the model. Hence, the proposed model becomes:

$$
(26) \begin{cases}
x_1(t+1) = \min(M_1, x_1(t)+Q_1(t)-U_1(t)) \\
x_2(t+1) = \min(M_2, x_2(t)+Q_2(t)-U_2(t)) \\
x_3(t+1) = \min(M_3, x_3(t)+Q_3(t)-U_3(t)) \\
x_4(t+1) = \min(M_4, x_4(t)+Q_4(t)-U_4(t)) \\
x_5(t+1) = \min(M_5, x_5(t)+Q_5(t)-U_5(t))
\end{cases}
$$

where Q_i(t) are defined by the equations (11).

The optimization problem has a Lagrange term as an objective function:

$$
(27) \quad I(t) = \sum_{s=t}^{N-1} L(s, x(s), u(s)) \text{ , where}
$$

$$
\begin{aligned}
L(s, x(s), u(s)) = & \\
& [\max(0, x_1(s)+Q_1(s)-U_1(s)-M_1)+ \\
&+\max(0, x_2(s)+Q_2(s)-U_2(s)-M_2)+ \\
&+\max(0, x_3(s)+Q_3(s)-U_3(s)-M_3)+ \\
&+\max(0, x_4(s)+Q_4(s)-U_4(s)-M_4)+ \\
&+\max(0, x_5(s)+Q_5(s)-U_5(s)-M_5)]
\end{aligned} \quad (28)
$$

The state equation of the discrete system may be written as below:

$$
x(t+1) = f(t, x(t), U(t)) =
$$

$$
(29) \quad = \begin{bmatrix}
\min(M_1, x_1(t)+Q_1(t)-U_1(t)) \\
\min(M_2, x_2(t)+Q_2(t)-U_2(t)) \\
\min(M_3, x_3(t)+Q_3(t)-U_3(t)) \\
\min(M_4, x_4(t)+Q_4(t)-U_4(t)) \\
\min(M_5, x_5(t)+Q_5(t)-U_5(t))
\end{bmatrix}
$$

where:

$$
(30) \quad x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \end{bmatrix}
$$

and Q_i(t) are defined by the equations (11). The control vector is defined as below:

$$
(31) \quad U(t) = \begin{bmatrix} U_1(t) \\ U_2(t) \\ U_3(t) \\ U_4(t) \\ U_5(t) \end{bmatrix}
$$

The problem has also bound constraints:

$$
(32) \quad 0 \le x_i(t) \le M_i, \; x_i(t) \in \boldsymbol{I} \quad i=1,\dots,5
$$

$$
(33) \quad 0 \le U_i(t) \le U_i^{\max}, \; U_i(t) \in \boldsymbol{I}, \quad i=1,\dots,5,
$$

where ***I*** is the integer number set.

Resuming, the proposed optimal control problem is defined by the following elements:

- the nonlinear discrete system (29), where Q_i(t) are defined by the equations (11);

- the initial conditions given by equation (18)

- the bound constraints (32) and (33);

- the objective function (27) with Lagrange term (28);

- the optimum criterion

$$\min_{U(t)\in U} I(t_0)\,, \quad (34)$$

where t_0 and N are initial data.

4. APPLICATION OF OPTIMAL CONTROL OF A SEWER NETWORK - CONCLUSION

The optimization problem defined at the end of the previous section can be solved in different ways, if we dispose of the estimated influent $\boldsymbol{D}(t)$ for the time horizon t=0,…,N.

In the paper (Minzu, *et all*., 2014) the optimal control of a SN is solved through a Multi-Agent Control System. Whatever is the way of solving the optimization problem, let's note that an optimal controller has as input the estimated influent $\boldsymbol{D}(t)$, while the SN evolves under the influence of the real influent $\boldsymbol{D}_{real}(t)$, like in Fig. 2. $Q_{over}(t)$ represents the total overflow at the moment t

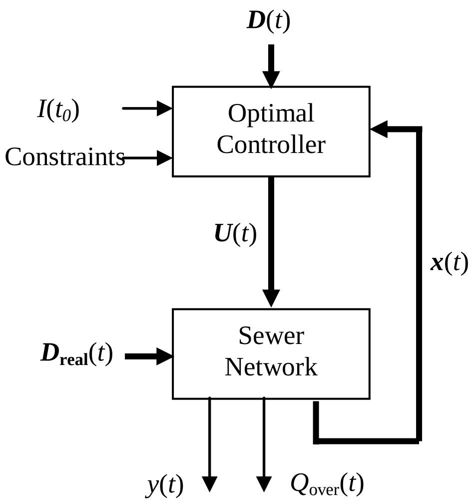

Fig.2. A real control structure for a SN

The control structure illustrated in Fig. 2 suggests a general strategy to implement a realistic control structure. The optimal controller solves the optimization problem for an established time horizon, using the sequence estimated influent $\boldsymbol{D}(t)$, $\boldsymbol{D}(t+1)$,…, $\boldsymbol{D}(t+N-1)$ and the real state vector $\boldsymbol{x}$(t) that gives the initial conditions for the tanks. The solution of the optimization problem consists in a sequence of optimal control inputs for the established time horizon: $\boldsymbol{U}(t)$, $\boldsymbol{U}(t+1)$,…, $\boldsymbol{U}(t+N-1)$. Only the first one - corresponding to the present moment t- is sent to the process. At the next sampling period, the controller solve the optimization problem considering the new initial states $\boldsymbol{x}$(t+1) and the new sequence of estimated influent. The solution is a new sequence of control inputs whose first element is sent to the process as the current control input $\boldsymbol{U}(t+1)$.

The future work will be devoted to found out an efficient way to solve this optimization problem that hasn't regularity properties.

5. REFERENCES

Rojas, J. D., Flores X., et all, (2012); Application of multivariate virtual reference feedback tuning for wastewater treatment plant control", *Control Engineering Practice*, Vol. 20, no. 5, pp. 499-510.

O'Brien, M., Mack, J., Lennox, B., et all (2011), Model predictive control of an activated sludge process: a case study, *Control Engineering Practice*, vol. 19, no. 1, pp. 54–61.

Carp, D.M. (2014) *Contribution to the Sewer Network Control and Waste Water Treatment* PhD Thesis, "Dunarea de Jos" University of Galati.

Carp, D., Barbu, M., Mînzu, V., (2013) *Sewer Network Discharge Control Using a Fuzzy Logic Approach*, The 4th International Symposium on Electrical and Electronics Engineering, Galaţi.

Aulinas, M., Nieves, J.C., Cortes, U., et all, (2010),Supporting decision making in urban wastewater systems usinga knowledge-based approach, Environmental Modelling & Software, No. 26, Pp. 562 - 572.

Murillo, J., Busquets, D., Dalmau, I., et all, (2011) "Improving urban wastewater management through an auction-based management of discharges", *Environmental Modelling & Software*, Vol. 26, Pp. 689 – 696,

Marinaki M., Papageorgiou M.: (2005) Optimal Real-Time Control of Sewer Networks. *Springer-Verlag*,.

Ocampo-Martinez C., (2010) Model Predictive Control of Wastewater Systems, *Springer-Verlag*, London,.

Dorsano, S., Labadie J.W., (2007); Neural-optimal control algorithm for real-time regulation of in-line storage in combined sewer systems, *Environmental Modelling & Software*, 22 (9), pp. 1349-1361.

Minzu, V. ; Barbu, M. ; *et all*,(2014); *Sewer network discharge control using a multiagent approach;* The 18th International Conference System Theory, Control and Computing (ICSTCC),;Page(s): 779 - 784