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NUMERICAL EVALUATION OF RELIABILITY INDICATORS UNDER OPTIMAL RENEWAL STRATEGIES

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Abstract: This paper investigates the opportunity of using a numerical procedure for computing the renewal parameters of an equipment. The reasons of this approach reside in the sampled way we measure the process variables and the possibility of on-line parameter computation, for both the renewal and the reliability models. The computation algorithm is presented, then the errors with respect to an analytical procedure are evaluated, considering a validation example. The paper shows that the errors of the sampled model are negligible, within the average operating time, with respect to a continuous model. The same property applies to the comparison with a hybrid model (continuous for the process model and sampled for the renewal model), so we conclude that numerical evaluation of the reliability and renewal parameters is possible and preferable.

Keywords: reliability, Markov model, renewal process, numerical procedures, preventive maintenance

1. Introduction

In order to obtain the best possible performance in the operation of technical equipment, it is necessary to implement optimal maintenance strategies to minimize maintenance costs of the equipments throughout their service lives. The research carried out (see [1]) showed that the costs determined by the measures needed to increase reliability and availability of equipment through preventive maintenance actions are incomparably less than the cost of the design and implementation of new equipment with improved performance. On the accuracy with which the maintenance strategy parameters are determined depend both safe operation of equipment - an essential factor in some applications - and getting an as low as possible maintenance cost. The correct determination of the optimal preventive renewal period keeps the equipment performance within admissible limits and maximizes its availability. It is recognized that any renewal made before or after the optimum time leads to increased maintenance costs. In case of late renewals, the increased cost is caused by increased risk of accidental damage and the operation of the equipment with a greater degree of wear than acceptable. The advantages of preventive maintenance are listed below:

-Preventive renewal being planned, all material and human resources necessary for quality interventions are available;

- Operating mode of the equipment on which the intervention is made preventively is led to a state which removes any losses on the technological flow and allows easier works for the maintenance team;

- Worn components are replaced/repaired exactly at the best time, before they get damaged which may further cause other components of the equipment to get damaged;

- Stationary periods, ranging from failure occurrence and time of intervention are eliminated.

Preventive maintenance has been addressed in many specialized works [2], [3], [4], [5], [7] and [8]. In most cases Markov models are used to determine the reliability indicators needed to establish optimal period of renewal. Due to the variant external conditions faced by equipment during operation it is necessary that Markov model parameters be adjusted in real time according to the actual development of the equipment wear condition (see [6], [7]). For example in [7], determining the optimal period for renewal is made in two phases: the first phase, off-line, is to determine the Markov parameters based on the information received from the sensors, and in the second stage, on-line, it is estimated the time left until the equipment failure.

This paper addresses the opportunity of developing numerical algorithms to evaluate the indicators used in the synthesis of an evolutionary strategy for renewal, using reliability Markov models. For this purpose, the errors introduced are assessed by using numerical algorithms for determining the reliability indicators for Markov models of 2, 3 and 4 order. The results are validated by comparing the probability indicators, obtained by integrating the analytical model and numerical simulation of a model. Based on these results, it is reasonable to

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employ numerical algorithms for the evaluation of the reliability model parameters.

2. Principle of evolutionary renewal strategy

Characterization of renewal processes can be done, briefly, with the average number of renewals made within a range (0, t) called the renewal function $H(t)$ and the renewal density $h(t)$. This is obtained by derivating the renewal function and is the probability of renewal occurrence around time t, regardless the order of the model. If you want renewals to take place at the optimum time it is required for these two functions to be determined based on a Markov model updated in real-time, a model able to better reflect the equipment wear condition. Adjusting the Markov model parameters (MM) can be done in two ways:

- based on data collected by the human operator during maintenance operations, because the parameters are constant between two interventions;
- in a renewal cycle, MM parameters are adjusted periodically based on data collected by a diagnosis system.

The structure proposed in [6] to determine optimal renewal period is shown in Figure 1.

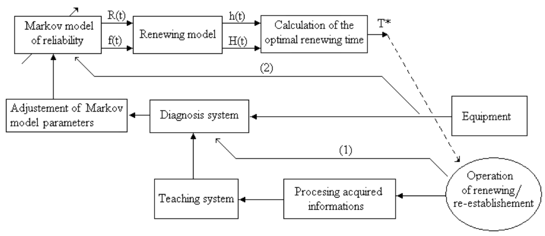

Fig. 1. Structure of the renewal evolutionary system

The evolutionary renewal strategy involves calculating the optimal period T^* based on information provided by the reliability MM, a model whose parameters $p_i(t)$, corresponding to the wear condition of equipment analyzed, are periodically adjusted according to path (2) by the diagnosis system. This system, in turn, is trained/adjusted after each renewal / restoration, on which occasion, the human operator establishes the real state of wear of that particular equipment (path 1 in Figure 1).

The following is a proposal of a numerical calculation procedure by means of which blocks MM and MR in Figure 1 provide the outputs in the sequence: $p_i^k(t) \rightarrow R^k(t) \rightarrow f^k(t) \rightarrow h^k(t) \rightarrow H^k(t)$, where the index k represents the current renewal cycle, $p_i^k(t)$ is the probability of the states of Markov reliability model, $R^k(t)$ is the reliability function of the equipment and $f^k(t)$ is the probability density of the equipment operating time. The procedure is expected to ensure:

- Recursive calculation of functions $p_i(t)$, $f(t)$, $R(t)$, $h(t)$ and $H(t)$, knowing that among them there is the following dependence:

$$R(t) = \sum_{j=0}^{n-1} p_j(t) \tag{1}$$

$$f(t) = -\frac{dR(t)}{dt} \tag{2}$$

$$h^*(s) = \frac{f^*(s)}{1 - f^*(s)} \quad \text{(Laplace transform)} \tag{3}$$

$$H(t) = \int_0^t h(\tau)\, d\tau \tag{4}$$

- Obtain after each adjustment of MM parameters, the solution for optimal renewal period T^{*k};
- Easily configuration of the calculation scheme based on the time structure of the graph that represents the MM of the equipment.

Although numerical procedures will provide approximate solutions to the reliability and renewal models, they have the advantage that they can be easily applied to variant models, as shown for periodic adjustment of MM. If adjustments of MM parameters are performed often enough, it can be admitted that changes of its parameters will be reduced in amplitude

3. Numerical procedures to determine the reliability indicators

For the recursive calculation of functions $p_i^k(t)$, $f^k(t)$ and $R^k(t)$ we shall consider an equipment characterized by MM in fig. 2, where the states 0, 1, 2 are operation states while state 3 is failure state.

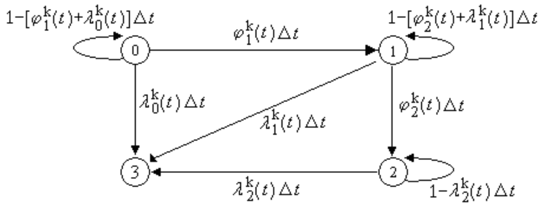

Fig. 2. Markov model of the equipment

Notations in fig. 2 have the following meanings:

- k – current renewal step;
- $\lambda_0^k(t)\Delta t$, $\lambda_1^k(t)\Delta t$, $\lambda_2^k(t)\Delta t$ – is the probability of failure of the equipment in states 0, 1, 2, over the period Δt;
- $\varphi_1(t)\Delta t$, $\varphi_2(t)\Delta t$ – is the probability of transition from state 0 into state 1, and from state 1 to 2, respectively.

The state equation of the equipment characterized by MM in fig. 2 is:

$$\frac{d}{dt}\begin{bmatrix} p_0^k(t) \\ p_1^k(t) \\ p_2^k(t) \end{bmatrix} = \begin{bmatrix} -(\lambda_0^k + \varphi_1^k) & 0 & 0 \\ \varphi_1^k & -(\lambda_1^k + \varphi_2^k) & 0 \\ 0 & \varphi_2^k & -\lambda_2^k \end{bmatrix} \cdot \begin{bmatrix} p_0^k(t) \\ p_1^k(t) \\ p_2^k(t) \end{bmatrix} \tag{5}$$

The state equation (5) corresponds to the model in Figure 3 where each of the three subsystems is described by a 1st order differential equation.

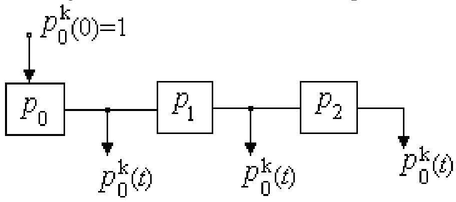

Fig. 3. Representation of Markov model block

The first subsystem is autonomous, meeting the nonzero initial condition $p_0^k(0)=1$. It is further applied the Laplace transform and equation (5) and the relations (1) and (2) are also considered.

Applying signal $p_0(t)*\delta(t)$ at the input of the first element is equal to initializing this nonperiodic element to the value $p_0(0)=1$, then this entry is considered invalid. Using Laplace transformation and the properties of the distribution $\delta(t)$, we obtain:

$$p_0^k(s) = (s+\lambda_0^k+\varphi_1^k)\cdot \mathcal{L}\left\{p_0^k(t)\cdot\delta(t)\right\} =$$
$$= (s+\lambda_0^k+\varphi_1^k)\cdot \mathcal{L}\left\{p_0^k(0)\cdot\delta(t)\right\} =$$
$$= (s+\lambda_0^k+\varphi_1^k)\cdot \mathcal{L}\left\{1\cdot\delta(t)\right\} = s+\lambda_0^k+\varphi_1^k \quad (6)$$

It further results:

$$p_0^k(t) = \exp\left[-(\lambda_0^k+\varphi_1^k)\cdot t\right] \quad (7)$$

Laplace transform of the probability of equipment failure can be obtained from equation (2) as follows:

$$F^k(s) = -\left[s\cdot R^k(s) - R^k(0)\right] = 1 - s\cdot R^k(s) \quad (8)$$

System structure diagram that provides the images of functions $R^k(s)$ and $F^k(s)$ is shown in Figure 4.

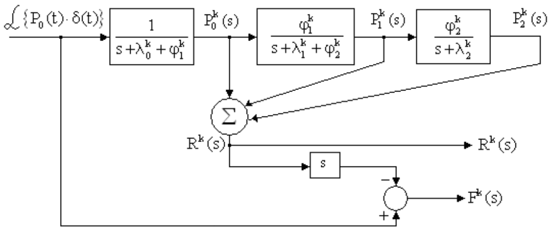

Fig. 4. Structure diagram of Markov model

By time discretization the model illustrated in fig. 5 is obtained, as equivalent of the system in fig. 4.

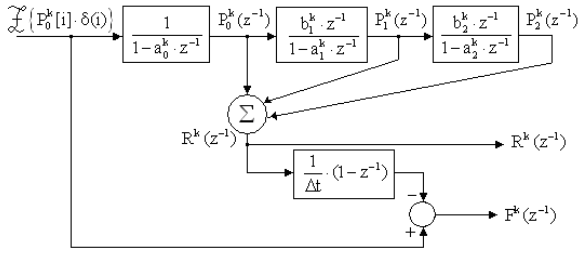

Fig. 5. Discretized Markov model

Difference equations that recursively provide the functions $R^k[i]$ and $f^k[i]$ are:

$$p_0^k[i] = a_0^k\cdot p_0^k[i-1] \quad \text{where} \quad p_0[0]=1 \quad (9)$$

$$p_1^k[i] = a_1^k\cdot p_1^k[i-1] + b_1^k\cdot p_0^k; \qquad p_1^k[0]=1 \quad (10)$$

$$p_2^k[i] = a_2^k\cdot p_2^k[i-1] + b_2^k\cdot p_1^k; \qquad p_2^k[0]=0 \quad (11)$$

$$R^k[i] = p_0^k[i] + p_1^k[i] + p_2^k[i]; \qquad R^k[0]=1 \quad (12)$$

$$f^k[i] = \frac{\left(R^k[i-1]-R^k[i]\right)}{\Delta t} \quad (13)$$

where:

$$a_0^k = \exp(-\Delta t\cdot(\lambda_0^k+\varphi_1^k)) \quad (14)$$

$$a_1^k = \exp(-\Delta t\cdot(\lambda_1^k+\varphi_2^k)); \; b_1^k = \frac{\varphi_1^k}{\lambda_1^k+\varphi_2^k}(1-a_1^k) \quad (15)$$

$$a_2^k = \exp(-\Delta t\cdot\lambda_2^k); \qquad b_2^k = \frac{\varphi_2^k}{\lambda_2^k}(1-a_2^k) \quad (16)$$

This scheme of calculating the reliability function and probability density of the operating time function, $f(t)$, can be easily generalized to various structures of the Markov model.

4. Determining the optimal period for equipment renewal

Achieving an optimal renewal strategy primarily involves minimizing maintenance costs of equipment per unit time. Average cost expression is given by (17), (see [1]).

$$C(T) = \frac{1\cdot H(T)+b}{T} \quad (17)$$

where:

- T is the period between two preventive renewals;
- $H(t)$ is the renewal function;
- b is the cost of preventive renewal expressed as a fraction of the cost of renewal in case of accidental failure.

Deduction of function $H(t)$ can be done in two ways: using the analytical model or using numerical methods. Getting function $H(t)$ analytically is unreasonable due to complexity of the model and the sample character of all measured quantities. Consequently, the question arises to predict it by numerical procedures (essential is the time discretization as quantization resolution of the numerical values can be made large enough that quantization noise should not matter).

To get the renewal function $H(t)$ it is necessary to determine first the renewal density $h(t)$. If we consider there is a simple renewal process, then the renewal density can be obtain from 18 [1].

$$h(t) = f(t) + \int_0^t h(\tau) \cdot f(t-\tau)\, d\tau \qquad (18)$$

By discretizing the relation (18) and considering that the discrete time $i \cdot \Delta t$ takes values for i=0, 1, 2, ..., it can be obtained:

- $h^k[0] = f^k[0]$ (19)
- $h^k[1] = f^k[1] + \Delta t\left(h^k[0] f^k[1] + h^k[1] f^k[0]\right)$ (20)
- $h^k[2] = f^k[2] + \Delta t \cdot \left(h^k[0] \cdot f^k[2] + h^k[1] \cdot f^k[1] + h^k[2] \cdot f^k[0]\right)$ (21)

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and step i is reached:

- $h^k[i] = f^k[i] + \Delta t \cdot (h^k[0] \cdot f^k[i] + h^k[1] \cdot f^k[i-1] + \ldots + h^k[i] \cdot f^k[0]$ (22)

Analyzing relations (19)...(22) it can be noticed that the sampled values of the renewal density can be recursively calculated within the same cycle in which the density function of the operating time is determined, $f(t)$. The calculation algorithm is as follows:

initializing $f(0)$

$$h^k[0] = f^k[0]$$

calculation: $f^k[1]$ (the values of f are computed within the reliability MM block, as in eqs. (9)-(13))

$$h^k[1] = \frac{1}{1 - f^k[0] \cdot \Delta t} \cdot \left(f^k[1] + \Delta t \cdot h^k[0] \cdot f^k[1]\right)$$

calculation: $f^k[2]$

$$h^k[2] = \frac{1}{1 - f^k[0] \cdot \Delta t} \cdot \left(f^k[2] + \Delta t \cdot (h^k[0] \cdot f^k[2] + h^k[1] \cdot f^k[1])\right)$$

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calculation: $f^k[i]$

$$h^k[i] = \frac{1}{1 - f^k[0] \cdot \Delta t} \cdot \left(f^k[i] + \Delta t \cdot \sum_{m=0}^{i-1} (h^k[m] \cdot f^k[i-m])\right) \qquad (23)$$

In order to compute the values according to eq. (23) it is necessary to establish the initial value of the operating time density $f^k[0]$. Since this is an important value in all relationships (23), it is recommended that the prediction/deduction should be made by a method that provides the exact value, i.e. to use the analytical way to determine this value. From the analysis of Markov model segment (Figure 2), corresponding to state "0" when the equipment was restored / renewed and enters a new cycle of operation, i.e. current cycle k, the initial value of the renewal density is equal to the equipment failure rate at time t=0. It follows:

$$h^k[0] = f^k[0] = \lambda_0^k \qquad (24)$$

5. Analysis of numerical errors in the procedure for calculating the renewal density function and the renewal function

For the numerical calculation of the renewal density the algorithm using eq. (23) has been developed including the recursive relationship for calculating the function $h(t)$ with discrete step Δt. Initializing the recursive relationship has been established through the relation (24) to the value $h[0]=\lambda_0$, where λ_0 is the parameter that defines the transition from Markov model between the initial state "0" and the corresponding failure situation. If we consider a simple renewal process, as described by the renewal equation (18), this equation can be put in correspondence with the block diagram in Figure 6, where we denote by S a dynamic system, with the output value of the renewal density $h(t)$ and input $f(t)$ and Σ is a summation block.

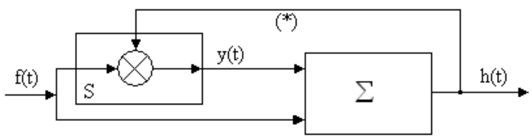

Fig. 6. Block diagram of renewal equation

The model of system S is given by the convolution product of functions $f(t)$ and $h(t)$:

$$y(t) = h(t) \otimes f(t) = \int_{-\infty}^{+\infty} h(\tau) \cdot f(t-\tau) \cdot d\tau = \int_0^t h(\tau) \cdot f(t-\tau) \cdot d\tau \qquad (25)$$

The discretization of model (25) can be done without difficulty, using discrete convolution. Recursive integration of the equation (16) by the discrete relations (23) features risks that are seen easily in Figure 6. Discretization errors in system S model are sent to the output and, hence, propagate back to the system S through the link marked with (*) in Figure 6. Under these circumstances the question is whether the algorithm (23) always ensures obtaining the renewal density $h(T)$ with a reasonable error. This analysis should take into account the specific problem as follows:

a) renewal process is assumed to be asymptotically stationary (Blackwell's theorem) [1], i.e.:

$$\lim_{t\to\infty} h(t) = const = \frac{1}{m} \qquad (26)$$

where m is the average of the operating time. It follows that the system S in Figure 6 is integrative.
b) input signal in system S is a signal of L_1. Since $f(t) \subset L_1$, it follows that the output $y(t)$ is bounded.

To test the calculation scheme in Figure 6, the input variable is generated by a block containing the reliability model of Markov type, simulated numerically. It means the probability density of the operating time function and has the property $f(t) \subset L_1$

To validate the proposed model simulations using Markov models of order 2, 3 and 4 were performed. For the 2nd order model the results obtained using numerical procedures were compared to those obtained analytically
For model of order 2, the state equations corresponding to Markov model are:

$$\frac{d}{dt}\begin{bmatrix} p_0^k(t) \\ p_1^k(t) \end{bmatrix} = \begin{bmatrix} -\lambda_0^k(t)+\varphi^k(t) & 0 \\ \varphi^k(t) & -\lambda_1^k(t) \end{bmatrix} \cdot \begin{bmatrix} p_0^k(t) \\ p_1^k(t) \end{bmatrix} \qquad (27)$$

Under initial conditions: $p_0^k(0)=1$, $p_1^k(0)=0$ and $\varphi^k(t)=const=\varphi^k$, $\lambda_0^k(t)=const=\lambda_0^k$ and $\lambda_1^k(t)=const=\lambda_1^k$ the following solutions are obtained:

$$p_0^k(t) = \exp[-(\lambda_0^k+\varphi^k)\cdot t] \qquad (28)$$

$$p_1^k(t) = \frac{\varphi^k}{\varphi^k+\lambda_0^k-\lambda_1^k} \cdot \left\{\exp(-\lambda_1^k \cdot t) - \exp[-(\lambda_0^k+\varphi^k)\cdot t]\right\} \qquad (29)$$

Further the reliability functions $R^k(t) = p_0^k(t) + p_1^k(t)$ can be calculated and then the other reliability indicators needed to determine the optimal renewal period.

The parameters of the 2nd order model used in simulation: λ_0=0.001, λ_1=0.005 and φ=0.005, resulting in $1/m$=0.00304 (m measured in hours, the other parameters measured in hrs^{-1}), a value that defines the horizontal asymptote of the renewal density function according to equation (26).

The performance of the numerical procedures were evaluated according to the relative errors made in calculating functions $h(t)$ and $H(t)$ and the precision with which relation (26) is satisfied when $h(t)$ is obtained by numerical procedure. Figure 7 presents the results for the pairs of functions $[f_a(t), f_d(t)]$, $[h_a(t), h_d(t)]$, $[H_a(t), H_d(t)]$, where the a and d indices denote the calculation procedures: analytical, or by discretization of the mathematical model. A discrete step Δt=1 and a zero-order extrapolator were used.

If using a discrete step Δt=0.5 and the same type of extrapolator almost identical results to those obtained for Δt=1 are obtained. The evolution of relative errors $|\varepsilon_{rf}(t)|$, $|\varepsilon_{rh}(t)|$ și $|\varepsilon_{rH}(t)|$ to determine the functions $f(t)$, $h(t)$ and $H(t)$ by numerical procedure using a discrete step Δt=1, are presented in Figures 8 and 9.

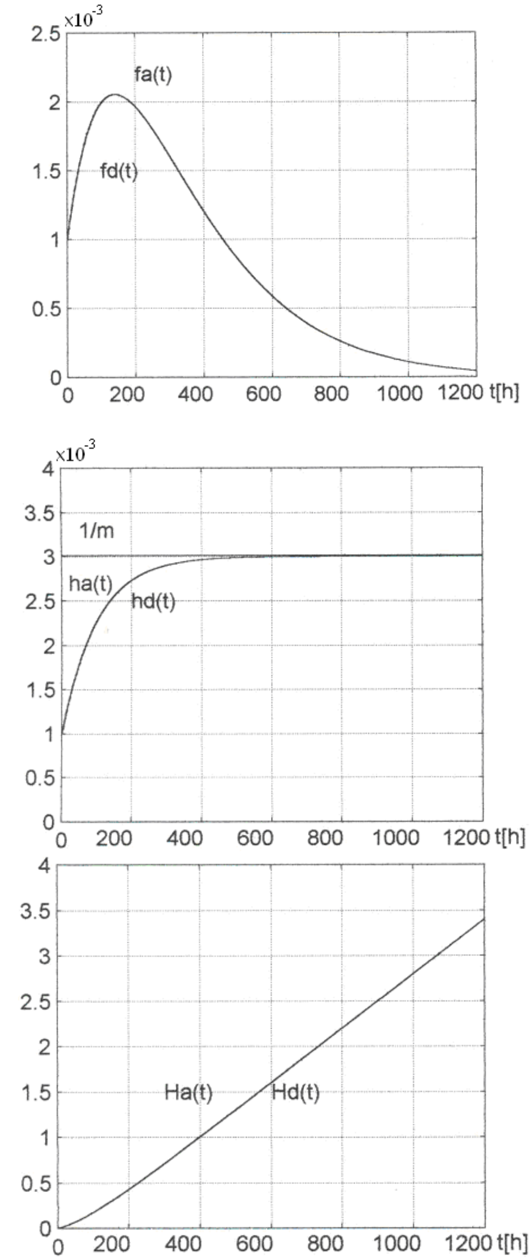

Fig. 7. Probability density (f), renewal density (h) and renewal function (H)

The results in Figure 8 were obtained using a zero-order extrapolator and for those in Figure 9 the first-order extrapolator was used. The curves labelled a, b, c correspond to functions $f(t)$, $h(t)$ and $H(t)$.

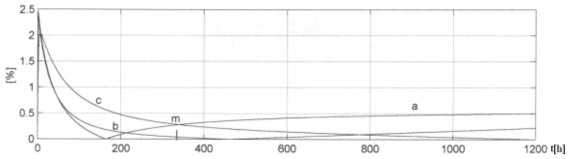

Fig. 8. Relative errors for Δ*t*=1 *and zero-order extrapolator*

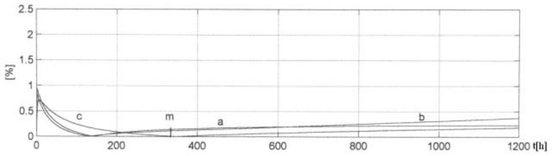

Fig. 9. Relative errors for Δ*t*=1 *and first-order extrapolator*

For a discrete step Δ*t*=0.5 the results in Figure 10 were achieved, where zero-order extrapolator was used and the results shown in Figure 11 for the first-order extrapolator.

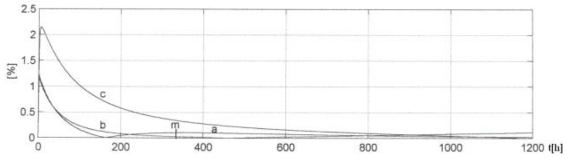

Fig. 10. Relative errors for Δ*t*=0.5 *and zero-order extrapolator*

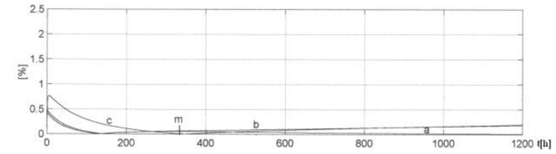

Fig. 11. Relative errors for Δ*t*=0.5 *and first-order extrapolator*

From the analysis of the results the following conclusions can be drawn:

- The instantaneous values of the relative errors is acceptable, even for discrete step Δ*t*=1. These errors are significantly reduced by decreasing the discrete step and using the first-order extrapolator (fig. 11).
- The integrating effect of the subsystem S in Figure 6 can be seen in all the results presented in Figures 8 ... 11, when the error $|\varepsilon_f(t)|$ is almost constant and error $|\varepsilon_h(t)|$ tends to evolve as a ramp. These developments are obtained at much higher values than *m* (average operating time) so that they do not influence the solution to the evolutionary renewal.
- For the range (0.5 ... 0.75)*m*, where *T** is the solution most likely to be located (obviously depending on the cost of a preventive renewal *b*), the relative error values, when using the first-order extrapolator, do not exceed 0.2% even when using a discrete step Δ*t*=1.

For the reliability Markov models of order 3 and 4 it is not possible to calculate the errors $|\varepsilon_h(t)|$ and $|\varepsilon_H(t)|$ because it is not possible to get the analytical expressions of the functions *h*(*t*) and *H*(*t*). In this case, the validation of numerical procedures can be done by verification of condition (26), where *h*(*t*) is calculated in accordance with the procedure concerned.

For the numerically simulated model of order 3 following parameters were adopted: λ_0=0.001, λ_1=0.003, λ_2=0.006, φ_1=0.008 and φ_2=0.005, yielding an average operating time *m*=312.46 hours. Figure 12 illustrates the relative errors $|\varepsilon_f(t)|$ in the following situations:

Δ*t*=1, zero-order extrapolator, curve (1)
Δ*t*=0.5, zero-order extrapolator, curve (2)
Δ*t*=1, first-order extrapolator, curve (3)
Δ*t*=0.5, first-order extrapolator, curve (4)

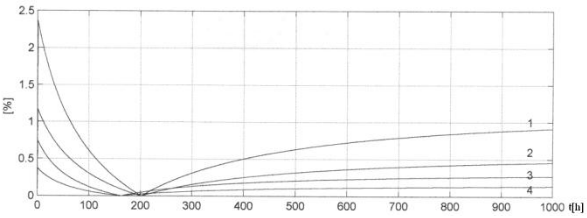

Fig. 12. Relative errors for probability density f(*t*)

To assess the effect of errors shown in Figure 12 on the function *h*(*t*) calculated numerically in Figure 13 the renewal density is presented where the error $|\varepsilon_f(t)|$ corresponds to curve 3 in Figure 12. From Figure 13 it is observed that the function *h*(*t*) calculated numerically converges to 1/*m* with very good accuracy.

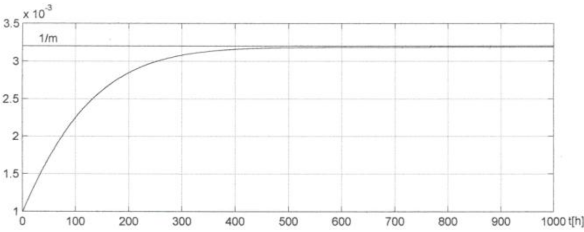

Fig. 13. Renewal density h(t), Δt=1, first-order extrapolator

For the 4th order model the parameters were adopted: λ_0=0.001, λ_1=0.003, λ_2=0.004, λ_3=0.005, φ_1=0.009 and φ_2=0.008, φ_3=0.006 resulting in an average operating time *m*=324.39 hours. Function *h*(*t*) calculated numerically represented in Figure 14, converges to 1/*m* with very good accuracy (for Δ*t*=1 and cardinal extrapolator).

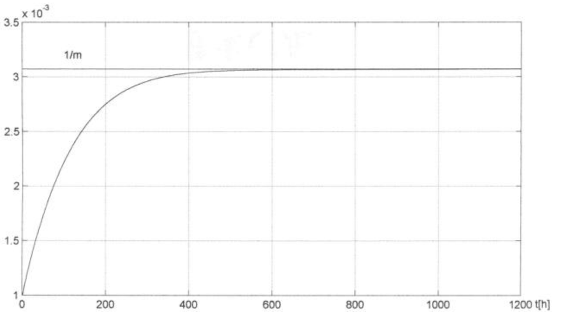

Fig. 14. Renewal density h(t), Δt=1, zero-order extrapolator

If one wants to reduce numerical errors the following solutions may be applied:

- Using a hybrid model: analytical for the Markov reliability model and numerical for renewal model.
- Using an efficient algorithm to discretize the renewal model. A solution could be to replace the relationship (23) by:

$$h[i] = \frac{1}{1 - f[0]\Delta t} \cdot \left\{ f[i] + \frac{\Delta t}{2}\left[h[0]f[i] + 2\sum_{m=1}^{i-2} h[m]f[i-m] + h[i-1]f[1] \right] \right\} \quad (30)$$

Figure 15 illustrates the relative errors for renewal density (curve 1) and the renewal function (curve 2).

$$e_h(t) = \left(h_a(t) - h_n(t)\right)/h_n(t) * 100 \quad [\%] \quad (31)$$

$$e_H(t) = \left(H_a(t) - H_n(t)\right)/H_n(t) * 100 \ [\%] \quad (32)$$

where $h_a(t)$ and $H_a(t)$ are calculated by a hybrid model (analytical Markov model and the renewal numerical number).

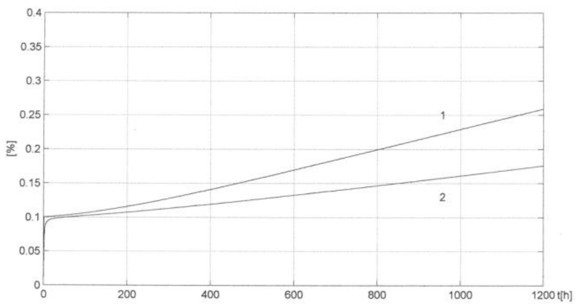

Fig. 15. 1- renewal density h(t), 2- renewal function H(t)

From Figure 15 it is observed that for time values less than the average operating time (m=324.39 hours) relative errors are insignificant (less than 0.15%), but comparable to those obtained with the numerical model, which shows that it is not reasonable to complicate the solution by the use of a hybrid model.

6. Conclusions

The analysis of the results shows that the proposed numerical procedures to calculate the density and renewal functions can be successfully used in the procedure for determining the optimal period of renewal. Errors made in determining these parameters are negligible in all cases analyzed, for time values less than the average operating time. In addition, using a hybrid model (continuous for the process model and sampled for the renewal model) does not bring a dramatic reduction of error, so it is favourable the numerical evaluation of both sets of parameters.

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