ON THE POTENTIAL OF USING FRACTIONAL-ORDER SYSTEMS TO MODEL THE RESPIRATORY IMPEDANCE

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Abstract: This contribution provides an analysis of the human respiratory system in frequency domain by means of estimating the respiratory impedance. Further on, analysis of several models for human respiratory impedance is done, leading to the conclusion that a fractional model gives a better description of the impedance than the classical theory of integer-order systems. A mathematical analysis follows, starting from the conclusions obtained heuristically. Correlation to the physiological characteristics of the respiratory system is discussed.

Keywords: modeling, respiratory system, non-parametric modeling, parametric modeling, Laplace operator, fractional derivatives, fractional integrals.

1. INTRODUCTION

Several parametric models describing the mechanics of human respiratory system have been discussed in the specialized literature (Farré *et al.*, 1989). However, none of them proved to be fully correlated to the anatomical structure of the respiratory tree. Although a physiological interpretation may have been given to some parameters, the models seem to provide un-satisfactory results from the medical point of view. The respiratory system characteristics were given in terms of resistance, elasticity and compliance properties of the airways and lung tissue, or comprise more specific information such as radial distance of airways or length. Such a characterisation of human respiratory system is interesting for assessing clinical diagnosis or helping locate the respiratory disorders (e.g. in main or secondary airways, in tissue, in alveoli). Nevertheless, amongst the models existing in the literature, none of them seemed to have been employed in standard clinical diagnosis using forced oscillation technique.

The present contribution is a step towards providing a solution to this open-problem by modeling the respiratory system as an approximation to a tree of (distributed) pipelines. The idea originating this investigation lies within the field of fractional-order systems. The latter have been proved to be able to explain surprisingly well the spectral behaviour of biological signals such as EEG, speech and ocular nerv (Ortigueira and Machado, 2006).

A brief description of respiratory signals analysis and estimation of respiratory impedance is given in the next section. A short analysis of classical integer- and fractional-order models is provided in section 3, followed by a mathematical analysis in section 4. A discussion upon the results and correlation to the anatomical structure of the respiratory system is detailed in section 5. Some conclusions and future directions summarize the main outcome of this contribution.

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2. ESTIMATING RESPIRATORY IMPEDANCE

Measuring the human respiratory impedance (Z_r) measurements using forced oscillation technique (FOT) (Oostveen *et al.*, 2003) is an attractive method to explore lung and chest mechanics. It is fast, non-invasive, and, requires but passive cooperation from the subject. The amount of information that may be extracted from the data is essentially the function of the experimental conditions and of the frequency range over which the measurements are done.

Typically, pressure and flow are measured at the mouth using the device schematically represented in figure 1. In this figure, the following notations apply: DSP – digital signal processing board; LS - loudspeaker; BT - bias-tube; BF - bias-flow; PN - pneumotachograph; bf - biological filter; A - amplifier; PT - pressure transducer; Q - flow and P – pressure, U_g – the generated input; U_r – the breathing signal; GUI – graphical user interface.

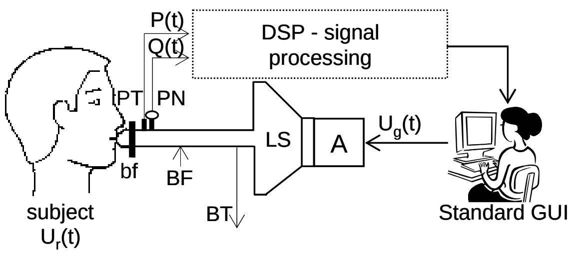

Fig. 1. Standard FOT set-up.

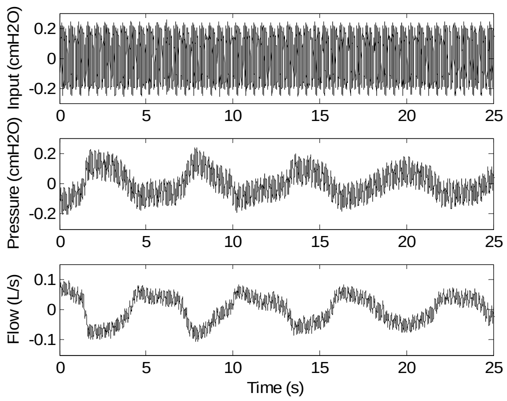

Fig. 2. Measured input and output time signals.

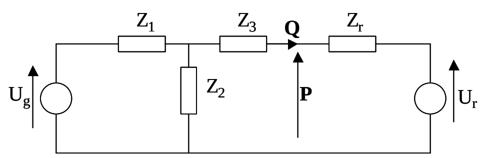

Fig. 3. Electrical scheme analogy.

This conventional / commercially available FOT set-up is based on superimposing a low-amplitude pressure oscillation at the mouth (in a range of frequencies from 4-48Hz) while the patient is breathing spontaneously. The oscillation pressure is generated by a loudspeaker (LS) fed with the oscillating signal generated by a computer. The movement of the loudspeaker cone generates a pressure oscillation inside the chamber, which is applied to the patient's respiratory system by means of a tube connecting the loudspeaker chamber and the mouthpiece or a biological filter (bf). As the patient breathes spontaneously through a bias tube (BT) - ideally presenting low impedance to the breathing frequency and high impedance to the forced oscillation frequency - a constant bias flow (BF) avoids re-breathing. During the measurements, the patient wears a nose-clip to avoid loss of air flow and keeps the cheeks firmly supported to avoid biased impedance values.

From the measured pressure and flow signals (see figure 2), the respiratory impedance can be extracted. Consider the equivalent circuit for the global setup, denoted by figure 3, with the notations as: U_g = generator test signal – driving signal (measured); U_r = effect of spontaneous breathing (respiratory system / unknown); Z_r = impedance of interest (to be estimated): the impedance of the total respiratory system (including the airways, lung tissues and chest wall); Z_1 = impedance (unknown) describing the transformation of driving voltage (U_g) to chamber pressure; Z_2 = impedance (unknown) of both bias tubes and loud-speaker chamber; Z_3 = impedance (unknown) of tube segment between bias tube and mouth piece (effect of pneumotachograph essentially); P = (measured) pressure; Q = (measured) flow. The corresponding equation is:

$$P(s) = Z_r(s)Q(s) + U_r(s) \qquad (1)$$

where s denotes the Laplace operator. If the excitation signal is taken into account and correlation analysis applied to the measured signals, one obtains that:

$$\mathrm{E}\left\{P(t)U_g(t+\tau)\right\} = Z_r \cdot \mathrm{E}\left\{Q(t)U_g(t+\tau)\right\} + \mathrm{E}\left\{U_r(t)U_g(t+\tau)\right\} \qquad (2)$$

where $\mathrm{E}\{*\}$ denotes the stochastic expectation operator. From the point of view of the forced oscillatory experiment, the signal components of respiratory origin, (U_r) have to be regarded as pure noise for the identification task. Considering the test signal U_g designed such that it is uncorrelated with the normal respiratory breathing signal U_r, then

$$\mathrm{E}\left\{U_r(t)U_g(t+\tau)\right\} \equiv 0 \qquad (3)$$

Applying Fast Fourier Transform we obtain the corresponding spectral power density functions and the respiratory impedance is given by the ratio:

$$Z_r(j\omega) = \frac{S_{PU_g}(j\omega)}{S_{QU_g}(j\omega)} \quad (4)$$

The impedance is therefore a complex variable, defined by a real and imaginary part. The typical characteristics are depicted for a healthy and an asthmatic person in figure 4.

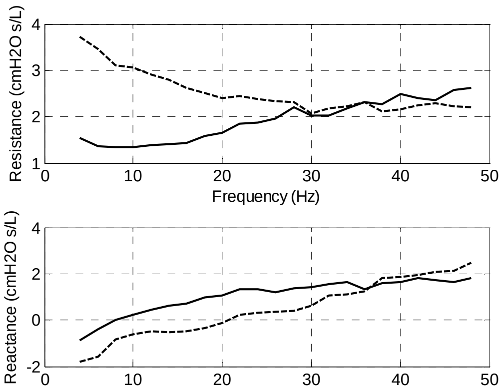

Fig. 4. Respiratory impedance: healthy (continuous line) and asthmatic (dashed line) subject.

The key concept of the forced oscillatory respiratory mechanics (Oostveen *et al.*, 2003) is the impedance (Z_r), the spectral (frequency domain) relationship between pressure (P) and airflow (Q). Since it embodies both the in-phase and out-of-phase relationships between P and Q, Z_r can be conceived as a generalisation of resistance. The in-phase component is called the real part of Z_r (or Resistance), whereas the out-of-phase relationship is expressed by the imaginary part (or Reactance), and both appear as functions of the frequencies of oscillation (*f*). In other words, the real part describes the dissipative mechanical properties of the respiratory system, whereas the imaginary part is related to the energy storage capacity.

3. MODELS OF RESPIRATORY SYSTEM

In this section, a comparison of classical integer-order systems and a fractional-order system is made. The models aim to provide a good fit to the frequency-dependent impedances from figure 4.

3.1. Integer-order models.

When both pressure is varied and flow measured at the airway opening (input impedance), the impedance data is best interpreted with a simple resistance-inertance-compliance (RLC) model, as in figure 5.

The equivalent equation of this circuit is given by:

$$Z(j\omega) = R + L(j\omega) + \frac{1}{C(j\omega)} \quad (5)$$

with $\omega = 2\pi f$ and *f* – the frequency. It is clear that such an approximation will not be able to characterise the frequency-dependence of the real part in the impedance shown in figure 4. Intuitively, a better model would be the one that contains a real part which is dependent of frequency. In Farré et al. 1989, such a model with a frequency-dependent airway resistance (R_{aw}) has been discussed. This model is a seven-coefficient model and therefore would not be useful when comparing to (5). Therefore, preserving the idea of a frequency-dependent resistance, a variation of it has been considered as being:

$$Z(j\omega) = R_{aw} + L(j\omega) + \frac{1}{C(j\omega)}, R_{aw} = R_o + A\omega \quad (6)$$

The results of these two models are depicted in figure 6. The RLC model from (5) is not able to characterise the frequency-dependance of the resistance. Adding a frequency-dependent resistance in (6) proved to be neccessary in order to obtain a good fit. A physiological interpretation of the *A* parameter could be that it stands for all the effects which could not be captured in R_0, with R_0 denoting the total resistance of the airways.

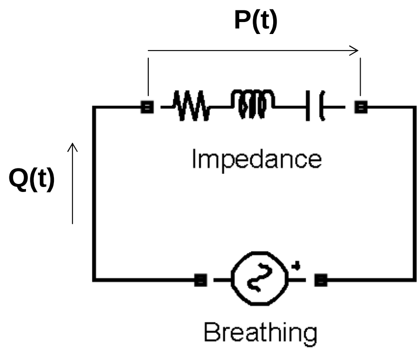

Fig. 5. Schematic representation of an RLC circuit.

3.2. A Fractional-order model.

Furthermore, a fractional-order model has been considered to model the impedance of the respiratory system.

A fractional model is based on concepts from the theory of Fractional Calculus. The non-integer order operators can describe dynamical behaviour of materials and processes over vast time and frequency scales.

There is a long and growing list of practical applications for the increased power of the fractional calculus. These include thermal engineering, acoustics, electromagnetism, robotics, viscoelasticity, diffusion, edge detection, turbulence and many other physical processes. There is evidence that most biological signals have spectra that do not increase or decrease by multiples of 20dB/dec. This happens, for example, with ECG, speech, music, etc. The electric line is a channel with such characteristics, conceptually modelled as transmission line (Machado and Jesus, 2005).

The choice of a fractional-order model lies within the fact that the respiratory system contains visco-elastic elements and can be equivalated to a pipeline tree or distributed transmission lines. Such a model could then be:

$$Z(j\omega)=R+L\cdot(j\omega)^{\alpha}+\frac{1}{C\cdot(j\omega)^{\beta}} \qquad (7)$$

with $0<\alpha\leq 1$ and $0<\beta\leq 1$. Using the definition of complex numbers, (7) becomes:

$$\begin{aligned}Z(j\omega)=&R+L\omega^{\alpha}\cos(\frac{\pi}{2}\cdot\alpha)+\frac{1}{C\omega^{\beta}}\cos(\frac{\pi}{2}\cdot\beta)+\\ &j[L\omega^{\alpha}\sin(\frac{\pi}{2}\cdot\alpha)-\frac{1}{C\omega^{\beta}}\sin(\frac{\pi}{2}\cdot\beta)]\end{aligned} \qquad (8)$$

Now it is possible to see that the real part of the impedance will depend on frequency and will comprise elastance and compliance effects. Further on, using a nonlinear least squares algorithm, the solution is a set of (*R*,*L*,*C*,*α*,*β*) parameters and the fitting to the impedance is shown in figure 6. The estimated respiratory impedance is compared between the non-parametric model from (4), the integer-order RLC model (5), integer-order *RLC-A* model (6) and the fractional-order *RLCαβ* model (7). Model (5) and model (6) give the same result for the imaginary part in figure 6 and for the real part in figure 7. As expected, the real part of the impedance must comprise the frequency-dependent effects from the elastance and compliance in order to obtain a good fit.

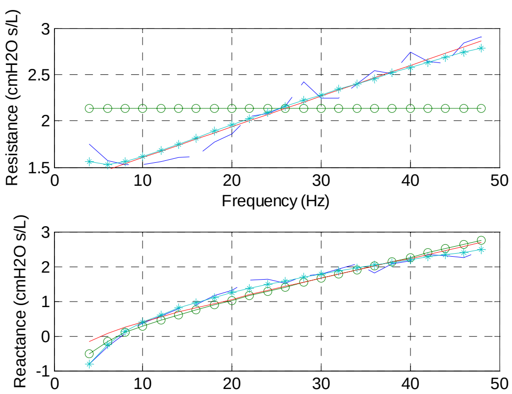

Fig. 6. Healthy subject case. (--): non-parametric model from (4); (o): the integer-order RLC model (5); (-): integer-order *RLC-A* model (6); (*) the fractional-order *RLCαβ* model (7).

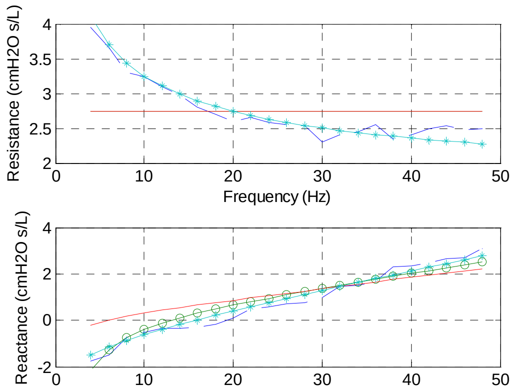

Fig. 7. Asthmatic subject case. (--): non-parametric model from (4); (o): the integer-order RLC model (5); (-): integer-order *RLC-A* model (6); (*) the fractional-order *RLCαβ* model (7).

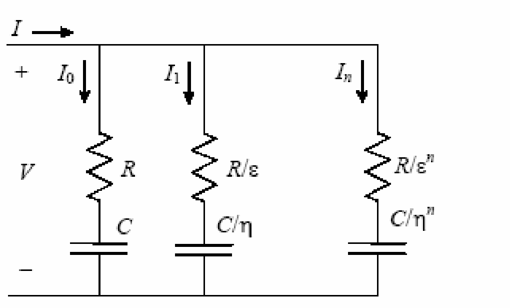

Fig. 8. Recursive circuit scheme.

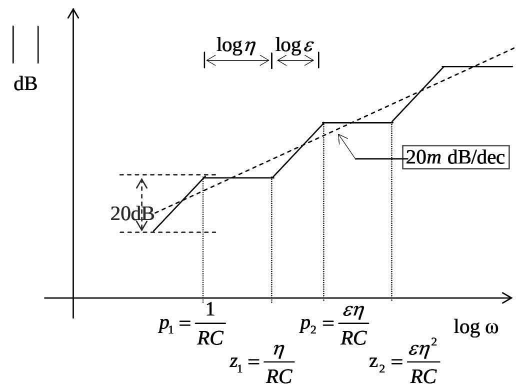

Fig. 9. Pole-zero location for the admitance.

4. A MATHEMATICAL INTERPRETATION

One way to interpret a *fractional-order derivative* is to consider it as a recursive circuit (Machado *et al.*, 2006) represented in figure 8, such that:

$$I=\sum_{i=0}^{n}I_i,\quad R_{i+1}=\frac{R_i}{\varepsilon},\quad C_{i+1}=\frac{C_i}{\eta} \qquad (9)$$

where ε and η are scale factors, I is the current due to an applied voltage V and R_i and C_i are the resistance and capacitance elements of the i^{th} branch of the circuit. The admitance is given by:

$$Y(j\omega) = \frac{I(j\omega)}{V(j\omega)} = \sum_{i=0}^{n} \frac{j\omega C\varepsilon^i}{j\omega CR + (\eta\varepsilon)^i} \quad (10)$$

where the location of the pole-zero (p,z) fulfill the follwing relationship:

$$\frac{z_{i+1}}{z_i} = \frac{p_{i+1}}{p_i} = \varepsilon\eta \quad (11)$$

Figure 9 shows the magnitude - Bode diagram corresponding to (10) whereas the average slope is a fraction of the 20dB/dec:

$$m = \frac{\log\varepsilon}{\log\varepsilon + \log\eta} \quad (12)$$

Equivalently, when calculating the impedance, the slope of the Bode plot in figure 9 will change in sign (decrease). Based on the relations between pole-zero in the system (11) it results that:

$$p_i = \frac{(\varepsilon\eta)^{i-1}}{RC} \text{ and } z_i = \frac{\varepsilon^{i-1}\eta^i}{RC} \quad (13)$$

with $i=0.....n$ and n the number of ladders. In order to have a good frequency resolution, we consider n ladders of the model (recall figure 8) between the range of frequencies of interest. Define as ω_c the cut-off frequency (by definition the maximum frequency) and by $\omega_{\min}$ the minimum value of frequency, in our case 48 Hz and 4 Hz respectively. Taking into account that the width of the ladder is given by

$$\log\varepsilon + \log\eta = \frac{\omega_c - \omega_{\min}}{n} \quad (14)$$

and using relation (12), results that

$$\log\varepsilon = m\frac{\omega_c - \omega_{\min}}{n} \quad (15)$$

Having the identified slope of the derivative term (7) as $m = \alpha$, the value of ε can be extracted as:

$$\varepsilon = 10^{\alpha\frac{\omega_c - \omega_{\min}}{n}} \quad (16)$$

From (14) and (16) we can obtain η. Since the first pole of the model will be at the location $\omega_{\min} = \frac{1}{RC}$, we have to determine the values for R and C. Supposing that the $R = 100R_{id}$ with R_{id} the value of resistance identified in (7), the value of C can be calculated as:

$$C = \frac{1}{\omega_{\min} R} \quad (17)$$

Now all necessary parameters are available. Notice that this procedure is defined for the calculus of admitance, therefore in case of equation (7) – the impedance – changing sign in figure 9 will result in a first point in frequency-domain being the value of a zero (instead of a pole). In order to do this, the capacitor must be replaced by an inductor.

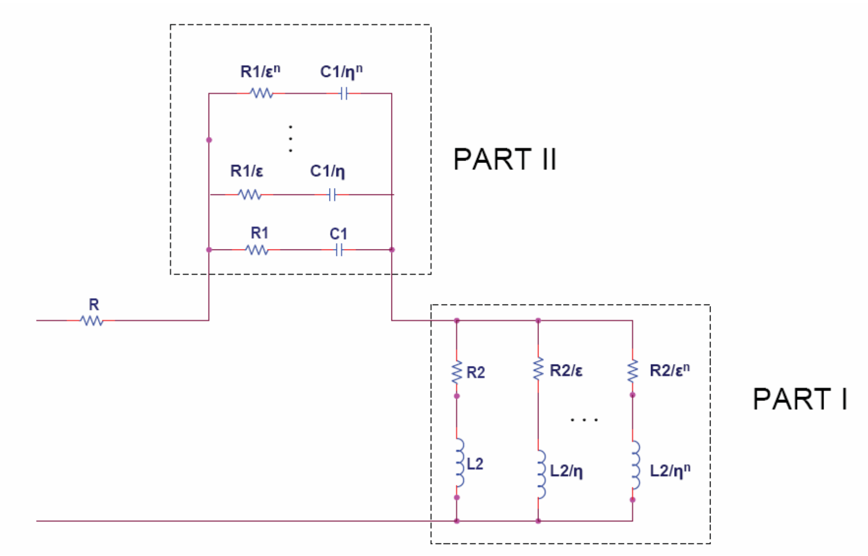

Fig. 10. The corresponding implementation of the fractional-order model is an equivalent higher order circuit.

Adding the inductor identified in (7) will result in the circuit equivalence denoted in figure 10 by part I and this concludes the implementation for the fractional-order derivative term.

Similarly, the implementation of the fractional integral in (7) is done using the β value of the fractional-order. Changing in figure 8 the first-point of the frequency-domain from a pole to a zero is equivalent to replacing the inductors by capacitors, as in figure 10 part II. Adding the identified compliance in (7) will complete the implementation.

5. DISCUSSION

The above described mathematical analysis corresponds to an approximation of the respiratory system as a pipeline model. This is correlated with the physiological properties of airways, that is, the air is circulating in and out of them as through a set of pipelines. Of course, the assumption that the respiratory tree can be approximated by a lumped parametric model of a pipeline is not fully correct.

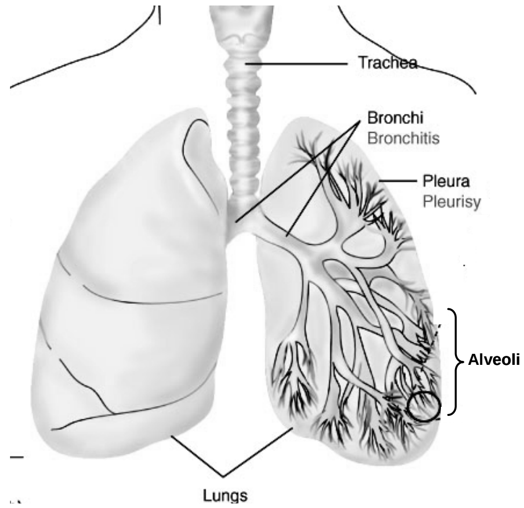

Fig. 11. The respiratory airways structure.

The anatomical structure of the respiratory tree corresponds to a length of about 50cm of distributed airways from the upper aiways (trachea) until the lower airways (alveoli), as depicted by figure 11.

A more realistic model in (7) would then correspond to a set of distributed pipelines in a similar structure as the real respiratory system. The proof that such a fractional-model can represent a bifurcation of pipelines is undergoing research. According to figure 11, this first bifurcation occurs when the air passes from trachea to the bronchi. Further-on, a generalization to a higher number of bifurcations (alveoli) would then complete the mathematical analysis of such a fractional-order model.

The final aim is to correlate the parameters of the model in (7) to the length and diameter of the airways. In this manner, the clinicians would have an insight view upon the mechanical parameters of the airways and tissue, such as resistance, inertance and compliance, all depending on the length and diameter of the airways.

6. CONCLUSION

A model of fractional-order has been compared to models of integer-order and used for characterising the human respiratory impedance. A preliminary mathematical analysis has been employed, suggesting that the fractional-order model corresponds to a model of a recursive circuit. Such circuits have been employed in modeling some distributed systems such as transmission lines. By approximating a network of transmission lines as a network of pipelines, a correlation could be possible between ladder networks and pipeline networks. Such a pipeline model would then be closer to the physical interpretation of the airways. Therefore, a next step is to check the correlation between fractional-order models and a set of pipelines similar to the anatomical structure of the respiratory system.

7. ACKNOWLEDGEMENTS

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8. REFERENCES

Farré, R., Peslin, R., Oostveen, E., Suki, B., Duvivier, C., Navajas, D., (1989). Human respiratory impedance from 8 to 256 Hz corrected for upper airway shunt, *American Journal of Applied Physiology*, **67**, 1973-1981

Ljung, L. (1999). *System Identification*,NJ: Prentice Hall.

Machado, J., Jesus, I., (2005). Fractional order dynamics in some distributed parameter systems, *Proc. IASTED Conf on Modeling, Identification and Control*, Innsbruck, Austria, 29-34

Machado, J., Jesus, I., Galhano, A., Cunha, B., (2006). Fractional order electromagnetics, *Signal Processing*, **86**, 2637-2644

Oostveen, E., Macleod, D., Lorino, H., Farré, R., Hantos, Z., Desager, K., Marchal, F., (2003). The forced oscillation technique in clinical practice: methodology, recommendations and future developments, *Eur Respir J*, **22**, 1026-1041

Ortigueira, M., Machado, J., (2006). Editorial. Special Issue on Fractional calculus applications in signals and systems, *Signal Processing*, **86**, 2503-2504