ROBOTS CONTROL BY ADAPTIVE GAIN SMOOTH SLIDING OBSERVER-CONTROLLER AND PARAMETER IDENTIFICATION

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Abstract: An adaptive gain, smooth sliding observer-controller are developed to control uncertain parameters, n-degree of freedom rigid robotic manipulators. Furthermore, an on-line, closed loop identification scheme, for time-varying parameters is proposed in order to obtain useful information despite loads, external disturbances and faults detection. In order to reduce the chattering, a smooth switching function (parameterised tangent hyperbolic function) is used instead of pure relay one, into the observer and the controller. The gains of the switching functions are adaptively updated, depending on the estimation error and tracking error, respectively. By using adaptive gains, the transient and tracking responses are improved. Simulation results with a two degree of freedom (DOF) robot manipulator are presented to show the interest of the approach.

Keywords: Robotic manipulators, adaptive gain, smooth sliding observer-controller, parameter identification.

1. INTRODUCTION

The state and parameter uncertainties in the model of the rigid robotic manipulators, considered as MIMO non-linear systems, as well as the deviations of the parameters from their nominal values and external disturbances lead to some problems in parameter identification and state estimation. All that makes absolutely necessary the design of the controller and/or the observer such as the closed loop robustness is achieved, stability with small tracking and estimation errors. It is well known that the robustness to model parameter uncertainties and external disturbances of the closed loop can be achieved with a variable structure controller. Maintaining the system on a sliding surface, weakens the influence of the uncertainties in the closed loop performances and quickly leads to an equilibrium point. In Filipescu (2003), an adaptive variable structure control with parameterized tangent hyperbolic as a switching function (denoted k-tanh) with adaptive modification of its magnitude (denoted as λ-modification) is used, instead of a pure relay one with constant gain. In this paper the parameterized tangent hyperbolic function is used as switching function in order to alleviate, or/and eliminate chattering. Decreasing the parameter k in the switching function makes the gain around zero smaller and the un-modelled dynamics are excited in a smaller measure in high frequency. Also, the delay due to the control input calculus and the finite rate of switching can lead to chattering. Using the λ-modification into the gain of k-tanh switching function, smoothes the response and increases the robustness to structural uncertainties. The adaptive gain is time depending, with the norm of the corresponding sliding surface, as input. Based on a time-varying parameters identification technique presented in Xu and Hashimoto (1993), Xu and Hashimoto (1996) and Xu, Pan and Lee (2003), we extend the scheme, by introducing, the observer, smooth switching function and adaptive gains. It is then applied to a general model of the robotic manipulator dynamics. The physical robot may have inside the joint, gears and clutches, through the torque supplied by the DC motor is transmitted in order to move the link. For this reason, the general model of the robotics manipulator is involved. We develop a variable structure observer-controller based

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on the work of Sanchis and Nijmeijer (1998). Extensions of sliding control to MIMO non-linear uncertain systems have been made in Khalil (1996) and Utkin (1992). Several applications of the variable structure control to robotic manipulator controlling point out the robustness to uncertainties and external disturbances of the closed loop (Slotine & Sastry, 1983; Canudas de Wit & Slotine1991). With the k-tanh switching function and the λ-modification in the observer-controller gains, the closed loop behaves like an approximate sliding mode, in the neighbourhood of the corresponding sliding surface.

The main contributions of this paper are concerned with: the adaptive smooth sliding observer-controller, the updating law of the variable structure gains, and finally the identification of the time-varying parameters and external disturbances.

The paper is organized as follows. In the Section 2, a general model for the n-degree of freedom robot manipulator and the sliding observer are presented. The smooth sliding observer is designed, the gain updating law is presented and a bound for the estimate error is computed. The design of the adaptive gain smooth sliding controller is performed in Section 3. An upper bound of the tracking error is provided, too. In Section 4, a stable identification scheme of time-varying parameters and external disturbances applied to a n-DOF robotic manipulator is presented. A 2-DOF vertical robotic manipulator, together with closed loop simulation results are presented in the Section5. Some conclusion remarks can be found in Section 6.

2. ADAPTIVE GAIN SMOOTH SLIDING OBSERVER

A very general model of the robotic manipulator can be expressed as a square non-linear MIMO model

$$\text{(1)}\quad \begin{aligned} &\dot{\mathbf{x}}_1 = \mathbf{x}_2, \quad \mathbf{x}_1 \in \Re^n, \\ &\dot{\mathbf{x}}_2 = \mathbf{h}(\mathbf{x}_1,\mathbf{p})^{-1}[\mathbf{f}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{p}) + \mathbf{g}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{p})\mathbf{u}]\ \mathbf{x}_2 \in \Re^n\ \mathbf{u} \in \Re^n, \\ &\mathbf{y} = \mathbf{x}_1, \quad \mathbf{p} \in \Re^{n_p}, \end{aligned}$$

where only the vector $\mathbf{x}_1$ is available for measurement, $\mathbf{u}$ and $\mathbf{y}$ are control input and measured output, respectively. The state space dimension is $2n$ and $\mathbf{x} = [\mathbf{x}_1^T \quad \mathbf{x}_2^T]^T \in \Re^{2n}$ is the state vector. The unknown time-varying parameter vector $\mathbf{p} \in \Re^{n_p}$ is supposed to be bounded. The matrices $\mathbf{f}$, $\mathbf{g}$ and $\mathbf{h}$ may be partially unknown, with some parameter uncertainties. If one assumes the partial knowledge of the model parameters, state estimates, time-varying parameters and disturbances, then one can define $\hat{\mathbf{f}} = \mathbf{f}(\mathbf{x}_1, \hat{\mathbf{x}}_2, \hat{\mathbf{p}})$, $\hat{\mathbf{g}} = \mathbf{g}(\mathbf{x}_1, \hat{\mathbf{x}}_2, \hat{\mathbf{p}})$ and $\hat{\mathbf{h}} = \mathbf{h}(\mathbf{x}_1, \hat{\mathbf{p}})$ as the estimates of the functions $\mathbf{f}$, $\mathbf{g}$ and $\mathbf{h}$. Moreover, if the matrices $\mathbf{g}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{p})$ and $\hat{\mathbf{g}}(\mathbf{x}_1, \hat{\mathbf{x}}_2, \hat{\mathbf{p}})$ are nonsingular for all $\mathbf{x}, \hat{\mathbf{x}}, \mathbf{p}, \hat{\mathbf{p}}$, then the system may be linearized via state feedback .

Let choose as the observer sliding surface $\mathbf{S}_o = \hat{\mathbf{x}}_1 - \mathbf{x}_1 = \mathbf{0}_n$. The observer can be written as

$$\text{(2)}\quad \begin{aligned} &\dot{\hat{\mathbf{x}}}_1 = -\mathbf{\Gamma}_1(\hat{\mathbf{x}}_1 - \mathbf{x}_1) + \mathbf{\Theta}_1(t)\tanh(k_o\mathbf{S}_o) + \hat{\mathbf{x}}_2, \\ &\dot{\hat{\mathbf{x}}}_2 = -\mathbf{\Gamma}_2(\hat{\mathbf{x}}_1 - \mathbf{x}_1) + \mathbf{\Theta}_2(t)\tanh(k_o\mathbf{S}_o) \\ &\quad + \hat{\mathbf{h}}(\mathbf{x}_1, \hat{\mathbf{p}})^{-1}[\hat{\mathbf{f}}(\mathbf{x}_1, \hat{\mathbf{x}}_2, \hat{\mathbf{p}}) + \hat{\mathbf{g}}(\mathbf{x}_1, \hat{\mathbf{x}}_2, \hat{\mathbf{p}})\mathbf{u}(\mathbf{x}_1, \hat{\mathbf{x}}_2, \hat{\mathbf{p}})], \end{aligned}$$

where $\mathbf{\Gamma}_1 = \text{diag}[\gamma_{11} \cdots \gamma_{1n}]$, $\mathbf{\Gamma}_2 = \text{diag}[\gamma_{21} \cdots \gamma_{2n}]$ with $\gamma_{ij} > 0, i = 1,2$ and $j = 1,n$, $k_o > 0$ being a design parameter. The gains $\mathbf{\Theta}_1 = \text{diag}[\theta_{11} \cdots \theta_{1n}]$, $\mathbf{\Theta}_2 = \text{diag}[\theta_{21} \cdots \theta_{21}]$, are time-varying and defined by (λ-modification is included)

$$\text{(3)}\quad \dot{\mathbf{\Theta}}_1(t) = -\boldsymbol{\lambda}_1\mathbf{\Theta}_1(t) - \boldsymbol{\rho}_1\text{diag}[|\hat{x}_{11} - x_{11}| \cdots |\hat{x}_{1n} - x_{1n}|],$$

$$\text{(4)}\quad \dot{\mathbf{\Theta}}_2(t) = -\boldsymbol{\lambda}_2\mathbf{\Theta}_2(t) - \boldsymbol{\rho}_2\text{diag}[|\hat{x}_{11} - x_{11}| \cdots |\hat{x}_{1n} - x_{1n}|],$$

where $\boldsymbol{\lambda}_1 = \text{diag}[\lambda_{11} \cdots \lambda_{1n}]$, $\boldsymbol{\lambda}_2 = \text{diag}[\lambda_{21} \cdots \lambda_{2n}]$, $\boldsymbol{\rho}_1 = \text{diag}[\rho_{11} \cdots \rho_{1n}]$, $\boldsymbol{\rho}_2 = \text{diag}[\rho_{21} \cdots \rho_{2n}]$, with λ_{1i}, λ_{2i}, ρ_{1i}, ρ_{2i}, $i = 1, \cdots, n$ positive constants.

Remark 1. The dynamics (3) and (4) of the switching function force the matrices $\mathbf{\Theta}_1$ and $\mathbf{\Theta}_2$ to the negative values. They are almost zero when the observer is in the neighbourhood of sliding surface. In order to satisfy the attractiveness condition $\dot{S}_{oi}S_{oi} < 0, \quad i = 1,\dots,n$, the gain $\mathbf{\Theta}_1$ must be chosen such that

$$\text{(5)}\quad -\theta_{1i}(t) > |\hat{x}_{2i}(t) - x_{2i}(t)|, i = 1,\dots,n, \forall t \in [0 \quad \infty).$$

By an appropriate choice of the matrices λ_1 and ρ_1, the above condition at $t = 0$ remains satisfied for any $t > 0$.
If the active torque delivered by the joint DC-motor is considered as the control input, the model of the n-DOF robotic manipulator is

$$\text{(6)}\quad \begin{aligned} &\mathbf{H}(\mathbf{q}, m_p)\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}, m_p)\dot{\mathbf{q}} + \mathbf{F}\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}, m_p) \\ &= \mathbf{u} + \mathbf{d}, \end{aligned}$$

where $\mathbf{q} = [q_1 \quad \dots \quad q_n]^T$ is the vector of link positions, $\mathbf{H}(\mathbf{q}, m_P) \in \Re^{nxn}$ is the positive definite inertia matrix, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}, m_p) \in \Re^{nxn}$ is the Coriolis and centripetal force matrix, $\mathbf{F} \in \Re^{nxn}$ is a positive semi-definite diagonal matrix with the viscous friction coefficients, $\mathbf{u} \in \Re^n$ is the vector of driving torques.

Define the unknown time-varying parameter vector $\mathbf{p}(t)=\left[m_p(t),\quad \mathbf{d}^T(t)\right]^T \in \Re^{n_p}$, where $m_p(t)$ is the payload and $\mathbf{d}(t)$ is an additive input disturbance. Let $\mathbf{q}=\mathbf{x}_1=\left[x_{11}\dots \quad x_{1n}\right]^T$, $\dot{\mathbf{q}}=\mathbf{x}_2=\left[x_{21}\dots \quad x_{2n}\right]^T$ be the angular positions and velocity vectors, respectively. The measurements only concern the link positions $\mathbf{y}=\mathbf{x}_1$. The robot state space representation can be written as

$$(7)\quad \begin{aligned}&\dot{\mathbf{x}}_1=\mathbf{x}_2\\ &\dot{\mathbf{x}}_2=-\mathbf{H}(\mathbf{x}_1,\mathrm{m_p})^{-1}\begin{bmatrix}\mathbf{C}(\mathbf{x}_1,\mathbf{x}_2,\mathrm{m_p})\mathbf{x}_2\\ +\mathbf{G}(\mathbf{x}_1,\mathrm{m_p})+\mathbf{F}\mathbf{x}_2-\mathbf{u}-\mathbf{d}\end{bmatrix}.\end{aligned}$$

Taking into account the uncertainties, one can define:

$$(8)\quad \hat{\mathbf{H}}(\mathbf{x}_1,\hat{m}_p)=\hat{\mathbf{H}}_1(\mathbf{x}_1)+\hat{\mathbf{H}}_2(\mathbf{x}_1)\hat{m}_p\ ,$$

$$(9)\quad \hat{\mathbf{C}}(\mathbf{x}_1,\hat{\mathbf{x}}_2,\hat{m}_p)=\hat{\mathbf{C}}_1(\mathbf{x}_1,\hat{\mathbf{x}}_2)+\hat{\mathbf{C}}_2(\mathbf{x}_1,\hat{\mathbf{x}}_2)\hat{m}_P\ ,$$

$$(10)\quad \hat{\mathbf{G}}(\mathbf{x}_1,\hat{m}_p)=\hat{\mathbf{G}}_1(\mathbf{x}_1)+\hat{\mathbf{G}}_2(\mathbf{x}_1)\hat{m}_p\ ,$$

as the estimates of function matrices: $\mathbf{H}(\mathbf{x}_1,m_p),\mathbf{C}(\mathbf{x}_1,\mathbf{x}_2,m_p),\mathbf{G}(\mathbf{x}_1,m_p)$. Without loss of the generality, the friction is considered as a positive constant uncertain diagonal matrix $\hat{\mathbf{F}}$.

The following assumptions have to be considered
Assumption 1. The reference signals $y_{ri}(t)$ $i=1,\cdots,n$ are C^n functions;
Assumption 2. The matrices $\hat{\mathbf{H}}(\mathbf{x}_1,\hat{m}_p)$ and $\mathbf{H}(\mathbf{x}_1,m_p)$ are non-singular for all $\mathbf{x}_1,m_p,\hat{m}_p$;
Assumptions 3. The time-varying vector $\mathbf{p}(t)$ is bounded all the time.

With the above notations the model (6) can be rewritten as

$$(11)\quad \begin{aligned}&\begin{bmatrix}\mathbf{I}_\mathrm{n} & \mathbf{0}\\ \mathbf{0} & \mathbf{H}\end{bmatrix}\begin{bmatrix}\dot{\mathbf{x}}_1\\ \dot{\mathbf{x}}_2\end{bmatrix}=-\begin{bmatrix}\mathbf{0} & \mathbf{I}_\mathrm{n}\\ \mathbf{0} & \mathbf{C}\end{bmatrix}\begin{bmatrix}\mathbf{x}_1\\ \mathbf{x}_2\end{bmatrix}\\ &-\begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{F}\end{bmatrix}\begin{bmatrix}\mathbf{x}_1\\ \mathbf{x}_2\end{bmatrix}-\begin{bmatrix}\mathbf{0}\\ \mathbf{G}\end{bmatrix}+\begin{bmatrix}\mathbf{0}\\ \mathbf{u}+\mathbf{d}\end{bmatrix}.\end{aligned}$$

The smooth sliding observer (k-tanh as switching function), with gains adaptively updated (λ-modification is included, as in (3) and (4)), is given by the equations

$$(12)\quad \begin{aligned}&\dot{\hat{\mathbf{x}}}_1=-\mathbf{\Gamma}_1(\hat{\mathbf{x}}_1-\mathbf{x}_1)+\mathbf{\Theta}_1(\mathrm{t})\tanh(\mathrm{k_o}\mathbf{S}_\mathrm{o})+\hat{\mathbf{x}}_2\\ &\dot{\hat{\mathbf{x}}}_2=-\mathbf{\Gamma}_2(\hat{\mathbf{x}}_1-\mathbf{x}_1)+\mathbf{\Theta}_2(\mathrm{t})\tanh(\mathrm{k_o}\mathbf{S}_\mathrm{o})\\ &-\hat{\mathbf{H}}^{-1}\left[\hat{\mathbf{C}}\hat{\mathbf{x}}_2+\hat{\mathbf{F}}\hat{\mathbf{x}}_2+\hat{\mathbf{G}}-\hat{\mathbf{u}}\right].\end{aligned}$$

The smooth switching function allows to consider that the approximate conditions: $\mathbf{S}_o\approx 0,\quad \dot{\mathbf{S}}_o\approx 0$ are satisfied during sliding. $\tanh(k_o\mathbf{S}_o)$ can be expressed from the first equation of (12) and replaced in the second. Hence, the estimate error equation can be written as

$$(13)\quad \begin{aligned}&\dot{\tilde{\mathbf{x}}}_2=-\mathbf{\Theta}_2\mathbf{\Theta}_1^{-1}\tilde{\mathbf{x}}_2-\hat{\mathbf{H}}^{-1}\left[\hat{\mathbf{C}}\hat{\mathbf{x}}_2+\hat{\mathbf{F}}\hat{\mathbf{x}}_2+\hat{\mathbf{G}}-\hat{\mathbf{u}}\right]\\ &+\mathbf{H}^{-1}\left[\mathbf{C}\mathbf{x}_2+\mathbf{F}\mathbf{x}_2+\mathbf{G}-\mathbf{u}-\mathbf{d}\right]\end{aligned}.$$

Above equation assures the stability of the observer and exponential convergence rate as how is proofed in Sanchis and Nijmeijer (1998). Let $\mathbf{Q}\in\Re^{nxn}$ be the time varying positive definite matrix defined as

$$(14)\quad \mathbf{Q}(t)=\hat{\mathbf{H}}(\mathbf{x}_1,\hat{m}_p)\mathbf{\Theta}_2(t)\mathbf{\Theta}_1^{-1}(t)+\hat{\vartheta}(\mathbf{x}_1,\hat{\mathbf{x}}_2)+\hat{\mathbf{F}}\ ,$$

where

$$(15)\quad \vartheta(\mathbf{x}_1,\hat{\mathbf{x}}_2)=\frac{\partial}{\partial\mathbf{x}_2}\left[\mathbf{C}(\mathbf{x}_1,\mathbf{x}_2,m_p)\ \hat{\mathbf{x}}_2\right]\Big|_{\mathbf{x}_2=\hat{\mathbf{x}}_2}$$

$$(16)\quad \hat{\mathbf{C}}(\mathbf{x}_1,\hat{\mathbf{x}}_2,\hat{m}_p)\hat{\mathbf{x}}_2=\hat{\mathbf{C}}(\mathbf{x}_1,\mathbf{x}_2,\hat{m}_p)\hat{\mathbf{x}}_2+\hat{\vartheta}(\mathbf{x}_1,\hat{\mathbf{x}}_2)\tilde{\mathbf{x}}_2\ .$$

Choosing large eigenvalues of $\mathbf{Q}$, the observation error can be globally ultimately bounded (Corollary 5.3 from Khalil 1996). The matrix $\mathbf{Q}$ determines the robustness of the observer to the parameter uncertainties. Taking V_2 as a Lyapunov function candidate

$$(17)\quad V_2=\frac{1}{2}\tilde{\mathbf{x}}_2^T\hat{\mathbf{H}}(\mathbf{x}_1,\hat{m}_p)\tilde{x}_2\ ,$$

the derivative can be expressed as

$$(18)\quad \begin{aligned}&\dot{\mathrm{V}}_2=\tilde{\mathbf{x}}_2^\mathrm{T}\hat{\mathbf{H}}\dot{\tilde{\mathbf{x}}}_2+\frac{1}{2}\tilde{\mathbf{x}}_2^\mathrm{T}\dot{\hat{\mathbf{H}}}\tilde{\mathbf{x}}_2=-\tilde{\mathbf{x}}_2^\mathrm{T}\\ &\begin{Bmatrix}\hat{\mathbf{H}}\mathbf{\Theta}_2\mathbf{\Theta}_1^{-1}\tilde{\mathbf{x}}_2+\hat{\vartheta}\tilde{\mathbf{x}}_2+\hat{\mathbf{F}}\tilde{\mathbf{x}}_2-\tilde{\mathbf{G}}-\tilde{\mathbf{C}}\mathbf{x}_2\\ -\tilde{\mathbf{F}}\mathrm{x}_2+\hat{\mathbf{H}}\tilde{\mathbf{H}}^{-1}\left[\mathbf{G}+\mathbf{C}\mathbf{x}_2+\mathbf{F}\mathbf{x}_2-\hat{\mathbf{u}}-\mathbf{d}\right]\end{Bmatrix}\end{aligned}$$

using the robot equations property

$$(19)\quad \tilde{\mathbf{x}}_2^\mathrm{T}\left(\dot{\hat{\mathbf{H}}}/2-\hat{\mathbf{C}}(\mathbf{x}_1,\mathbf{x}_2)\right)\tilde{\mathbf{x}}_2=0,\quad \forall\tilde{\mathbf{x}}_2\in\Re^\mathrm{n}$$

and the notations:

$$(20)\quad \tilde{\mathbf{H}}^{-1}=\hat{\mathbf{H}}^{-1}-\mathbf{H}^{-1},$$

$$(21)\quad \tilde{\mathbf{C}}(\mathbf{x}_1,\mathbf{x}_2)=\hat{\mathbf{C}}(\mathbf{x}_1,\mathbf{x}_2,\hat{m}_p)-\mathbf{C}(\mathbf{x}_1,\mathbf{x}_2,m_p),$$

$$(22)\quad \tilde{\mathbf{G}}=\hat{\mathbf{G}}-\mathbf{G}\ ,\ \tilde{\mathbf{F}}=\hat{\mathbf{F}}-\mathbf{F}\ .$$

Let define the vector $\boldsymbol{\mu}=\boldsymbol{\mu}(\mathbf{x}_1,\mathbf{x}_2,\hat{\mathbf{x}}_2)$ as

$$(23)\quad \begin{aligned}&\boldsymbol{\mu}=-\tilde{\mathbf{G}}-\tilde{\mathbf{F}}\mathbf{x}_2-\tilde{\mathbf{C}}\mathbf{x}_2\\ &-\hat{\mathbf{H}}\tilde{\mathbf{H}}^{-1}\left[\mathbf{G}+\mathbf{C}\mathbf{x}_2-\hat{\mathbf{u}}-\mathbf{d}+\mathbf{F}\mathbf{x}_2\right]\end{aligned}$$

and assume that μ is linearly bounded by $\tilde{x}_2$:

$$\|\boldsymbol{\mu}\| \le \beta + \gamma \|\tilde{\mathbf{x}}_2\|, \quad \forall t > 0 . \tag{24}$$

for some $\beta, \gamma > 0$. The derivative of the Lyapunov function is bounded by

$$\begin{aligned} \dot{V}_2 &\le -\lambda_{\min \mathbf{Q}} \|\tilde{\mathbf{x}}_2\|^2 + \|\tilde{\mathbf{x}}_2\| \|\boldsymbol{\mu}\| \\ &\le \left(-\lambda_{\min \mathbf{Q}} + \gamma\right) \|\tilde{\mathbf{x}}_2\|^2 + \beta \|\tilde{\mathbf{x}}_2\| \le -\varepsilon \|\tilde{\mathbf{x}}_2\|^2 \end{aligned}, \tag{25}$$

where ε is a positive constant satisfying

$$\varepsilon \le \lambda_{\inf Q} - \gamma . \tag{26}$$

If at $t = 0$, the switching gain $\boldsymbol{\Theta}_1$ satisfies (5), both gains $\boldsymbol{\Theta}_1, \boldsymbol{\Theta}_2$ follow the adaptation laws (3) and (4), respectively, and the vector $\boldsymbol{\mu}$ is bounded, then there exists $t_1 \ge 0$ such that the velocity estimation error satisfies the inequality

$$\|\tilde{\mathbf{x}}_2(t)\| \le \sqrt{\frac{\lambda_{\max \hat{\mathbf{H}}}}{\lambda_{\min \hat{\mathbf{H}}}}} \|\tilde{\mathbf{x}}_2(0)\| e^{\frac{-\varepsilon}{2\lambda_{\max \hat{\mathbf{H}}}} t}, \quad \forall t < t_1 . \tag{27}$$

More, in finite time, the estimation error enters into the ball $B(0, r)$. That means

$$\|\tilde{\mathbf{x}}_2(t)\| \le \sqrt{\frac{\lambda_{\max \hat{\mathbf{H}}}}{\lambda_{\min \hat{\mathbf{H}}}}} \frac{\beta}{\lambda_{\min \mathbf{Q}} - \gamma - \varepsilon} \le r, \quad \forall t \ge t_1 , \tag{28}$$

where the ball radius satisfies the inequality

$$r \ge \sqrt{\frac{\lambda_{\max \hat{\mathbf{H}}}}{\lambda_{\min \hat{\mathbf{H}}}}} \frac{\beta}{\lambda_{\min \mathbf{Q}} - \gamma - \varepsilon} . \tag{29}$$

Remark 2. The adaptation law (3), starting from nonzero initial condition, assures the non-singularity of the gain matrix $\boldsymbol{\Theta}_1$ during sliding. Hence, the matrix $\mathbf{Q}$ can be computed all the time using the expression (14). The ultimate bound r satisfying (29) is smaller if $\lambda_{\min \mathbf{Q}}$ is greater, i.e., if the initial value of $\boldsymbol{\Theta}_1$ is chosen smaller than $\boldsymbol{\Theta}_2$.

3. ADAPTIVE GAIN SMOOTH SLIDING CONTROLLER

The controller is defined assuming only that the state $\mathbf{x}_1$ is known and that the state $\mathbf{x}_2$ is provided by the observer. Corresponding to the n-dimensional control input, the controller sliding surface is

$$\hat{\mathbf{S}}_c(\mathbf{x}_1, \hat{\mathbf{x}}_2) = \hat{\mathbf{x}}_2(t) - \dot{\mathbf{y}}_r(t) + \boldsymbol{\psi}(\mathbf{x}_1(t) - \mathbf{y}_r(t)), \tag{30}$$

where $\mathbf{y}_r(t)$ represents the trajectory to be tracked. The matrix $\boldsymbol{\psi} = \operatorname{diag}[\psi_1 \ldots \ \psi_n]$, with positive constants, ψ_i, $i = 1, \cdots, n$, determines the dynamics during sliding. The sliding surface is attractive if the following condition holds

$$\dot{\hat{S}}_{ci} \hat{S}_{ci} < 0, \quad i = 1, \ldots, n . \tag{31}$$

The time derivative of the sliding surface can be written as

$$\begin{aligned} \dot{\hat{\mathbf{S}}}_c &= \dot{\hat{\mathbf{x}}}_2 - \ddot{\mathbf{y}}_{1r} + \boldsymbol{\psi}(\hat{\mathbf{x}}_2 - \dot{\mathbf{y}}_{1r}) = \hat{\mathbf{h}}(\mathbf{x}_1, \hat{m}_p)^{-1} \\ &\left[\hat{\mathbf{f}}(\mathbf{x}_1, \hat{\mathbf{x}}_2, \hat{\mathbf{p}}) + \hat{\mathbf{g}}(\mathbf{x}_1, \hat{\mathbf{x}}_2, \hat{\mathbf{p}}) \hat{\mathbf{u}}(\mathbf{x}_1, \hat{\mathbf{x}}_2, \hat{\mathbf{p}})\right] - \ddot{\mathbf{y}}_r + \boldsymbol{\psi}(\hat{\mathbf{x}}_2 - \dot{\mathbf{y}}_r) \end{aligned} \tag{32}$$

If k-tanh is used as switching function and the diagonal matrix $\boldsymbol{\eta} = \operatorname{diag}[\eta_1 \cdots \eta_n]$ is taken time depending

$$\dot{\boldsymbol{\eta}}(t) = -\boldsymbol{\lambda}_c \boldsymbol{\eta}(t) - \boldsymbol{\rho}_c \operatorname{diag}\left[\left|\hat{S}_{c1}\right| \ \cdots \ \left|\hat{S}_{cn}\right|\right], \tag{33}$$

the controller which fulfils the sliding condition $\dot{\hat{\mathbf{S}}}_c = 0$ can be expressed as

$$\begin{aligned} \hat{\mathbf{u}} &= -\hat{\mathbf{f}}(\mathbf{x}_1, \hat{\mathbf{x}}_2, \hat{\mathbf{p}}) + \hat{\mathbf{g}}^{-1}(\mathbf{x}_1, \hat{\mathbf{x}}_2, \hat{\mathbf{p}}) \hat{\mathbf{h}}(\mathbf{x}_1, \hat{\mathbf{p}}) \\ &\left[-\boldsymbol{\psi} \hat{\mathbf{S}}_c + \boldsymbol{\eta}(t) \tanh(k_c \hat{\mathbf{S}}_c) + \ddot{\mathbf{y}}_r - \boldsymbol{\psi}(\hat{\mathbf{x}}_2 - \dot{\mathbf{y}}_r)\right] \end{aligned}, \tag{34}$$

where the matrices: $\boldsymbol{\lambda}_c = \operatorname{diag}[\lambda_{c1} \cdots \lambda_{cn}]$, $\boldsymbol{\rho}_c = \operatorname{diag}[\rho_{c1} \cdots \rho_{cn}]$ are positive definite. The term $-\boldsymbol{\psi} \hat{\mathbf{S}}_c$ is introduced in order to reduce the controller to classical feedback linearization one (Marino and Tomei 1995), if the switching term is set to zero.

Despite the calculus of the control input for n-DOF robotic manipulator, to fulfil the attractiveness condition (31), it is necessary to express the derivative of the sliding surface (30)

$$\begin{aligned} \dot{\hat{\mathbf{S}}}_c &= \dot{\hat{\mathbf{x}}}_2 - \ddot{\mathbf{y}}_r + \boldsymbol{\psi}(\hat{\mathbf{x}}_2 - \dot{\mathbf{y}}_r) = -\hat{\mathbf{H}}(\mathbf{x}_1, \hat{m}_p)^{-1} \\ &\left[\hat{\mathbf{C}}(\mathbf{x}_1, \hat{\mathbf{x}}_2, \hat{m}_p) \hat{\mathbf{x}}_2 + \mathbf{F} \hat{\mathbf{x}}_2 + \hat{\mathbf{G}}(\mathbf{x}_1, \hat{m}_p) - \hat{\mathbf{u}}\right] \\ &- \ddot{\mathbf{y}}_r + \boldsymbol{\psi}(\hat{\mathbf{x}}_2 - \dot{\mathbf{y}}_r) \end{aligned} . \tag{35}$$

Similarly as to the observer, using k-tanh as switching function and λ-modification into the gain, the sliding condition is fulfilled, if the control input is chosen as:

$$\begin{aligned}&\hat{\mathbf{u}} = \hat{\mathbf{C}}(\mathbf{x}_1, \hat{\mathbf{x}}_2, \hat{m}_p)\hat{\mathbf{x}}_2 + \hat{\mathbf{F}}\hat{\mathbf{x}}_2 + \hat{\mathbf{G}}(\mathbf{x}_1, \hat{m}_p) \\ &+ \hat{\mathbf{H}}(\mathbf{x}_1, \hat{m}_p)\begin{bmatrix} -\boldsymbol{\psi}\hat{\mathbf{S}}_c + \boldsymbol{\eta}(t)\tanh(k_c\hat{\mathbf{S}}_c) \\ + \ddot{\mathbf{y}}_r - \boldsymbol{\psi}(\hat{\mathbf{x}}_2 - \dot{\mathbf{y}}_r)\end{bmatrix}\end{aligned} \quad (36)$$

The controller switching gain $\boldsymbol{\eta}(t)$ is adaptively updated as in (33).

Remark 3.The observer error is nonzero if a k-tanh function is used as a switching function in the observer equations. The controller sliding surface $\hat{\mathbf{S}}_c$ can still be attractive by choosing sufficiently large initial values for the switching gains $\boldsymbol{\Theta}_1$ and $\boldsymbol{\Theta}_2$. Moreover, the tracking error does not go to zero on controller sliding surface, because the smooth controller is used (k-tanh switching function).

Remark 4. In order to reduce the influence of velocity estimation error in the control input, the relative weight of the states $\hat{\mathbf{x}}_2$ in the definition of the sliding surface should be decreased. This explains the introduction of the supplementary term $-\boldsymbol{\psi}\hat{\mathbf{S}}_c$ in the control input. The increasing of the parameter $\boldsymbol{\psi}$ is limited by the switching frequency and possible measurement noise.

Using (13), the derivative of the sliding surface (30) can be expressed as

$$\begin{aligned}&\dot{\hat{\mathbf{S}}}_c = \boldsymbol{\eta}(t)\tanh(k_c\hat{\mathbf{S}}_c) - \boldsymbol{\psi}\hat{\mathbf{S}}_c \\ &+ (\boldsymbol{\Theta}_2(t)\boldsymbol{\Theta}_1^{-1}(t) - \boldsymbol{\psi})\tilde{\mathbf{x}}_2\end{aligned} \quad (37)$$

If the gain $\boldsymbol{\eta}(t)$ fulfils the inequality

$$\begin{aligned}&\psi_i|\hat{S}_{ci}| - \eta_i(t) \\ &\geq \left|\left[(\Theta_2(t)\Theta_1^{-1}(t) - \psi)\tilde{x}_2\right]_i\right|, \forall t, i = 1\ldots n\end{aligned} \quad (38)$$

then the attractiveness condition is achieved. Because $\boldsymbol{\Theta}_1$ and $\boldsymbol{\Theta}_2$ are diagonal matrices, the inequality (38) can be written as

$$\psi_i|\hat{S}_{ci}| - \eta_i(t) \geq \left|\left(\frac{\theta_{2i}(t)}{\theta_{1i}(t)} - \psi_i\right)\tilde{x}_{2i}\right|, \forall t, i = 1\ldots n. \quad (39)$$

Remark 5. The initial value of the switching controller gain must be defined to guarantee the sliding condition after convergence of the observer, when the error in state estimates is bounded by (28). The term $\boldsymbol{\psi}\hat{\mathbf{S}}_c$ maintains the sliding variable bounded during the observer transient. This leads to

$$-\eta_i(t_o) \geq \left|\frac{\theta_{2i}(t_o)}{\theta_{1i}(t_o)} - \psi_i\right| \frac{\lambda_{\max \hat{\mathbf{H}}}}{\lambda_{\min \hat{\mathbf{H}}}} \frac{\beta}{(\lambda_{\min \mathbf{Q}} - \gamma)}. \quad (40)$$

By an appropriate choice of $\boldsymbol{\lambda}_c$ and $\boldsymbol{\rho}_c$ with respect to $\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2$, $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$, the above condition can be satisfied all the time.

Expressing the control input sliding condition as

$$\mathbf{x}_2 - \dot{\mathbf{y}}_r + \boldsymbol{\psi}(\mathbf{x}_1 - \mathbf{y}_r) = -\tilde{\mathbf{x}}_2, \quad (41)$$

where the true velocity state is introduced, taking into account (28), a bound of the tracking error can be obtained

$$|x_i - y_{ri}| \leq \frac{1}{\psi_i} \frac{\lambda_{\max \hat{\mathbf{H}}}}{\lambda_{\min \hat{\mathbf{H}}}} \frac{\beta}{(\lambda_{\min \mathbf{Q}} - \gamma)}, \forall t > t_1. \quad (42)$$

Remark 6. The actual value of t_1 depends on the convergence rate of the observer, and on the time defined by the gain matrix $\boldsymbol{\psi}$. The, observer and the controller, both of them into a smoothed form, can achieve high performance. Choosing the value of the constant k_o greater than k_c, the smooth switching function of the observer is closer to a pure relay than the smooth switching function of the controller. Therefore, the observer converges faster than the controller with small estimate error. The state estimates could be chattering-free, independent by of the value of the gains $\boldsymbol{\Theta}_1$ and $\boldsymbol{\Theta}_2$. More, choosing the matrices $\boldsymbol{\Theta}_1$ and $\boldsymbol{\Theta}_2$ adaptively updated as in (3) and (4), the magnitudes of the switching function go to small values while link position errors go to small values.

Remark 7. During sliding, the error $\mathbf{S}_o = \hat{\mathbf{x}}_1 - \mathbf{x}_1$. is approximately zero. The derivate is not exactly zero, but it is a high frequency signal of average approximately zero, with an amplitude depending of $\boldsymbol{\Theta}_1$. If the gain $\boldsymbol{\Theta}_1$ goes to zero, the derivative of the velocity estimation error goes to zero or becomes very small. That means a reduced observation error even in the presence of parameter uncertainties. Also, the behaviour of the controller is similar with that of the full state measurements if its switching is based on a smooth variable. The smooth controller means a reduced or free chattering for the control input law and/or the output.

4. PARAMETER IDENTIFICATION BASED ON SMOOTH SLIDING OBSERVER-CONTROLLER

The way followed for the time-varying parameter identification is quite different from that proposed by Xu, Pan and Lee (2003). Firstly, it is based on the state estimates and on the faster convergence of the observer than the controller. Secondly, it is based on smooth sliding observer-controller, both of them having adaptive switching gain. Zero or small state estimate error leads to zero or small tracking error and small gains of the corresponding switching function. Consequently, during sliding, the weight of the switching term is negligible with respect to the compensation part.

Define as the parameter vector estimate with $\hat{\mathbf{p}}$. If the functions $\mathbf{f}$, $\mathbf{g}$ and $\mathbf{h}$ are linear in thetime varying parameters, each term of the system (1) can be expressed as follows:

$$(43)\quad \begin{aligned}&\begin{bmatrix}\mathbf{I}_n & \mathbf{0}_{nxn}\\ \mathbf{0}_{nxn} & \hat{\mathbf{h}}(\mathbf{x}_1,\hat{\mathbf{p}})\end{bmatrix}\begin{bmatrix}\dot{\mathbf{x}}_1\\ \dot{\hat{\mathbf{x}}}_2\end{bmatrix}=\begin{bmatrix}\dot{\mathbf{x}}_1\\ \hat{\mathbf{h}}_1(\mathbf{x}_1)\dot{\hat{\mathbf{x}}}_2\end{bmatrix}\\ &+\begin{bmatrix}\mathbf{0}_n & \mathbf{0}_{nx(n_p-1)}\\ \hat{\mathbf{h}}_2(\mathbf{x}_1)\dot{\hat{\mathbf{x}}}_2 & \varphi^1_{nx(n_p-1)}\end{bmatrix}\hat{\mathbf{p}}(t)\end{aligned},$$

$$(44)\quad \begin{aligned}&\begin{bmatrix}\hat{\mathbf{x}}_2\\ \hat{\mathbf{f}}(\mathbf{x}_1,\hat{\mathbf{x}}_2,\hat{\mathbf{p}})\end{bmatrix}=\begin{bmatrix}\hat{\mathbf{x}}_2\\ \hat{\mathbf{f}}_1(\mathbf{x}_1,\hat{\mathbf{x}}_2)\end{bmatrix}\\ &+\begin{bmatrix}\mathbf{0}_n & \mathbf{0}_{nx(n_p-1)}\\ \hat{\mathbf{f}}_2(\mathbf{x}_1,\hat{\mathbf{x}}_2) & \varphi^2_{nx(n_p-1)}\end{bmatrix}\hat{\mathbf{p}}(t)\end{aligned},$$

$$(45)\quad \begin{aligned}&\begin{bmatrix}\mathbf{I}_n & \mathbf{0}_{nxn}\\ \mathbf{0}_{nxn} & \hat{\mathbf{g}}(\mathbf{x}_1,\hat{\mathbf{x}}_2,\hat{\mathbf{p}})\end{bmatrix}\begin{bmatrix}\mathbf{0}_n\\ \hat{\mathbf{u}}\end{bmatrix}=\begin{bmatrix}\mathbf{0}_n\\ \hat{\mathbf{g}}_1(\mathbf{x}_1,\hat{\mathbf{x}}_2,\mathbf{u})\end{bmatrix}.\\ &+\mathbf{\Phi}_3(\mathbf{x}_1,\hat{\mathbf{x}}_2,\hat{\mathbf{u}})\hat{\mathbf{p}}(t)\end{aligned}$$

Define the followings function matrices and vectors:

$$(46)\quad \hat{\mathbf{H}}_0(\mathbf{x}_1,\hat{\mathbf{p}})=\begin{bmatrix}\mathbf{I}_n & \mathbf{0}_{nxn}\\ \mathbf{0}_{nxn} & \hat{\mathbf{h}}(\mathbf{x}_1,\hat{\mathbf{p}})\end{bmatrix},$$

$$(47)\quad \hat{\mathbf{h}}_0(\dot{\mathbf{x}}_1,\dot{\hat{\mathbf{x}}}_2,\mathbf{x}_1,\hat{\mathbf{x}}_2)=\begin{bmatrix}\dot{\mathbf{x}}_1\\ \hat{\mathbf{h}}_1(\mathbf{x}_1)\dot{\hat{\mathbf{x}}}_2\end{bmatrix},$$

$$(48)\quad \hat{\mathbf{\Phi}}_1(\dot{\mathbf{x}}_1,\dot{\hat{\mathbf{x}}}_2,\mathbf{x}_1,\hat{\mathbf{x}}_2)=\begin{bmatrix}\mathbf{0}_n & \mathbf{0}_{nx(n_p-1)}\\ \hat{\mathbf{h}}_2(\mathbf{x}_1)\dot{\hat{\mathbf{x}}}_2 & \varphi^1_{nx(n_p-1)}\end{bmatrix},$$

$$(49)\quad \hat{\mathbf{f}}_0(\mathbf{x}_1,\hat{\mathbf{x}}_2,\hat{\mathbf{p}})=\begin{bmatrix}\hat{\mathbf{x}}_2\\ \hat{\mathbf{f}}(\mathbf{x}_1,\hat{\mathbf{x}}_2,\hat{\mathbf{p}})\end{bmatrix},\ \hat{\mathbf{f}}_{01}(\mathbf{x}_1,\hat{\mathbf{x}}_2)=\begin{bmatrix}\hat{\mathbf{x}}_2\\ \hat{\mathbf{f}}_1(\mathbf{x}_1,\hat{\mathbf{x}}_2)\end{bmatrix},$$

$$(50)\quad \hat{\mathbf{\Phi}}_2(\mathbf{x}_1,\hat{\mathbf{x}}_2)=\begin{bmatrix}\mathbf{0}_n & \mathbf{0}_{nx(n_p-1)}\\ \hat{\mathbf{f}}_2(\mathbf{x}_1,\hat{\mathbf{x}}_2) & \varphi^2_{nx(n_p-1)}\end{bmatrix},$$

$$(51)\quad \hat{\mathbf{G}}_0(\mathbf{x}_1,\hat{\mathbf{x}}_2,\hat{\mathbf{p}})=\begin{bmatrix}\mathbf{I}_n & \mathbf{0}_{nxn}\\ \mathbf{0}_{nxn} & \hat{\mathbf{g}}(\mathbf{x}_1,\hat{\mathbf{x}}_2,\hat{\mathbf{p}})\end{bmatrix},$$

$$(52)\quad \hat{\mathbf{g}}_o(\mathbf{x}_1,\hat{\mathbf{x}}_2,\hat{\mathbf{u}})=\begin{bmatrix}\mathbf{0}_n\\ \hat{\mathbf{g}}_1(\mathbf{x}_1,\hat{\mathbf{x}}_2,\hat{\mathbf{u}})\end{bmatrix},$$

In the relationships (46),…,(52), $\hat{\mathbf{H}}_0$, $\hat{\mathbf{G}}_0$ are $2nx2n$ matrices, $\hat{\mathbf{\Phi}}_1$, $\hat{\mathbf{\Phi}}_2$, $\hat{\mathbf{\Phi}}_3$ are $2nxn_p$ matrices and $\hat{\mathbf{f}}_0$, $\hat{\mathbf{f}}_{01}$, $\hat{\mathbf{g}}_0$, $\hat{\mathbf{h}}_0$ are $2n$ vectors. With the above notations the robot model can by expressed compactly by:

$$(53)\quad \hat{\mathbf{H}}_0(\mathbf{x}_1,\hat{\mathbf{p}})\dot{\mathbf{x}}=\hat{\mathbf{f}}_0(\mathbf{x}_1,\hat{\mathbf{x}}_2)+\hat{\mathbf{G}}_0(\mathbf{x}_1,\hat{\mathbf{x}}_2,\hat{\mathbf{p}})\hat{\mathbf{u}}_0,$$

where $\hat{\mathbf{u}}_0=\begin{bmatrix}\mathbf{0}_n^T & \hat{\mathbf{u}}^T\end{bmatrix}^T$.

Assumption 4. To each element $p_i(t), i=1\ldots n_p$ of the unknown time varying parameter vector $\mathbf{p}(t)$, there exist the values $p_{i_{\min}}, p_{i_{\max}}$, a priori known, such that

$$(54)\quad p_{i_{\min}}\le p_i\le p_{i_{\max}}.$$

Assumption 5. There exist bounding functions $\alpha(x_1)$, $\hat{\alpha}(x_1)$ such that

$$(55)\quad \begin{aligned}&\left\|\mathbf{h}^{-1}(\mathbf{x}_1,\mathbf{p})\right\|\le\left\|\mathbf{H}_0^{-1}(\mathbf{x}_1,\mathbf{p})\right\|\le\alpha(\mathbf{x}_1),\\ &\left\|\hat{\mathbf{h}}^{-1}(\mathbf{x}_1,\hat{\mathbf{p}})\right\|\le\left\|\hat{\mathbf{H}}_0^{-1}(\mathbf{x}_1,\hat{\mathbf{p}})\right\|\le\hat{\alpha}(\mathbf{x}_1), \forall\mathbf{x}_1\in\Re^n,\\ &\forall\,\mathbf{p}\in\Re^{n_p}, \forall\,\hat{p}_i\in\left[p_{i_{\min}}\quad p_{i_{\min}}\right], \forall t\in[0\quad\infty).\end{aligned}$$

Assumption 6. There exist $2nx2n$ function matrices, structured as follows:

$$(56)\quad \hat{\mathbf{G}}_{01}(\mathbf{x}_1,\hat{\mathbf{x}}_2)=\begin{bmatrix}\mathbf{I}_n & \mathbf{0}_{nxn}\\ \mathbf{0}_{nxn} & \hat{\mathbf{g}}_{01}(\mathbf{x}_1,\hat{\mathbf{x}}_2)\end{bmatrix}$$

of full rank, and

$$(57)\quad \hat{\mathbf{G}}_{02}(\mathbf{x}_1,\hat{\mathbf{x}}_2,\hat{\mathbf{p}})=\begin{bmatrix}\mathbf{0}_{nxn} & \mathbf{0}_{nxn}\\ \mathbf{0}_{nxn} & \hat{\mathbf{g}}_{02}(\mathbf{x}_1,\hat{\mathbf{x}}_2,\hat{\mathbf{p}})\end{bmatrix},$$

with $g_{02ij}(\mathbf{x}_1,\hat{\mathbf{x}}_2,\hat{\mathbf{p}})=g_{02ij}(\mathbf{x}_1,\hat{\mathbf{x}}_2)\hat{\mathbf{p}}$, such that

$$(58)\quad \begin{aligned}&\hat{\mathbf{G}}_0(\mathbf{x}_1,\hat{\mathbf{x}}_2,\hat{\mathbf{p}})\\ &=\hat{\mathbf{G}}_{01}(\mathbf{x}_1,\hat{\mathbf{x}}_2)\left[\mathbf{I}_{2n}+\hat{\mathbf{G}}_{02}(\mathbf{x}_1,\hat{\mathbf{x}}_2,\hat{\mathbf{p}})\right]\end{aligned}.$$

Assumption 7. There is a positive constant σ such that

$$(59)\quad \mathbf{v}^T\hat{\mathbf{h}}^{-1}\hat{\mathbf{g}}\mathbf{v}\ge\sigma\|\mathbf{v}\|^2, \forall\mathbf{v}\in\Re^n.$$

Define the matrix

(60) $\hat{\mathbf{\Phi}}\left(\dot{\mathbf{x}}_1, \dot{\hat{\mathbf{x}}}_2, \mathbf{x}_1, \hat{\mathbf{x}}_2, \mathbf{u}\right) = -\hat{\mathbf{\Phi}}_1 + \hat{\mathbf{\Phi}}_2 + \hat{\mathbf{\Phi}}_3$

and the vector

(61) $\hat{\boldsymbol{\omega}}\left(\dot{\mathbf{x}}_1, \dot{\hat{\mathbf{x}}}_2, \mathbf{x}_1, \hat{\mathbf{x}}_2, \mathbf{u}\right) = \hat{\mathbf{h}}_0 - \hat{\mathbf{f}}_0 - \hat{\mathbf{g}}_0$

of $2nxn_p$ and $2n$-dimension, respectively. Suppose that $\hat{\mathbf{\Phi}}^T\hat{\mathbf{\Phi}}$ is a nonsingular matrix, then the parameter estimate $\hat{\mathbf{p}}$ can be computed as the minimum residuum solution of the system

(62) $\hat{\mathbf{\Phi}}\hat{\mathbf{p}} = \hat{\boldsymbol{\omega}}$.

In order ensure the boundedness of $\hat{\mathbf{p}}$, the following scheme is used for computing the parameter estimate

(63) $$\hat{p}_i(t) = \begin{cases} p_{i_{\min}} \text{ if } \left[\left(\hat{\mathbf{\Phi}}^T\hat{\mathbf{\Phi}}\right)^{-1}\hat{\mathbf{\Phi}}^T\hat{\boldsymbol{\omega}}\right]_i < p_{i_{\min}}, \\ \left[\left(\hat{\mathbf{\Phi}}^T\hat{\mathbf{\Phi}}\right)^{-1}\hat{\mathbf{\Phi}}^T\hat{\boldsymbol{\omega}}\right]_i \text{ if } \left[\left(\hat{\mathbf{\Phi}}^T\hat{\mathbf{\Phi}}\right)^{-1}\hat{\mathbf{\Phi}}^T\hat{\boldsymbol{\omega}}\right]_i \\ \in \left[p_{i_{\min}} \quad p_{i_{\min}}\right], \\ p_{i_{\max}} \text{ if } \left[\left(\hat{\mathbf{\Phi}}^T\hat{\mathbf{\Phi}}\right)^{-1}\hat{\mathbf{\Phi}}^T\hat{\boldsymbol{\omega}}\right]_i > p_{i_{\max}} \end{cases}.$$

With the observer (2) and the control law (34), both of them having smooth switching term and gains adaptively updated, the neighborhood of the controller sliding surface (30) can be reached in finite time.

The Lyapunov function candidate is chosen as

(64) $V = \hat{\mathbf{S}}_c^T\hat{\mathbf{S}}_c/2$.

The controller sliding surface (30) depends on the tracking error vector (reference tracking and velocity tracking)

(65) $\mathbf{e} = \mathbf{x}_r - \hat{\mathbf{x}} = \left[(\mathbf{y}_r - \mathbf{x}_1)^T \quad (\dot{\mathbf{y}}_r - \hat{\mathbf{x}}_2)^T\right]^T$.

The derivative of the Lyapunov function can be expressed as

(66) $\dot{V} = \hat{\mathbf{S}}_c^T\dot{\hat{\mathbf{S}}}_c = \hat{\mathbf{S}}_c \dfrac{\partial\hat{\mathbf{S}}_c}{\partial\mathbf{e}}\dot{\mathbf{e}} = \hat{\mathbf{S}}_c^T \dfrac{\partial\hat{\mathbf{S}}_c}{\partial\mathbf{e}}\left(\dot{\mathbf{y}}_r - \dot{\hat{\mathbf{x}}}\right)$.

Using (53), the above derivative function can be written as

(67) $\dot{V} = \hat{\mathbf{S}}_c^T \dfrac{\partial\hat{\mathbf{S}}_c}{\partial\mathbf{e}}\left(\dot{\mathbf{y}}_r - \hat{\mathbf{H}}_0^{-1}\hat{\mathbf{f}}_0 - \hat{\mathbf{H}}_0^{-1}\hat{\mathbf{G}}_0\hat{\mathbf{u}}\right)$.

The smooth sliding controller (32) can be expressed as the sum of two terms

(68) $\hat{\mathbf{u}} = \hat{\mathbf{u}}_c + \hat{\mathbf{u}}_s$,

where

(69) $$\hat{\mathbf{u}}_c = -\hat{\mathbf{f}}(\mathbf{x}_1, \hat{\mathbf{x}}_2, \hat{\mathbf{p}}) + \hat{\mathbf{g}}^{-1}(\mathbf{x}_1, \hat{\mathbf{x}}_2, \hat{\mathbf{p}})\hat{\mathbf{h}}(\mathbf{x}_1, \hat{\mathbf{p}}) \left[-\boldsymbol{\psi}\hat{\mathbf{S}}_c + \ddot{\mathbf{y}}_r - \boldsymbol{\psi}(\hat{\mathbf{x}}_2 - \dot{\mathbf{y}}_r)\right]$$

is the compensation part,

(70) $\hat{\mathbf{u}}_s = \hat{\mathbf{g}}^{-1}(\mathbf{x}_1, \hat{\mathbf{x}}_2, \hat{\mathbf{p}})\hat{\mathbf{h}}(\mathbf{x}_1, \hat{\mathbf{p}})\boldsymbol{\eta}(t)\tanh\left(k_c\hat{\mathbf{S}}_c\right)$

being the switching part one. Using (58) and the block diagonal form of the matrices, the compensation part can be further expressed as

(71) $$\hat{\mathbf{u}}_c = \left[\hat{\mathbf{g}}_{01}\left[\mathbf{I}_n + \hat{\mathbf{g}}_{02}\right]\right]^{-1} \left[\ddot{\mathbf{y}}_r - \hat{\mathbf{f}}_1 + \hat{\mathbf{h}}_1\dot{\hat{\mathbf{x}}}_2 - \left(\hat{\mathbf{\Phi}}_2^2 - \hat{\mathbf{\Phi}}_1^2\right)\hat{\mathbf{p}}\right],$$

where $\hat{\mathbf{\Phi}}_1^2 = \left[\hat{\mathbf{h}}_2(\mathbf{x}_1)\dot{\hat{\mathbf{x}}}_2 \quad \varphi^1_{nx(n_p-1)}\right]$,
$\hat{\mathbf{\Phi}}_2^2 = \left[\hat{\mathbf{f}}_2(\mathbf{x}_1, \hat{\mathbf{x}}_2) \quad \varphi^2_{nx(n_p-1)}\right]$ are nxn_p matrices which hold the second block row of the matrices $\hat{\mathbf{\Phi}}_1$ and $\hat{\mathbf{\Phi}}_2$, respectively.

In order to re-write the variable structure term, the expression $\dfrac{\partial\hat{\mathbf{S}}_c^T}{\partial\mathbf{e}}\hat{\mathbf{S}}_c$ can be replaced with smooth switching function $\tanh\left(k_c\hat{\mathbf{S}}_c\right)$ whilst the system evolution is in a neighbourhood of the sliding surface, the attractiveness condition is satisfied, the switching gain $\boldsymbol{\eta}(t)$ is closed to zero and the parameter k_c is sufficiently large.

Defining the vector

(72) $\boldsymbol{\pi} = \left[p_{1_{\max}} - p_{1_{\min}} \quad \cdots \quad p_{n_p\max} - p_{n_p\min}\right]^T$

and using the relationships (64), (55), and (59), there exists a positive constant ξ such that

(73) $$\left\|\hat{\mathbf{g}}^{-1}(\mathbf{x}_1, \hat{\mathbf{x}}_2, \hat{\mathbf{p}})\hat{\mathbf{h}}(\mathbf{x}_1, \hat{\mathbf{p}})\boldsymbol{\eta}(t)\right\| \le \frac{\left(\hat{\alpha}\left\|\hat{\mathbf{\Phi}}\right\|\left\|\boldsymbol{\pi}\right\| + \xi\right)\left\|\tanh\left(k_c\hat{\mathbf{S}}_c\right)\right\|}{\sigma\left[\tanh\left(k_c\hat{\mathbf{S}}_c\right)\right]^T\tanh\left(k_c\hat{\mathbf{S}}_c\right)}, \forall t \in [0 \quad \infty)$$

The variable structure part can be re-expressed as

$$\text{(74)}\quad \hat{\mathbf{u}}_s = \frac{\left(\hat{\alpha}\left\|\hat{\boldsymbol{\Phi}}\right\|\left\|\boldsymbol{\pi}\right\| + \xi\right)\left\|\tanh\left(k_c\hat{\mathbf{S}}_c\right)\right\|}{\sigma\left[\tanh\left(k_c\hat{\mathbf{S}}_c\right)\right]^T \tanh\left(k_c\hat{\mathbf{S}}_c\right)}\tanh\left(k_c\hat{\mathbf{S}}_c\right).$$

With these components of the controller, taking into account the particular structure of the matrices and vectors (46)…(52), the derivative of the Lyapunov function may be expressed as

$$\begin{aligned}
\dot{V} &= \left[\tanh\left(k_c\hat{\mathbf{S}}_c\right)\right]^T \hat{\mathbf{h}}^{-1}\begin{bmatrix}\hat{\mathbf{h}}_1\dot{\hat{\mathbf{x}}}_2 + \hat{\boldsymbol{\Phi}}_1^2\hat{\mathbf{p}} - \hat{\mathbf{f}}_1 - \hat{\boldsymbol{\Phi}}_2^2\hat{\mathbf{p}} \\ -\left[\hat{\mathbf{g}}_{01}\left[\mathbf{I}_n + \hat{\mathbf{g}}_{02}\right]\right]\hat{\mathbf{u}}_c\end{bmatrix} \\
&- \left[\tanh\left(k_c\hat{\mathbf{S}}_c\right)\right]^T \hat{\mathbf{h}}^{-1}\hat{\mathbf{g}}\hat{\mathbf{u}}_s = \left[\tanh\left(k_c\hat{\mathbf{S}}_c\right)\right]^{-1}\hat{\mathbf{h}}^{-1} \\
&\begin{bmatrix}\hat{\mathbf{h}}_1\dot{\hat{\mathbf{x}}}_2 - \hat{\mathbf{f}}_1 + \left(\hat{\boldsymbol{\Phi}}_1^2 - \hat{\boldsymbol{\Phi}}_2^2\right)\hat{\mathbf{p}} - \left[\hat{\mathbf{g}}_{01}\left[\mathbf{I}_n + \mathbf{g}_{02}\right]\right]\hat{\mathbf{u}}_c \\ + \left[\hat{\mathbf{g}}_{01}\left[\mathbf{g}_{02} - \hat{\mathbf{g}}_{02}\right]\right]\hat{\mathbf{u}}_c\end{bmatrix} \\
&- \left[\tanh\left(k_c\hat{\mathbf{S}}_c\right)\right]^T \hat{\mathbf{h}}^{-1}\hat{\mathbf{g}}\hat{\mathbf{u}}_s = \left[\tanh\left(k_c\hat{\mathbf{S}}_c\right)\right]^T \hat{\mathbf{h}}^{-1} \\
&\left[\hat{\boldsymbol{\Phi}}_2^2 + \hat{\boldsymbol{\Phi}}_3^2 - \hat{\boldsymbol{\Phi}}_1^2\right]\left(\mathbf{p} - \hat{\mathbf{p}}\right) \\
&- \frac{\left(\hat{\alpha}\left\|\hat{\boldsymbol{\Phi}}\right\|\left\|\boldsymbol{\pi}\right\| + \xi\right)\left\|\tanh\left(k_c\hat{\mathbf{S}}_c\right)\right\|}{\sigma\left[\tanh\left(k_c\hat{\mathbf{S}}_c\right)\right]^T \tanh\left(k_c\hat{\mathbf{S}}_c\right)}\left[\tanh\left(k_c\hat{\mathbf{S}}_c\right)\right]^T \\
&\hat{\mathbf{h}}^{-1}\hat{\mathbf{g}}\tanh\left(k_c\hat{\mathbf{S}}_c\right) \le \hat{\alpha}\left\|\tanh\left(k_c\hat{\mathbf{S}}_c\right)\right\|\left\|\hat{\boldsymbol{\Phi}}\right\|\left\|\boldsymbol{\pi}\right\| \\
&- \left(\hat{\alpha}\left\|\hat{\boldsymbol{\Phi}}\right\|\left\|\boldsymbol{\pi}\right\| + \xi\right)\left\|\tanh\left(k_c\hat{\mathbf{S}}_c\right)\right\| \\
\text{(75)}\quad &\frac{\left[\tanh\left(k_c\hat{\mathbf{S}}_c\right)\right]^T \hat{\mathbf{h}}^{-1}\hat{\mathbf{g}}\tanh\left(k_c\hat{\mathbf{S}}_c\right)}{\sigma\left[\tanh\left(k_c\hat{\mathbf{S}}_c\right)\right]^T \tanh\left(k_c\hat{\mathbf{S}}_c\right)} \le -\xi\left\|\tanh\left(k_c\hat{\mathbf{S}}_c\right)\right\|.
\end{aligned}$$

Defining the set $\left\{\left\|\hat{\mathbf{S}}_c\right\| \le \frac{1}{k_c}\right\}$, we can say that there exists some $T \ge 0$ such that $\forall t \in [0, T)$, $\left\|\hat{\mathbf{S}}_c(t)\right\| > \frac{1}{k_c}$ and $\left\|\hat{\mathbf{S}}_c(t)\right\|$ will be strictly decreasing until it reaches the set in finite time and remains inside thereafter (for $t \ge T$).

Particularizing the above relationships for n-degree of freedom robot manipulator, considering the estimates of the velocities and the uncertainties in the parameters, the robot model (11) becomes

$$\text{(76)}\quad \begin{aligned}&\begin{bmatrix}\dot{\mathbf{x}}_1 \\ \hat{\mathbf{H}}_1\dot{\hat{\mathbf{x}}}_2\end{bmatrix} + \begin{bmatrix}\mathbf{0}_n \\ \hat{\mathbf{H}}_2\end{bmatrix}\hat{m}_p = -\begin{bmatrix}\hat{\mathbf{x}}_2 \\ \hat{\mathbf{C}}_1\hat{\mathbf{x}}_2 + \hat{\mathbf{F}}\hat{\mathbf{x}}_2 + \hat{\mathbf{G}}_1\end{bmatrix} \\ &-\begin{bmatrix}\mathbf{0}_n \\ \hat{\mathbf{C}}_2\hat{\mathbf{x}}_2 + \hat{\mathbf{G}}_2\end{bmatrix}\hat{m}_p + \begin{bmatrix}\mathbf{0}_n \\ \hat{\mathbf{u}}\end{bmatrix} + \begin{bmatrix}\mathbf{0}_n \\ \mathbf{d}\end{bmatrix}\end{aligned}.$$

Define the $2nx2n$ matrices and $2n$ vectors, respectively

$$\text{(77)}\quad \hat{\mathbf{H}}_0 = \begin{bmatrix}\mathbf{I}_n & \mathbf{0}_{nxn} \\ \mathbf{0}_{nxn} & \hat{\mathbf{H}}\end{bmatrix},\ \hat{\mathbf{h}}_0 = \begin{bmatrix}\dot{\mathbf{x}}_1 \\ \hat{\mathbf{H}}_1\dot{\hat{\mathbf{x}}}_2\end{bmatrix},$$

$$\text{(78)}\quad \hat{\boldsymbol{\Phi}}_1 = \begin{bmatrix}\mathbf{0}_n & \mathbf{0}_{nx(n_p-1)} \\ \hat{\mathbf{H}}_2\dot{\hat{\mathbf{x}}}_2 & \mathbf{0}_{nx(n_p-1)}\end{bmatrix},$$

$$\text{(79)}\quad \hat{\mathbf{f}}_0 = \begin{bmatrix}\hat{\mathbf{x}}_2 \\ -\hat{\mathbf{C}}\hat{\mathbf{x}}_2 - \hat{\mathbf{F}}\hat{\mathbf{x}}_2 - \hat{\mathbf{G}}\end{bmatrix}, \hat{\mathbf{f}}_{01} = \begin{bmatrix}\hat{\mathbf{x}}_2 \\ -\hat{\mathbf{C}}_1\hat{\mathbf{x}}_2 - \hat{\mathbf{F}}\hat{\mathbf{x}}_2 - \hat{\mathbf{G}}_1\end{bmatrix},$$

$$\text{(80)}\quad \hat{\boldsymbol{\Phi}}_2 = \begin{bmatrix}\mathbf{0}_n & \mathbf{0}_{nx(n_p-1)} \\ -\hat{\mathbf{C}}_2\hat{\mathbf{x}}_2 - \hat{\mathbf{G}}_2 & \mathbf{1}_{nx(n_p-1)}\end{bmatrix},$$

$$\text{(81)}\quad \hat{\mathbf{G}}_0 = \begin{bmatrix}\mathbf{I}_n & \mathbf{0}_{nxn} \\ \mathbf{0}_{nxn} & \mathbf{I}_n\end{bmatrix},\ \hat{\mathbf{g}}_0\left(\mathbf{x}_1, \hat{\mathbf{x}}_2, \hat{\mathbf{u}}\right) = \begin{bmatrix}\mathbf{0}_n \\ \hat{\mathbf{u}}\end{bmatrix},$$

$$\text{(82)}\quad \hat{\boldsymbol{\Phi}}_3 = \begin{bmatrix}\mathbf{0}_{nxn} & \mathbf{0}_{nx(n_p-1)} \\ \mathbf{0}_n & \mathbf{0}_{nx(n_p-1)}\end{bmatrix}.$$

This allows to re-write as

$$\text{(83)}\quad \hat{\mathbf{H}}_0\begin{bmatrix}\dot{\mathbf{x}}_1 \\ \dot{\hat{\mathbf{x}}}_2\end{bmatrix} = \hat{\mathbf{f}}_0 + \hat{\mathbf{G}}_0\hat{\mathbf{u}}$$

or equivalently as

$$\text{(84)}\quad \hat{\mathbf{h}}_0 + \hat{\boldsymbol{\Phi}}_1\hat{\mathbf{p}} = \hat{\mathbf{f}}_{01} + \hat{\boldsymbol{\Phi}}_2\hat{\mathbf{p}} + \hat{\mathbf{g}}_0 + \hat{\boldsymbol{\Phi}}_3\hat{\mathbf{p}}.$$

Remark 8. The smooth sliding controller allows the using of the compensation part as equivalent control input signal during sliding. The adaptive gain of the controller switching term goes to zero or becomes very small, depending on the error in the state estimate. Therefore, the influence of the noise induced by control input acquisition is very small in the parameter estimate.

Remark 9. In closed loop, the robustness to uncertainties makes insensitive the stability to phase lag induced by the filters used to compute the derivatives of the state estimate.

Remark 10. As emphasized in Xu, Pan and Lee (2003), the reference signal has to be chosen in order to avoid the singularity of the matrix $\hat{\boldsymbol{\Phi}}^T\hat{\boldsymbol{\Phi}}$.

5. CLOSED LOOP SIMULATION

A two degree of freedom vertical robot with two rigid revolute joints, two rigid links, a time varying payload $m_p(t)$ and an additive disturbance $d(t)$ on the control input has been considered in order to test the smooth variable structure observer-controller with time-varying parameter identification, developed in this paper.

The vectors of position and velocities are $\mathbf{x}_1 = [x_{11} \quad x_{12}]^T$ and $\mathbf{x}_2 = [x_{21} \quad x_{22}]^T$, respectively.

The trajectory to be tracked is defined as

$$\text{(84)}\quad \mathbf{y}_r = [-0.5 + 0.3\sin(t-0.3) \qquad 0.7\sin(2t+0.3)]^T$$

The parameter vector to be identified is

$$\text{(85)}\quad \mathbf{p}(t) = \left[3 + e^{-0.5t} \qquad -1 + 0.7\sin(3t)\right]^T,$$

where $m_p(t) = 3 + e^{-0.5t}$ is the payload and $d(t) = -1 + 0.7\sin(3t)$ is the additive disturbance.
The corresponding robot model matrices and vectors are the following:

$$\text{(86)}\quad \mathbf{H}(\mathbf{x}_1, \mathrm{m_p}) = \begin{bmatrix} 9.77 + 2.02\cos(\mathrm{x}_{12}) & 1.26 + 1.01\cos(\mathrm{x}_{12}) \\ 1.26 + 1.01\cos(\mathrm{x}_{12}) & 1.12 \end{bmatrix} + \begin{bmatrix} 2 + 2\cos(\mathrm{x}_{12}) & 1 + \cos(\mathrm{x}_{12}) \\ 1 + \cos(\mathrm{x}_{12}) & 1 \end{bmatrix}\mathrm{m_p},$$

$$\text{(87)}\quad \mathbf{C}(\mathbf{x}_1, \mathbf{x}_2, \mathrm{m_p}) = \sin(\mathrm{x}_{12})\left\{\begin{bmatrix} -\mathrm{x}_{22} & -\mathrm{x}_{21} - \mathrm{x}_{22} \\ \mathrm{x}_{21} & 0 \end{bmatrix}1.01 + \begin{bmatrix} -\mathrm{x}_{22} & -\mathrm{x}_{21} - \mathrm{x}_{22} \\ \mathrm{x}_{21} & 0 \end{bmatrix}\mathrm{m_p}\right\},$$

$$\text{(88)}\quad \mathbf{G}(\mathbf{x}_1, \mathrm{m_p}) = \mathrm{g}\begin{bmatrix} 8.1\sin(\mathrm{x}_{11}) + 1.13\sin(\mathrm{x}_{11} + \mathrm{x}_{12}) \\ 1.13\sin(\mathrm{x}_{11} + \mathrm{x}_{12}) \end{bmatrix} + \mathrm{g}\begin{bmatrix} \sin(\mathrm{x}_{11}) + \sin(\mathrm{x}_{11} + \mathrm{x}_{12}) \\ \sin(\mathrm{x}_{11} + \mathrm{x}_{12}) \end{bmatrix}\mathrm{m_p}$$

$$\text{(89)}\quad \mathbf{F} = \mathrm{diag}[10 \quad 10],$$

$$\text{(90)}\quad \mathbf{h}_0 = \begin{bmatrix} \dot{\mathrm{x}}_{11} \\ \dot{\mathrm{x}}_{12} \\ [9.77 + 2.02\cos(\mathrm{x}_{12})]\dot{\mathrm{x}}_{21} + [1.26 + 1.01\cos(\mathrm{x}_{12})]\dot{\mathrm{x}}_{22} \\ [1.26 + 1.01\cos(\mathrm{x}_{12})]\dot{\mathrm{x}}_{21} + 1.12\dot{\mathrm{x}}_{22} \end{bmatrix},$$

$$\text{(91)}\quad \mathbf{\Phi}_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ [2 + 2\cos(x_{12})]\dot{x}_{21} + [1 + \cos(x_{12})]\dot{x}_{22} & 0 \\ [1 + \cos(x_{12})]\dot{x}_{21} + \dot{x}_{22} & 0 \end{bmatrix}$$

$$\text{(92)}\quad \mathbf{f}_{01} = \begin{bmatrix} \mathrm{x}_{21} \\ \mathrm{x}_{22} \\ 1.01\sin(\mathrm{x}_{12})\mathrm{x}_{22}\mathrm{x}_{21} + (\mathrm{x}_{21} + \mathrm{x}_{22})1.01\sin(\mathrm{x}_{12})\mathrm{x}_{22} - 8.1\mathrm{g}\sin(\mathrm{x}_{11}) - 1.13\mathrm{g}\sin(\mathrm{x}_{21} + \mathrm{x}_{22}) - 10\mathrm{x}_{21} \\ -1.01\sin(\mathrm{x}_{12})\mathrm{x}_{21}^2 - 1.13\mathrm{g}\sin(\mathrm{x}_{21} + \mathrm{x}_{22}) - 10\mathrm{x}_{22} \end{bmatrix},$$

$$\text{(93)}\quad \mathbf{\Phi}_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \sin(\mathrm{x}_{12})\mathrm{x}_{22}\mathrm{x}_{21} + (\mathrm{x}_{21} + \mathrm{x}_{22})\sin(\mathrm{x}_{12})\mathrm{x}_{22} - \mathrm{g}\sin(\mathrm{x}_{11}) - \mathrm{g}\sin(\mathrm{x}_{21} + \mathrm{x}_{22}) & 0 \\ -\sin(\mathrm{x}_{12})\mathrm{x}_{21} - \mathrm{g}\sin(\mathrm{x}_{21} + \mathrm{x}_{22}) & 1 \end{bmatrix},$$

$$\text{(94)}\quad \hat{\mathbf{g}}_0 = [0 \quad 0 \quad u_1 \quad u_2]^T, \ \hat{\mathbf{\Phi}}_3 = [\mathbf{0}_{4x2}].$$

The initial conditions are:

$$\mathbf{x}_1(0) = \mathbf{x}_2(0) = \hat{\mathbf{x}}_1(0) = [0 \quad 0]^T; \ \hat{\mathbf{x}}_2(0) = [-1 \quad 2]^T,$$
$$\mathbf{\Theta}_1(0) = \begin{bmatrix} -10 & 0 \\ 0 & -10 \end{bmatrix}; \ \mathbf{\Theta}_2(0) = \begin{bmatrix} -100 & 0 \\ 0 & -200 \end{bmatrix};$$
$$\mathbf{\eta}(0) = \begin{bmatrix} -5 & 0 \\ 0 & -10 \end{bmatrix}.$$

The following constants are chosen as: $\mathbf{\lambda}_1 = \mathbf{\lambda}_2 = \mathbf{\lambda}_c = \mathrm{diag}[1 \quad 1]$, $\mathbf{\Gamma}_1 = \mathrm{diag}[10 \quad 10]$, $\mathbf{\Gamma}_2 = \mathrm{diag}[5000 \quad 5000]$, $\mathbf{\rho}_1 = \mathbf{\rho}_2 = \mathbf{\rho}_c = \mathrm{diag}[1 \quad 1]$, $\mathbf{\psi} = \mathrm{diag}[20 \quad 20]$.

In the figure 1, the closed loop simulated manipulator response is shown. Adaptive gains, smooth sliding observer-controller and time varying parameter have been introduced into the loop. Small parameter uncertainties (2%) have been considered. By choosing k_o greater than k_c, a faster sliding observer convergence than that of the sliding controller has been obtained. The response is free of chattering, although limitations have been introduced into control input ($|u_1| \le 150$; $|u_2| \le 75$). Even if the system evolutes, during sliding, in a neighbourhood of the corresponding sliding surface, the output tracking is achieved. In the figure 2, the identification of time-varying parameters $m_p(t)$ and $d(t)$ is shown. The reference signal was chosen in order to avoid the singularity of the matrix $\hat{\mathbf{\Phi}}^T \hat{\mathbf{\Phi}}$. In order to compute the derivatives of the state estimate the first order numerical difference has been used. The phase lag does not lead to instability and fluctuation in the parameter estimates.

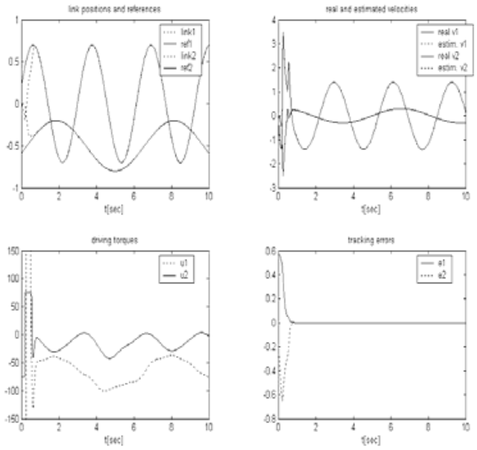

Fig. 1. Closed loop robot response, smooth sliding observer and controller, parameterized tangent hyperbolic switching function $k_o = 10$, $k_c = 1$.

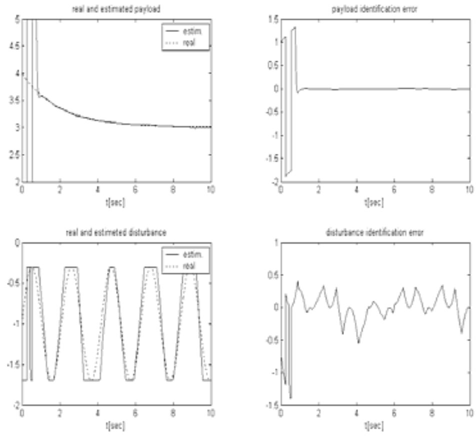

Fig. 2. Closed loop, smooth sliding observer-controller, on-line time varying parameters identification.

6. CONCLUSIONS

A robotic manipulator closed loop control with adaptive gains, smooth variable structure observer-controller and time varying parameter identification has been designed and tested by simulation. The output tracking, the robustness to uncertainties and external disturbances are increased by the use of parameterised switching functions with gains adaptively updating. The parameterised k-tanh switching function assures an alleviated or completely elimination of chattering. An appropriate choice of the parameters in the observer and controller switching functions, allows a faster convergence rate of the observer than that of the controller can be obtained. The gains adaptively updated lead the system to output tracking with smooth transient response. With some conditions on the robot model, reference input and a priori information, the identifier of time-varying parameters converges. The error in the parameter estimates depends on the error in the estimated state and on the tracking error.

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