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SEWER NETWORK DISCHARGE OPTIMIZATION USING THE DYNAMIC PROGRAMMING

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Abstract: It is necessary to adopt an optimal control that allows an efficient usage of the existing sewer networks, in order to avoid the building of new retention facilities. The main objective of the control action is to minimize the overflow volume of a sewer network. This paper proposes a method to apply a solution obtained by discrete dynamic programming through a realistic closed loop system.

Keywords: sewer network; nonlinear systems, optimal control; dynamic programming.

1. INTRODUCTION

The objective of avoiding the water pollution is today a very important one and it involves some major actions to be achieved. These actions concern mainly the construction of a sufficient number of wastewater treatment plants and sewer networks allowing the treatment of a larger percentage of wastewater. These issues have been addressed in many publications in the filed (Rojas, *et al.*, 2012; O’Brien, *et al.,* 2011; Aulinas, *et al.*, 2010.).

A realistic solution is to have an optimal control that allows an efficient usage of the existing sewer networks, avoiding the building of new retention facilities.

There are two classes of control techniques that have been used in this domain. The first class contains the intelligent techniques: fuzzy logic control (Carp *et al*, 2013; Carp, 2014), expert systems (Aulinas, 2010) or even multi-agent systems (Murillo, *et al.*, 2011; Minzu, *et al.*, 2014.). The second one contains optimal control techniques: nonlinear optimal control (Marinaki,and Papageorgiou, 2005), neural-optimal control algorithm (Dorsano and Labadie, 2007) predictive control for hybrid systems (Ocampo-Martinez, 2010) or discrete dynamic programming (Carp, 2014).

This paper is a continuation of the work presented in (Carp, 2014) and it is a thorough point of view regarding the possibility to use the discrete dynamic programming in an efficient way. The main objective of the control action is to minimize the overflow volume of a sewer network. Section 2 is a review of the nonlinear discrete model for a sewer network presented in (Minzu, 2014a). The off-line solution of the optimization problem based on the discrete dynamic programming is described in Section 3. A realistic method to apply this solution by a closed loop system is presented in Section 4. The Conclusion section underlines the cases when the proposed solution can by applied in an efficient way.

2. OPTIMIZATION PROBLEM FOR A SEWER NETWORK DISCHARGE

In the paper (Minzu, 2014), a nonlinear discrete model for a sewer network (SN) is presented. A

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particular structure of a SN is illustrated in Fig.1. Let n be the number of tanks. Here, we have n=4. A rectangular element represents a retention tank for the wastewater and a circular element is a collector for all the flow capacities that are the inputs of the tanks. The influent that affects the SN is represented by the flow capacities $d_i(t)$, i=1,…,n, from the four catchment areas. The discharge of the tank i caused by its evacuation pump it is denoted by $u_i(t)$, i=1,…, n. A characteristic of the SN is the fact that any tank has only one downstream tank. When the tanks reach their maximum capacity M_i, i=1, …, n, expressed by volume units, there will be overflows denoted by $q^{over}{}_i(t)$.

Let $q_i(t)$ be the total flow capacity of the wastewater entering in the retention tank i. In order to simplify the model, the variables $d_i(t)$, $u_i(t)$, $q_i(t)$ can be expressed by integer values. All the water columns have the same evacuation power. This may be expressed by the wastewater volume evacuated in a sampling period T. Let ΔV be this volume. Hence, all the variables have values multiple of ΔV. Now, these values are denoted by $D_i(t)$, $U_i(t)$, $Q_i(t)$, $Q^{over}{}_i(t)$, i=1,…, n and are integer values. The sampling period is denoted by T.

The values Q_i, i=1, …, n, are the reflection of the sewer network' s structure. For our example, we have

$$(1)\quad \begin{aligned} &Q_1(t) = D_1(t);\\ &Q_2(t) = D_2(t) + U_1(t);\\ &Q_3(t) = D_3(t)\\ &Q_4(t) = D_4(t) + U_2(t) + U_3(t); \end{aligned}$$

The estimated influent is known for all tanks and over the entire time horizon:

$$(2)\quad \boldsymbol{D}(t) = [D_1(t), D_2(t), \cdots, D_n(t)],\ t=t_0,\dots,N$$

One can consider the output variable, $y(t)$, as being the wastewater volume that exits from the last tank and goes to the wastewater treatment process (WTP, see Fig. 1). It is also denoted by $Q^{\text{effluent}}(t)$

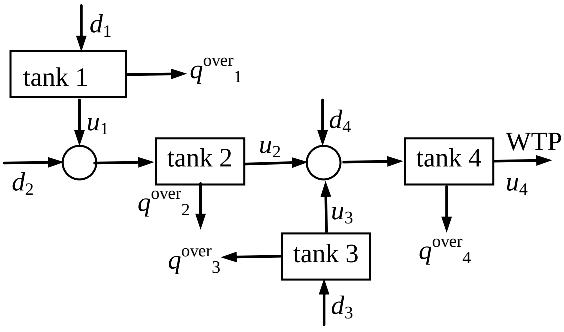

Fig.1. – Sewer network structure

In our example, we have:

$$y(t) = Q^{\text{effluent}}(t) = U_4(t)$$

In the same paper (Minzu, 2014), the discharge of the SN is treated like an optimization problem. Resuming, the proposed optimal control problem is defined by the following elements:

♦ *the nonlinear discrete system* (3):

$$(3)\quad x(t+1) = f(t, x(t), U(t)) = \begin{bmatrix} \min(M_1, x_1(t) + Q_1(t) - U_1(t)) \\ \cdots\cdots \\ \min(M_n, x_n(t) + Q_n(t) - U_n(t)) \end{bmatrix},$$

where:

$$(4)\quad x(t) = [x_1(t)\, x_2(t)\, x_3(t) \cdots x_n(t)]^T$$

$$(5)\quad U(t) = [U_1(t)\, U_2(t)\, U_3(t) \cdots U_n(t)]^T$$

and Q_i(t) are defined by the equations (1)

♦ *the initial conditions*:

$$(6)\quad x_i(0) = x_{i0},\ i=1,\dots,n$$

♦ *the bound constraints*:

$$(7)\quad 0 \le x_i(t) \le M_i,\ x_i(t) \in \boldsymbol{I} \quad i = 1,\dots,n$$

$$(8)\quad 0 \le U_i(t) \le U_i^{\max},\ U_i(t) \in \boldsymbol{I}, \quad i = 1,\dots,n,$$

♦ *the objective function*:

$$(9)\quad \begin{aligned} I(t) = \sum_{s=t}^{N-1} [&\max(0,\ x_1(s) + Q_1(s) - U_1(s) - M_1) + \\ &+ \cdots + \max(0, x_n(s) + Q_n(s) - U_n(s) - M_n)] \end{aligned},$$

where $0 \le t \le N-1$.
The objective function has a Lagrange form:

$$(10)\quad I(t) = \sum_{s=t}^{N-1} L(s, x(s), u(s)),\ \text{where}$$

$$(11)\quad \begin{aligned} &L(s, x(s), u(s)) = \\ &\max\{0, x_1(s) + Q_1(s) - U_1(s) - M_1\} + \cdots \\ &+ \max\{0, x_n(s) + Q_n(s) - U_n(s) - M_n\} \end{aligned}$$

♦ *the optimum criterion*:

$$(12)\quad \min_{U(t) \in U} I(t_0),$$

The values t_0 and N are known initial data.

3. OFF-LINE SOLUTION USING DYNAMIC PROGRAMMING

The state equation (2) and the Lagrange term (10) show that there is lack of the derivative in some points. Hence, the dynamic programming is an appropriate technique for solving our optimization problem.

$$(13)\quad V(t,x(t)) = \min_{v\in \mathbf{U}(t)} \left\{ \begin{array}{l} \max(0,\ x_1(t)+D_1(t)-v_1-M_1)+ \\ +\max(0,\ x_2(t)+D_2(t)+v_1-v_2-M_2)+ \\ +\max(0,\ x_3(t)+D_3(t)-v_3-M_3)+ \\ +\max(0,\ x_4(t)+D_4(t)+v_2+v_3-v_4-M_4)+ \\ \\ +V\left(t+1, \begin{bmatrix} \min(x_1(t)+D_1(t)-v_1,\ M_1) \\ \min(x_2(t)+D_2(t)+v_1-v_2,\ M_2) \\ \min(x_3(t)+D_3(t)-v_3,\ M_3) \\ \min(x_4(t)+D_4(t)+v_2+v_3-v_4,\ M_4) \end{bmatrix} \right) \end{array} \right\}$$

$$(14)\quad u^*(t) = v = [v_1 \quad v_2 \quad v_3 \quad v_4]^T \in \mathbf{U}(t)$$

Computation_V(t_0, H)

Algorithm for the computation of optimal costs and optimal control inputs using the DDP

Date de intrare: $\boldsymbol{U} = [0,\ U^1_{\boldsymbol{max}}]\times\cdots\times[0,\ U^n_{\boldsymbol{max}}]$

$D_1(t), D_2(t),\cdots, D_n(t),\ t = t_0,\ldots,N$: the estimated influent

$M_1, M_2,\ldots, M_n$: the tanks' volumes

t_0: initial time

H: the horizon of the DDP

<u>*Begin*</u>

♦ Initialize data corresponding to the discretization of state and control input variables;

♦ Initialize data structures for memorizing the values for $V(t,x(t))$ and optimal control inputs;

♦ $t_f \leftarrow t_0+H$; /* t_f : the final moment

♦ $t \leftarrow t_f-1$;

♦ <u>*while*</u> $t >= t_0$

<u>*for*</u> all the accessible states *x(t)* at moment *t*

<u>*for*</u> all admissible control inputs in the state variable *x(t)*

• compute and memorize *V(t,x(t))* using equation (14)

• compute and memorize the optimal control input *u*(t)=v* giving the minimum value from equation (14)

■

■

t=t-1;

■

♦ From the initial state variable $x(t_0)$ apply the memorized control inputs, for $t=t_0, \ldots, t_f$, in order to generate the optimal trajectory.

<u>*End*</u>

Fig.2. Algorithm for the computation of optimal costs, optimal control inputs and the generation of optimal trajectory

Some elements of this classical technique are reviewed hereafter (Ionescu *and* Popeea, 1981).

Minimum cost function associated with the optimal problem is:

$$(15)\quad V(t,x) = \min_{\substack{u(s)\in U(s) \\ t\le s\le t_f-1}} \{I(t)\}.$$

If $v=u(t)$ is the optimal control input, then the optimal cost is given by the *discrete dynamic programming equation*:

$$V(t,x) = \min_{v\in U(t)} \left\{ L(t,x(t),v) + V(t+1,\ f(t,x(t),v)) \right\}$$

and the optimal control input $u^*(t)$ is given by

$$u^*(t) = k(t,x(t)) \stackrel{\Delta}{=} \arg \min_{v\in U(t)} \{L(t,x(t),v) + V(t+1,\ f(t,x(t),v))\}$$

The function $k(t, x(t))$ is the optimal control law in open loop for the given optimization problem.

For our example, the discrete dynamic programming (DDP) equation has a more explicit form (13).

Fig. 2 presents the essential part of the algorithm described in pseudo code, which computes the optimal costs and optimal control inputs using the DDP. This algorithm is organized such that it would describe the MATLAB function Computation_V(t_0, H). It is a flexible function because the DDP may have any initial moment and any time horizon:

$$t_0 \geq 1\ ; H \leq N - 1$$

Remark: The function Computation_V(t_0, H) computes any optimal trajectory going from $x(t)$ and memorizes in V(t, $x(t)$) the minimum cost and in $k(t, x(t))$ the first optimal control input of the optimal sequence.

The running of this function yields the two data structures, $V(t, x(t))$ and $u^*(t)=k(t, x(t))$, and generates the optimal trajectory for any given initial state. These two data structures can be saved into a data file, in order to be used later in an on-line application.

The estimated influent $\boldsymbol{D}(t)$ is loaded from a data file that gives an influent scenario for the simulated time interval.

A realistic approach is to consider that all the control inputs are limited to 0 or 1 values. That means all the water columns evacuate $0 \cdot \Delta V$ or $1 \cdot \Delta V$ unities of volume. That is way we have adopted

$$(16)\ \ U_i^{\max} = \{0, 1\},\ i = 1, \ldots, n$$

The results of a MATLAB simulation, for the SN described before, are presented hereafter. The input data are:

(17) x_0=[3, 3 ,3, 3]^T; T=120 sec; N=40;

(18) M_1 =10, M_2 =14, M_3 =6, M_4 =20

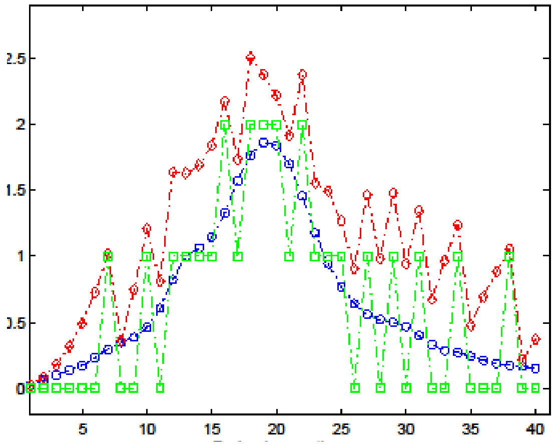

Fig.3. Influent scenario over the problem's horizon

The influent scenario for the catch area of tank 2 is presented in Fig. 3. The blue curve gives the volume of the influent in real values, the green one shows the integer values of the influent and the red one gives the corrected value of the influent. The later is used by the optimal algorithm.

The evolution of the optimal control input for the tank 2 is presented in Fig. 4

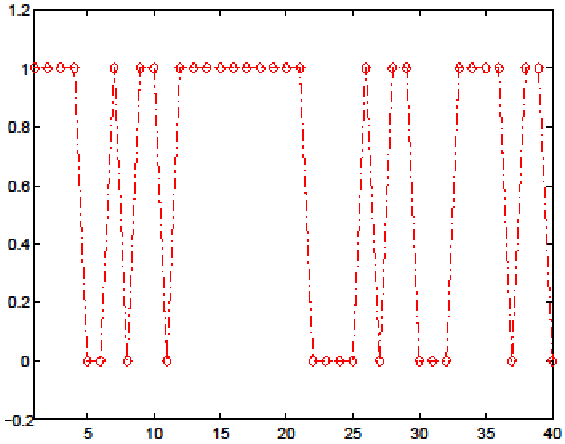

Fig.4. Evolution of the optimal input for tank 2

The optimal state trajectories for all tanks are illustrated in Fig. 5. The blue, red, green and magenta trajectories correspond to the tank 1, 2, 3 and 4 respectively.

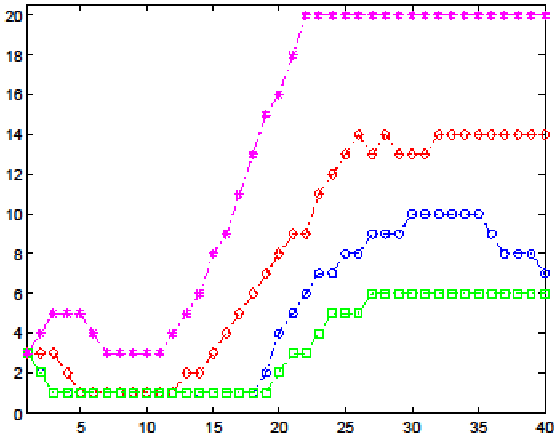

Fig.5. Evolution of the state variables for all tanks

For this influent and for the initial state x_0, the minimum total overflow volume is

$$(19)\ I(t_0)=0.$$

The simulations have proved that the behavior of the implemented program is quite acceptable, for $T \geq 120$ sec and $n \leq 5$. The computation of the next optimal control inputs ends before the duration of the sampling period *T*.

Therefore the program can be used even in a real time application.

4. IMPLEMENTATION OF A REAL OPTIMAL CONTROLLER

If we want to make an optimal controller utilizable in a closed loop system, we have to take into account some aspects mentioned below.

The real SN process is not affected by the estimated influent, but the real one. If the influent changes then the optimal sequence of control inputs changes even for the same initial state variables. Therefore, with a real influent, the sequence of control inputs and consequently the sequence of state variables are different.

Hence, if at the moment *t* we have two different state vectors

$$(20)\ x^1(t) \neq x^2(t),$$

then the optimal control inputs are different:

$$(21)\ u^*(t, x^1(t)) \neq u^*(t, x^2(t))$$

A realistic approach is to use an optimal control structure like in Fig. 6. The sewer network process is under the influence of the real influent $\boldsymbol{D}_{real}(t)$. The current state vector is sent to the optimal controller in order to select an appropriate control input $U(t)$. This one is the first element of an optimal sequence of control inputs associated with the initial state $x(t)$ and the estimated influent

$$(22)\ \boldsymbol{D}=[D(t),\ldots, D(N+t-1)].$$

In the previous section, it has been mentioned that the data structure giving $u^*(t)=k(t,x(t))$ can be obtained and memorized off-line by an execution of the optimization program. This data structure can by loaded afterwards in the optimal controller. For our example, when n=4, the size of this data structure is 35 kB. Any equipment used in the hardware implementation of the optimal controller has a memory big enough to store this data structure.

In order to validate optimal control structure presented in Fig. 6, the simulations have used a real influent scenario defined by this equation:

$$(23)\ \mathbf{D}_{real}(t)= \mathbf{D}(t)+\Delta\mathbf{D}.$$

$\Delta\boldsymbol{D}$ is a random variable with uniform distribution in the interval [0, *L*], where *L* is the upper bound of the deviation from the estimated influent. In our tests we have chosen L equal to 10% of the estimated influent average, for each tank. Obviously, the value of L is established by the simulation program. The real influent is also subject to a discretization and correction operations.

The results of the optimal control structure's simulation are presented in Fig. 7, Fig.8 and Fig.9 with the same initial data (17) and (18).

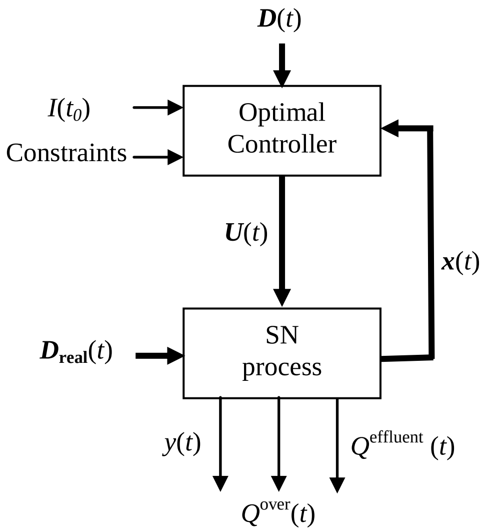

Fig.6. Implementation of a real control structure

For the same estimated influent, but also with a real influent affecting the SN process, and for the same initial state x_0 given by (17), the minimum total overflow volume is

$$(24)\ I(t_0)=7 \text{ (units)}$$

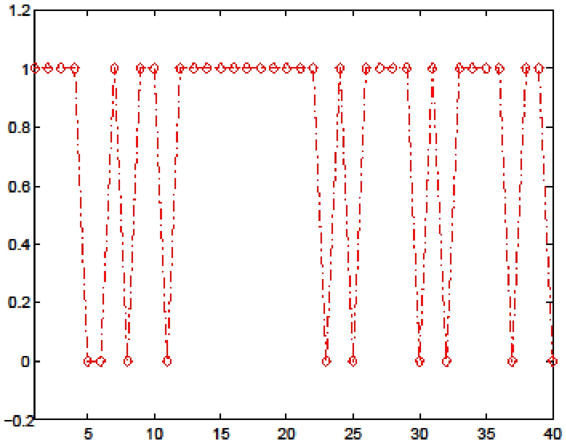

Fig.7. Evolution of the control input for the tank 2

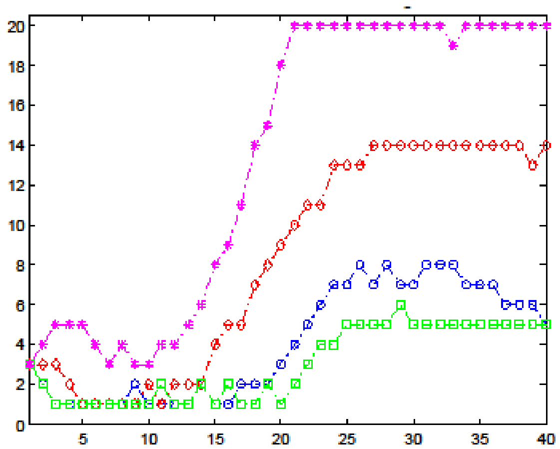

Fig.8. Evolution of the state variables with an optimal controller and a real influent

From the initial data, it results that the entire influent volume for all the tanks and over the all time horizon is 74 units (1 unit=120 m^3) that is 8880 m^3.

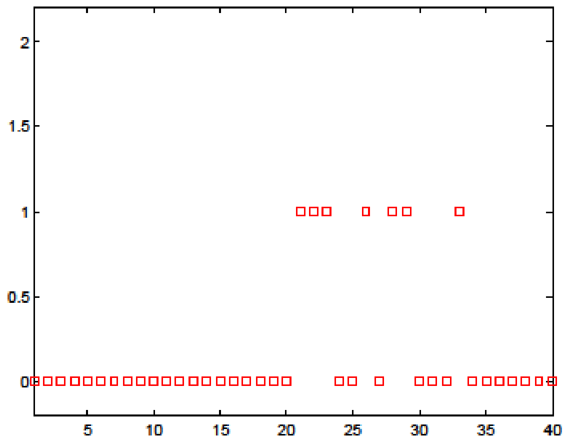

Fig.9. Distribution of the total overflow versus the sampling period

Obviously, from (23) it holds

(25) $\mathbf{D}_{real}(t) > \mathbf{D}(t)$

The equations (19) and (24) mark a difference between the total overflows obtained by the off-line simulation program and the real control structure. The main explanation of this difference is the fact that the real influent is bigger than the estimated one, even when the control loop keeps the optimal character.

5. CONCLUSION

The optimal controller proposed in section 4 can be a realistic solution of the described optimization problem, but there are some limitations. It is well known that the dynamic programming has a very high complexity. That is why the proposed solution is appropriate in the case when the number of tanks is limited: $n \leq 5$. It can by shown that the numerical complexity of the algorithm yielding the two data structures, $V(t,\ x(t))$ and $u^*(t)=k(t,\ x(t))$ is proportional with

$$N \cdot M_1 \cdot M_2 \cdots M_n$$

There are two ways to diminish this complexity. The first method is to decrease the number of discrete states by decreasing the number of possible states for each tank: $M'_1 < M_1 \cdots M'_n < M_n$. This is a technological solution because it needs to adopt more powerful water column. So, a greater quantity of wastewater is evacuated in a sampling time. Hence, the discrete states number of each tank decreases.

The second one is to decrease N. The time horizon of the optimization problem is $N \cdot T$. Hence, to decrease N means to increase T. In our simulations, for the SN treated before, we have adopted N=40 & T=120, but also N=20 & T=240. With initial data

$$N=20;\ T=240;\ x_0=[2\ 1\ 3\ 2]^T$$

the simulation results are:

$$I(t_0)=480\ \text{m}^3;\ \sum_{t=0}^{N-1} Q^{\text{effluent}}(t) = 4560\ \text{m}^3$$

If initial data are

$$N=40;\ T=120;\ x_0=[4\ 2\ 6\ 4]^T$$

the algorithm yields

$$I(t_0)=480\ \text{m}^3;\ \sum_{t=0}^{N-1} Q^{\text{effluent}}(t) = 4680\ \text{m}^3$$

The first case has a complexity 2×16 times grater than the second one, but yields almost the same results.

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