THE ANNALS OF "DUNAREA DE JOS" UNIVERSITY OF GALATI
FASCICLE III, 2004 ISSN 1221-454X
ELECTROTEHNICS, ELECTRONICS, AUTOMATIC CONTROL, INFORMATICS

SYNTHESIS METHODOLOGY FOR ACTIVE ELEMENT

USING IMPEDANCES –BASED BOND GRAPH METHODS

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Abstract: This paper introduces a synthesis methodology for active elements within systems that uses frequency response function as a basis for describing required behavior. The method is applicable in the design of a new system or in the retrofit of an existing system in which an active element is required or desired. The two basis principles of bond graph modeling are the "reticulation principle" which decomposes a physical into elemental physical laws represented as network elements interacting through ports and the "power postulate" which assembles the complete model through a network of power flows representing the exchange of energy between the elements. Moreover the bond graph model leads to a rigorous definitions of the structure of the associated dynamical equations.

Keywords: bond graph, active element, reticulation principle, power postulate.

1. INTRODUCTION

It is possible to concentrate and to separate certain properties of on object and to describe this object as system of interrelated properties or interconnected elements. The outcome of this approaches called the *bond graph*. From a mathematical point of view a bond graph is defined as an oriented graph with a special connection topology, whose edges and vertices have special meanings.

2. BASIC CONCEPT FROM BOND GRAPH THEORY

A bond graph is a directed graph $\vec{G}$ without isolated vertices whose underlying undirected graph is simple. This means that its underlying undirected graph does not contain parallel edges and loops. The edges of the bond graph are called *bonds* and its vertices are called *multi – port elements.*

Every edge b of a bond graph $\vec{G}$ is labeled by a pair of two signals: *flow signal,* f*,* and *effort signal,* e, that is

$$b = (f, e)$$

Bond graph are a powerful tools for modeling physical systems in because they facilitate the modeling of systems involve multiple energy domains. Systems state equations contain variables from multiple energy domains that can be extracted from a specific bond graph. We obtained set of structural dynamic equations with respect to the nonlinear model and its linearization is discussed. Finally a bond graph for a element active is obtained.

The signals f, e are defined on a set T, called time axis, which is an interval of R. The value of f, e at any instant of time t belong to two real vector spaces: F – *flow spaces* and E – *effort spaces*, respectively. Elements of the space F are called *flows*, f, and elements of the space E are called *efforts,* e.

Consequently b(t) belongs to B: F x E for any $t \in T$, and B is called the *bond space.*

In the usual formulation of a bond graph the spaces F and E are assumed to be *power conjugate.* This means that there exists a pairing

This paper was recommended for publication by Gheorghe PUSCASU and Adrian FILIPESCU

$$< . \mid . > E \times F \rightarrow R$$

called *power product* satisfying the following three properties:

1. It is a linear function of each coordinate, that is
2.

$$< \alpha e_1 + \beta e_2 \mid f >= \alpha < e_1 \mid f > + \beta < e_2 \mid f >$$
$$< e \mid \alpha f_1 + \beta e_2 \mid >= \alpha < e \mid f_1 > + \beta < e \mid f_2 >$$
$$\forall f, f_1, f_2 \in F; \quad \forall e, e_1, e_2 \in E; \forall \alpha, \beta \in R$$

3. It is non – degenerate, that is

$$< e \mid f >= 0; \ \forall e \in E \Rightarrow f = 0$$
$$< e \mid f >= 0; \ \forall f \in F \Rightarrow e = 0$$

4. The expression <e | f > has the physical dimension of power.

Note: Flow f and effort e correspond to some physical quantities (e.g. current and voltage; velocity and force; entropy flow and temperature, and so on). By inserting their physical dimensions into < f | e> we obtain the dimension of power.

The direction of a bond determines the positive flow of power exchanges between the vertices incident on the bond. The direction of a bond $b \in \vec{B}$ with respect to a multi – port element $M \in \mathbb{M}$ (That is a vertex) is defined by the map

$$\sigma : \vec{B} x M \rightarrow \{-1, 1\}$$

as follows

$$\sigma(b, M): \begin{Bmatrix} 1, & \text{if M is the tail of b} \\ -1, & \text{if M is the head of b} \end{Bmatrix}$$

The *net power* of the multi – port element M at any time instant t is defined by

$$P_{net}^{M}(t) = \sum_{i=1}^{k} \sigma(b_i, M) < e_i(t) \mid f_i(t) >_{B_1}$$

Here, <. | .> is the power product associated to the bond b_i.

Definition: A bond graph is a labeled direct graph $\vec{G} = \left(M, \vec{B} \right)$ without isolated vertices whose underlying undirected graph is simple and where to every edge $b \in \vec{B}$ there is associated a pair (f, e) $\in$F x E with F, E power conjugate real vector spaces, and where to every vertex $M \in \mathbb{M}$ there is associated a behavior in the flow and effort signals of the edges incident to M.

Remark: If the spaces F and E are not power conjugate then the directed graph $\vec{G}$ is called a *pseudo – bond graph.* Nevertheless all mathematical results developed in this paper also apply to pseudo – bond graph. On the other hand, in this case the net power of a vertex can not be always meaningfully defined.

Definition: A bond graph is standard if it is canonical and the vertices which do not belong to the junction structure are: C – elements, I –elements, R – elements, SE – elements, or SF – elements.

Definition: A bond graph is a scalar bond graph if the following two conditions are satisfied:
1. The bond space of every edge is R x R;
2. The ports of all multi – port elements except the 0 – junction, 1 – junctions, TF – elements and GY – elements, have independent causality.

Definition: Consider a scalar bond graph. The subgraph of scalar bond graph obtained by deleting all multi-port elements which are not ideal junctions, TF – elements and GY –elements is called *the junction structure*.

Definition: A scalar bond graph is canonical if the following conditions are satisfied:
1. At last one end vertex of every bond is an ideal junction;
2. If both end vertices of a bond are ideal junctions then on of them is a 1 – junction and the other ia a 0 – junction;
3. Any ideal junction is adjacent to one and only one multi – port element which does not belong to the junctions structure;
4. If and end vertex of a bond is a multi – port element that does not belong to the junction structure then the bond is incoming with respect to the multi – port element;
5. Every TF – element is adjacent to a 1 – junction and to a 0 – junction;
6. Every GY – element is adjacent to either two 1 – junctions or two 0 – junctions;
7. The number of different paths of length one and two between any two ideal junctions of the underlaying undirected graph of the bond graph is less than or equal one.

Definition: If a scalar bond graph is canonical then its junction structure is called a canonical junction structure.

The collection of all bonds incident to Z – elements is denoted by b_z where

$$Z \in \{ SE, SF, C, R, I\}$$

The bond b_z is the pair of signals f_z and e_z.

3. IMPORTANCE OF ACTIVE ELEMENT SYNTHESIS

This methodology aids in the determination of control schemes that are necessary to control the active element, such that required frequency domain behavior are met.

While other impedance – based method, such as electric circuit equivalent models, could be used to model system behavior, bond graph were selected as the modeling method to be used in this work for a number of reasons:

1. The bond graph methodology yields a clear mapping of the topology of a system;
2. It easily lends it self to impedance – based modeling;
3. There exists a straightforward method by which state equations can be extracted from the bond graph;
4. It allows for causality information to be contained in the system model;
5. It easily handles the modeling of systems that involve multiple energy domains, and it easily allows for the modeling of system elements that exhibit nonlinear behavior.

4. CASE – STUDY - PIPELINE MODELS [2]

4.1 Non-linear pipeline model with distributed parameters.

The non – linear pipeline model with distributed parameters is obtained by using the equations for continuity, momentum and energy. These equations correspond to the physical principles of mass conservation, Newton's second law and energy conservation. Applying these equation leads under the assumptions that the fluid is compressible, viscous, isentropic, homogenous and one – dimensional to the following coupled non –linear set of partial differential equations:

$$(1)\quad \frac{A}{a^2}\frac{\partial p}{\partial t} = -\frac{\partial q}{\partial x}$$

$$(2)\quad \frac{1}{A}\frac{\partial q}{\partial t} + \bar{\rho}\, g.\sin\alpha + \frac{\lambda(q)}{2DA^2\bar{\rho}}q^2 = -\frac{\partial p}{\partial x}$$

where p is the pressure, q is the flow, A the cross-section of the pipeline, a the velocity of sound, $\bar{\rho}$ the constant density of the homogenous fluid, α the pipeline inclination, λ the dimensionless friction coefficient and D the diameter of the pipeline.

The continuity and momentum equations 1 and 2 form a pair of quasilinear hyperbolic partial differential equations in term of two dependent variables, mass flow rate q(x, t) and pressure p(x, t), and two independent variables, distance along the pipeline x and time t. A general solution is not available, however, a transformation into four ordinary differential equations grouped to two pairs of equations by the characteristics method is possible.

4.2 Linear pipeline model with distributed parameters.

Nonlinear equations(1, 2) are linearised and written in a form using notations common in the analysis of electrical transmission lines. Also, the gravity effect can be included into the working point so $\alpha = 0$ is supposed. The corresponding system of linear partial differential equations is

$$(3)\quad L\frac{\partial q}{\partial t} + Rq = -\frac{\partial p}{\partial x}$$

$$(4)\quad C\frac{\partial p}{\partial t} = -\frac{\partial q}{\partial x}$$

where L =1/A, R = $\frac{\lambda\left(\bar{\bar{q}}\right)\bar{\bar{q}}}{A^2\bar{\rho}D}$ ($\bar{\bar{q}}$ is the flow at the working point) and C= A/a^2 are the inertance (inductivity), resistance and capacitance per unit length, respectively. We perform a complex – plan curve – fitting procedure in order to obtain the corresponding transfer function in analytical form, namely as a ratio of two polynomials. The results of the curve fitting rocedure yields what we will refer to in this case as the "data –based" transfer function, dTF(s). Introducing the characteristic impedance $Z_K = \sqrt{\frac{Ls+R}{Cs}}$ and $n = \sqrt{(Ls+R).Cs}$ the linearised model of the pipeline can be written in one of the following two causal forms which differ from each other with respect to the model inputs (independent quantities) and outputs (dependent quantities) and where P and Q are the double Laplace transformations of the pressure p and flow q respectively and indexes 0 and L denote the inlet and the outlet of the pipeline respectively.

1. Inputs Q_0, P_L, outputs Q_L, P_0:

1.1. Transfer function model

The derivation of a transfer function model for PDEs follows the same steps as for the scalar case.

1. Apply the Laplace transformation with respect to time. This removes the time derivatives and turns the initial – boundary –

value problem into a boundary value problem for the space variable.
2. Construct a suitable transformation for the space variable which removes the spatial derivatives and turns the boundary value problem into an algebraic equation.
3. To obtain a multidimensional function, solve the algebraic equation for the transfer of the solution of the PDE

We obtain:

$$(5)\quad Q_L = \frac{1}{\cosh(nL_p)}Q_0 - \frac{1}{Z_K}\tanh(nL_p)P_L$$

$$(6)\quad P_0 = Z_K \tanh(nL_p)Q_0 + \frac{1}{\cosh(nL_p)}P_L$$

Block diagram is shown in Figure 1.

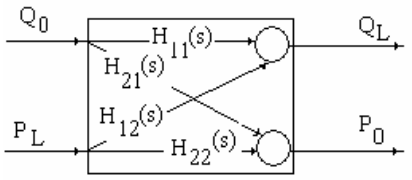

Fig. 1. Block diagram

2. Inputs Q_L, P_0, outputs Q_0, P_L:

The transfer functions are determined from following equations:

$$(7)\quad Q_0 = \frac{1}{\cosh(nL_p)}Q_L - \frac{1}{Z_K}\tanh(nL_p)P_0$$

$$(8)\quad P_L = -Z_K \tanh(nL_p)Q_L + \frac{1}{\cosh(nL_p)}P_0$$

The block diagram is shown in Figure 2.

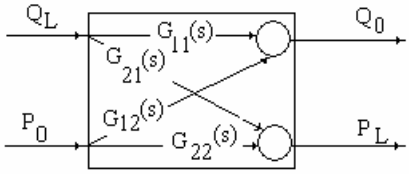

Figure 2. Block diagram

4.3 Linear pipeline model with lumped parameters.

The pipeline as a lumped parameter system can be presented as a second order transfer function in the form:

$$(9)\quad G(s) = \frac{b_2 s^2 + b_1 s + b_0}{a_2 s^2 + a_1 s + 1} e^{-sT_d}$$

where T_d is the dead time. The transcendent transfer functions are approximated by a rational transfer function with dead time however only for a class of well damped pipelines.

5. THE TEST OF A NEW CONTOLLER IN A CONTROL LOOP WITH A SOFTWARE MODEL

We consider the closed loop system shown in figure 3.

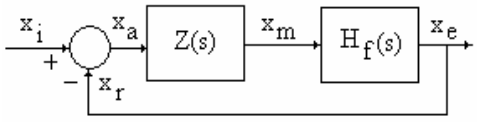

Fig. 3. The closed loop system

The transfer function of the closed loop system is:

$$(10)\quad H_0(s) = \frac{Z(s)H_f(s)}{1 + Z(s)H_f(s)}$$

The transfer function of the controller is:

$$(11)\quad Z(s) = \frac{H_0(s)}{1 - H_0(s)} \cdot \frac{1}{H_f(s)}$$

4.3.1. *Pressure → pressure and flow → flow transfer function*

$$(12)\quad H_{11}(s) = \frac{1}{\cosh(nL_p)}$$

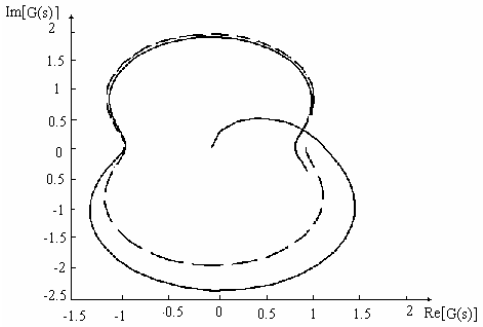

Figure 4. Nyquist plots- Curve fit of "experimental" frequency response data:
___ nonparametric model;
- - - parametric model.

Given that we have experimental data, we use the following transfer function

(13) $$H_f(s) = \frac{s^5 - 3.702s^4 - 0.7591s^3 + 121.3s^2 + 907.6s - 6556}{s^5 + 2.037s^4 + 25.72s^3 + 25.6s^2 + 55.6s + 1.33}$$

As accurate as possible curve fit is desired, however the designer must exercise caution, since selecting too high a degree of polynomials can cause the curve fitting algorithm to yield unstable transfer functions. An illustration of the curve fit to the "experimental data" is shown in Figure 3, where original data points are represented by solid line and the curve fit is represented by dashed line

Step responses for system consist in pipeline model and system desired are shown in Figure 5.

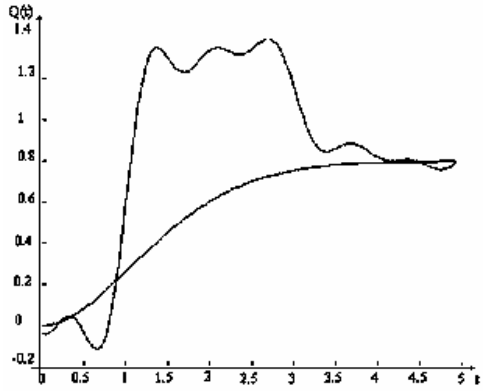

Fig. 5. Step response

Following the procedure, we can determine the impedance of the unknown filter by equating the desired transfer function with theoretical transfer function and solving for Z(s), as shown below.

The bond graph structure that represents the synthesis system impedance is shown in Figure 6. Note the representation of the negative bond graph elements.

(14) $$Z(s) = \frac{s^3 + 1.073s^2 + 22.4s + 0.7764}{1.1447s^4 - 11.4887s^3 + 62.2038s^2 - 124.5207s + 1}$$

(15) $$Z(s) = 1 + \frac{1.6039}{s - 3.4233} + \cfrac{1}{0.1362s - 0.3249 + \cfrac{1}{0.3403s - 1.4357 + \cfrac{1}{1409.1s - 11.1}}}$$

Application of the impedance decomposition procedure expression, whose terms consist of basic impedance only.

Note that some of the impedance terms are not positive real, thus indicating the need for an active element.

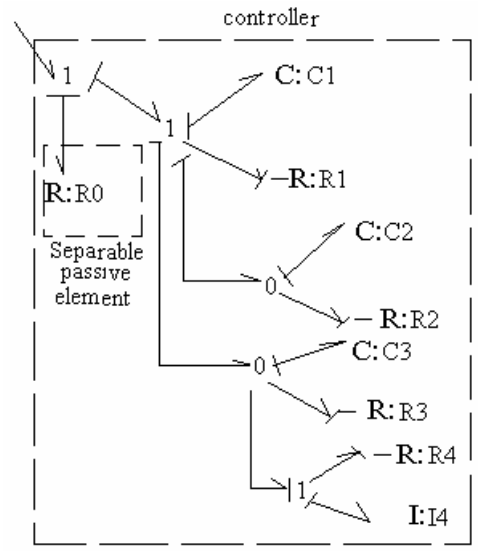

Fig. 6. Bond graph representation

A signal flow diagram was also developed, which provided information about the nature of the control system needed such that the system exhibited the desired frequency response behavior.

4.3.2. Pressure → flow transfer function

(16) $H_{12}(s) = 1/Z_K . \tanh(nLp)$

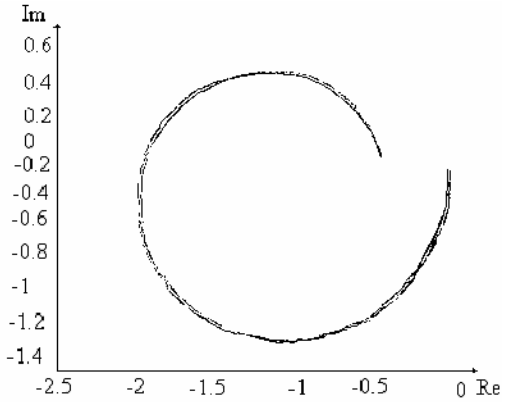

Figure 7. Nyquist plots:
___ nonparametric model;
- - - parametric model

(17) $$H_f(s) = \frac{-0.075s^5 - 1.87s^4 - 3.217s^3 - 17.3s^2 - 0.63s - 0.8}{1.08s^4 + 7.09s^3 + 9.37s^2 + 15.74s + 1}$$

(18) $$H_0(s) = \frac{s^2 + 1.3764s + 9.7016}{s^2 + 5.5243s + 0.3603}$$

(19) $$Z(s) = \frac{s^2 + 1.0405s + 2.5696}{s^3 + 23.4225s^2 + 0.7052s + 1.1146}$$

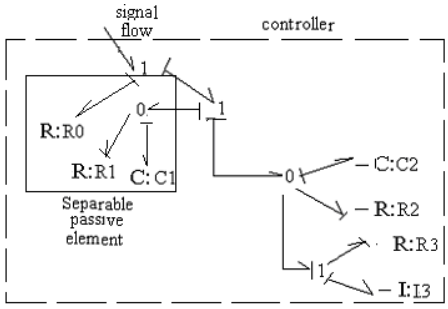

Fig. 8. Bond graph representation

4.3.3.Flow → pressure transfer function

(20) $H_{21}(s) = Z_K . \tanh(nLp)$

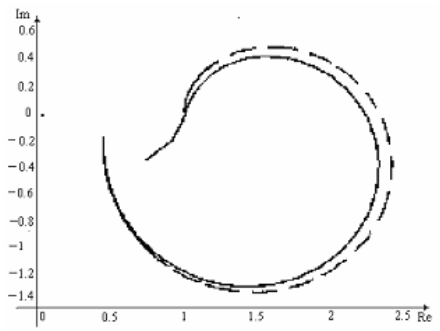

Fig. 9. Nyquist plots:
___ nonparametric model;
- - - parametric model

(21) $$H_f(s) = \frac{0.37s^6 + 0.65s^5 + 26.88s^4 + 48.58s^3 + 249.5s^2 + 226.6s + 1.247}{3.935s^5 + 6.593s^4 + 103.7s^3 + 98.47s^2 + 226s + 1}$$

(22) $$H_0(s) = \frac{s + 0.9905}{s^3 + 2.0062s^2 + 3.47s + 0.0164}$$

(23) $$Z(s) = \frac{s^3 + 0.6747s^2 + 23.21s + 0.1286}{s^4 + 0.7283s^3 + 70.48s^2 + 58.5s + 602.4}$$

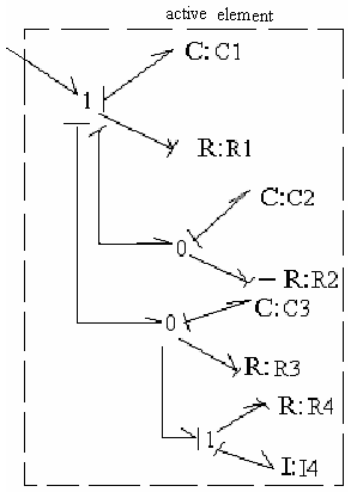

Fig. 10. Bond graph representation

Where:

C1 = 1; R1 = 1/0.0536; C2 = 0.0212;
R2 = - 0.0113; C3 = 1/4.2518;
R3 = -1/0.0113; I4 = 0.019; R4 = 0.0119

6. ACTIVE ELEMENT SYNTHESIS PROCEDURE

1. Obtain desired transfer function
   - given in analytical form or
   - determine from curve fit of experimental data.
2. Determine theoretical transfer function, use impedance – based bond graph.
3. Solve for unknown impedance, Z(s).
4. Decompose Z(s) into basic impedances – polynomial division and partial fraction expansion.
5. Cast into bond graph framework – using positive and negative basic impedances
6. Incorporate into system bond graph model – replace Z(s).
7. Determine state equations from bond graph model – identify virtual state variables.
8. Identify separable bond graph elements – select hybrid or fully active system
9. Select active devices physical realization – add idealized bond graph representation to model
10. Determine control signal diagram – graphical representation of the differential equations of virtual state variable
11. Perform simulations – obtain force – velocity profile and controlled source history

Add real (parasitic) actuator effects – repeat synthesis process.

Returning our attention to the nature of each impedance term in Equation (15), we see that the first impedance term is basic, which means that its impedance corresponds to a primitive bond graph elements, namely a resistive element. It is also positive definite, thus it is physically realizable element. The second term is composite, which means that its impedance does not correspond to a primitive bond graph element and must be further decomposed. It is also not positive definite, thus its resulting terms will not correspond to physically realizable elements. The third terms is composite, however no conclusion can be made about positive definiteness of its resulting terms until a further decomposition is carried out. The two composite impedance terms cannot be broken down any further. However, we can invert them to form admittances.
There is a distinction between the term active element, which refers to the model representation (including negative bond graph elements etc.), and

the term actuator, which refer to the actual physical realization of the active element

As a subset of these elements, the bond graph element with negative impedances are enclosure in a rectangle. Recall that, at this stage, the portion of the bond graph that represents the synthesized active element is strictly a representation of the behavior of the active element, and no topological or physical representation is yet implied.

The next task is to analyze a physical representation for the negative bond graph elements that are associated with the virtual state variables.

Using bond graph impedance methods, we form a symbolic expression for this transfer function. For this transfer function we need only to examine the bond graph

7. CONCLUSIONS

This paper demonstrated the use of the active element synthesis method in determining the design parameters of an active device.

The active element impedance was determining using impedance – based bond graph techniques and the desired frequency response function. The impedance was then decomposed into basic impedance terms in order to develop a bond graph model of the active element behavior. Preliminary simulation using this model showed that the synthesized active element met the desired frequency response behavior.

Its impedance – based approach is well suited to systems whose required behavior is expressed in terms of frequency response. Combined with the use of bond graph, this methodology easily allows for simulations to be run for various active device physical realizations and design parameters, such that active device operating regimes are obtained.

The results of these simulations allow a system designer to perform comparisons and make decisions regarding the preliminary design of an active device and its control system.

As an example, the active element chosen was a pipeline. After identifying the separable bond graph elements of the system, a bond graph model of the pipeline with a controlled was developed. A signal flow diagram was also developed, which provided information about the nature of the control system needed such that the system exhibited the desired frequency response behavior. This demonstrated the utility the synthesis method within a simulation – based design environment.

The second objective is to provide a validation of the methodology. Based strictly on the experimental data, we wish to determine if the synthesis method yields an active element whose behavior matches that of the existing active element in the system.

Thus, it can be concluded that the bond graph model of the reticulated filter produces the desired response characteristics.

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