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FLEXIBLE JOINTS ROBOTIC MANIPULATOR CONTROL BY ADAPTIVE GAIN SMOOTH SLIDING OBSERVER-CONTROLLER

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Abstract: An adaptive gain sliding observer for uncertain parameter nonlinear systems together with an adaptive gain sliding controller is proposed in this paper. It considered nonlinear, SISO affine systems, with uncertainties in steady-state functions and parameters. A further parameter term, adaptively updated, has been introduced in steady state space model of the controlled system, in order to obtain useful information despite fault detection and isolation. By using of the sliding observer with adaptive gain, the robustness to uncertainties is increased and the parameters adaptively updated can provide useful information in fault detection. Also, the state estimation error is bounded accordingly with bound limits of the uncertainties. The both of them, the sliding adaptive observer and sliding controller are designed to fulfill the attractiveness condition of its corresponding switching surface. An application to a single arm with flexible joint robot is presented. In order to alleviate chattering, a parameterized tangent hyperbolic has been used as switching function, instead of pure relay one, to the observer and the controller. Also, the gains of the switching functions, to the sliding observer and sliding controller are adaptively updated depending of estimation error and tracking error, respectively. By the using adaptive gains, the transient and tracking response can be improved.

Keywords: sliding adaptive observer-controller, nonlinear systems, flexible joint robots

1. WHY ADAPTIVE GAIN, SMOOTH SLIDING OBSERVER-CONTROLLER

The state and parameter uncertainties in the model of the rigid joints-rigid links robotic manipulators, as SISO non-linear systems and the deviations of the parameters from their nominal values lead to the difficulties of parameter identification and state estimation. All of these, do absolutely necessary the designing of the controller and/or the observer such as the closed loop to be robust. That means stability with small tracking and estimation errors. It is well known the robustness to model and parameter uncertainties and external disturbances of the closed loop with variable structure controller. Maintaining the system on sliding surface, the influence of the uncertainties into the closed loop performances is alleviated and the evolution is quickly to an equilibrium point. In [2] is used adaptive variable structure control with parameterized sigmoid as switching function (denoted k-sigmoid) with adaptive modifications of its amplitude (denoted λ-modification), instead of a pure relay one with constant gain. In this paper is used a parameterized tangent hyperbolic function (denoted k-tanh) as switching function in order to alleviate, or/and eliminate chattering. Robust variable structure observers for nonlinear systems could be found in [9]. The combinations of variable structure observer-controller for several particular nonlinear systems with application to robot manipulators are presented in [1] and [8]. Results concerning the exponential convergence of adaptive observer under persistent excitation conditions applied to a class of non-linear systems are shown in [5] and [6]. In [7], the persistent excitation condition is relaxed in the adaptive observer design and an extension to non-linear external perturbed systems is considered. A further parameter term, which could be adaptively updated, has been considered to the state model, in order to obtain information concerning parameters and their deviations from nominal values.

The main contributions of this paper are with the sliding observer and controller design, the choosing of the gains (the gains of linear part and variable structure part, respectively), updated law of variable structure gains, state and tracking error bounds. Variable structure gains, adaptively updated, become initial values into updating law.

The paper is organized as follows. In the Section 2 preliminary notions and assumptions concerning nonlinear affine systems in adaptive observer form are presented. In Section 3 is proposed an adaptive sliding observer together with possibilities for the choosing of them. State error bounds are provided, too. The design of the sliding controller will be done in Section 4. In Section 5 an application to a flexible joint, rigid link robot manipulator and additionally simulation results are presented. Some conclusions remark can be found in Section 6.

2. SISO NONLINEAR SYSTEMS IN ADAPTIVE OBSERVER FORM

Let the SISO nonlinear system be

$$(1)\quad \begin{aligned} \dot{x} &= f(x) + g(x)u + \sum_{i=1}^{p} q_i(x,u)\pi_i \\ y &= h(x) \end{aligned}$$

where: $x \in \Re^n$, $u, y \in \Re$, $f, g : \Re^n \to \Re^n$, $h : \Re^n \to \Re$, $\pi \in \Re^p$, $\pi = \begin{bmatrix}\pi_1 & \cdots & \pi_p\end{bmatrix}^T$, $q_i : \Re^n x \Re \to \Re^n$.

If the followings assumptions are hold:

A.2.1. $\operatorname{rank}\left[dh(x),\ dL_f h(x), \cdots,\ d\left(L_f^{n-1}h(x)\right)\right] = n$ $\quad \forall x \in \Re^n$

where $L_f^i h$ is the i-order Lie derivative of the smooth function h along the vector field f;

A.2.2. let r be the vector field which satisfies

$$(2)\quad \left\langle \begin{bmatrix} dh \\ \vdots \\ d\left(L_f^{n-1}h\right) \end{bmatrix}, r \right\rangle = \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix}$$

and $\left[ad_f^i r,\ ad_f^j r\right] = 0,\quad 0 \le i, j \le n-1$, where $ad_f^i r$ represents the i-order Lie bracket $[f, r]$ of two vector fields f and r, i.e.

$[f, r](h) = f(r(h)) - r(f(h)) = L_f L_r h - L_r L_f h$;

A.2.3. $\left[g, ad_f^j r\right] = 0,\quad 0 \le j-2 \le n-2$;

A.2.4. the vector fields $ad_f^i r,\ 0 \le i \le n-1$ are complete;

A.2.5. $\left[q_i,\ ad_f^i r\right] = 0,\quad 0 \le i \le p, 0 \le j \le n-2$;

A.2.6. $(3)\quad q_i(x,u) = \beta_i(h(x),u)\sum_{j=1}^{n} b_{n-j+1} ad_{(-f)}^{j-1} r(x),\quad 1 \le i \le p$

then, according to Lemma II.1 from [6], there exists a global space diffeomorphism

$$(4)\quad \begin{aligned} \zeta &= T(x) = \begin{bmatrix} h(x) & L_f h(x), & \cdots, & L_f^{n-1}h(x) \end{bmatrix}^T \\ T(x_o) &= 0 \end{aligned}$$

which transforms the system (1) into

$$\dot{\zeta} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \zeta + \psi_o(y)u + \sum_{i=1}^{p} \psi_i(y,u)\pi_i$$

$$(5)\quad \begin{aligned} &= A_c \zeta + \psi_o(y)u + \Psi(y,u)\pi \\ y &= \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \end{bmatrix}\zeta = c_c^T \zeta \end{aligned}$$

where $\psi_o : \Re \to \Re^n$, $\Psi_i : \Re \times \Re \to \Re^n$, $i = 1, \ldots, p$ are smooth functions.

Moreover, following [4] and [6], by using a filtered transformation

$$(6)\quad z = \zeta - M(t)\pi ,$$

the system (5) can be transformed into the adaptive observer form

$$(7)\quad \begin{aligned} \dot{z} &= A_c z + \psi_o(y)u + b\beta^T(t)\pi \\ y &= c_c^T z \end{aligned}$$

where $M, b,$ and β are expressed hereafter. Note that the transformation (6) does not change the term corresponding of the control input in (5). The matrix $M \in \Re^{nxp}$ can be expressed as $M = \begin{bmatrix} 0 \\ N \end{bmatrix}$, with $N \in \Re^{(n-1)xp}$ unique solution of the matrix differential equation

$$(8)\quad \dot{N} = A_N N + B_N \Psi(y,u), \quad N(0) = N$$

where

$$(9)\quad \begin{aligned} A_N &= \begin{bmatrix} -b_2 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -b_{n-1} & 0 & \cdots & 0 & 1 \\ -b_n & 0 & \cdots & 0 & 0 \end{bmatrix} \in \Re^{(n-1)x(n-1)} \\ B_N &= \begin{bmatrix} -b_2 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -b_{n-1} & 0 & \cdots & 1 & 0 \\ -b_n & 0 & \cdots & 0 & 1 \end{bmatrix} \in \Re^{(n-1)xn} \end{aligned}$$

The vector $b \in \Re^n, b = [1 \quad b_2, \cdots, \quad b_n]^T$ has constant elements which are the coefficients of Hurwitz polynomial: $s^{n-1} + b_2 s^{n-2} + \cdots + b_n$.

Replacing (6) in (8) and using the above notations, the matrix M can be written as the unique solution of the differential equation

$$\begin{aligned} \dot{M} &= \left(A_c - bc_c^T A_c\right)M + \left(I - bc_c^T\right)\Psi(y,u) \\ M(0) &= \begin{bmatrix} 0 \\ N(0) \end{bmatrix} \end{aligned} \quad (10)$$

The matrix A_N being a Hurwitz matrix, then the matrices $N(t)$ and $M(t)$ are bounded if the control input u, in (8), and the function $\Psi(y)$ are bounded.

The vector $\beta \in \Re^P$ is a continuous bounded function and can be expressed as:

$$(11)\quad \begin{aligned} \beta(y,u,t) &= \beta(t) = [\beta_1(t), \cdots, \quad \beta_p(t)]^T \\ &= c_c^T A_c M + c_c^T \Psi(y,u) \end{aligned}$$

Remark 2.1. If the assumptions A.2.2 and A.2.3 hold, then the each element $\psi_{oi}(y)u, \quad i = 1, \ldots, n$ is independent of x. If not, then some or all $\psi_{oi}(y)$ can depend of z (i.e. $\psi_{oi}(z,y)$). In this case the system (7) can be written as

$$(12)\quad \begin{aligned} \dot{z} &= A_c z + \psi_o(z,y)u + b\beta^T(y,u,t)\pi \\ y &= c_c^T z \end{aligned}$$

and, obviously, it is not in adaptive observer form.

Remark 2.2. If assumption A.2.6 holds the system (1) can be transformed directly in adaptive observer form (7), by using the global diffeomorphism (4), without the passing trough the intermediary transformed form (5).

Let ρ be the integer defined as the global relative degree of the system (1). According to the definition 4.1.2, from [5], the global relative degree is the integer such that

$$(13)\quad \begin{aligned} & L_g L_f^i h(x) = 0, \quad \forall x \in \Re^n, 0 \le i \le \rho - 2 \\ & L_g L_f^{\rho-1} h(x) \ne 0, \quad \forall x \in \Re^n \end{aligned}$$

Obviously, the transformed systems (4) and (7) are the same relative degree with the original system (1). Taking into account the relative degree, the elements of the vector term $\psi_o(y)u$, from (7) or (12), can be written as:

$$(14)\quad \begin{aligned} \psi_{oi} u &= 0, \quad i = 1, \ldots, \rho - 1 \\ \psi_{oj} u &= L_g L_f^{j-1} h(x) u, \quad j = \rho, \ldots, n-1 \\ \psi_{on} u &= L_f^n h(x) + L_g L_f^{n-1} h(x) u \end{aligned}$$

3. ADAPTIVE GAIN, SMOOTH SLIDING OBSERVER

In the followings, the attention is focused on the system (1) and on its transformed form (12). A sliding observer, with constant and/or adaptive gain is proposed in this section. All the uncertainties are considered on the function f and g. Define with $\hat{f}, \hat{g}$ the estimates of corresponding functions. No uncertain is considered into parameter vector π. The sliding observer is robust despite the third term from the right side member of differential equation (1) and (12).

Remark 3.1. The transformed system (12) is more general than the system (7), although the last one is in adaptive observer form.

The sliding mode evolution is performed on the sliding surface $z_1 - \hat{z}_1$ and will proof that the first ρ state estimate errors are ultimately bounded, while the others $n-\rho$ errors are bounded in the presence of the model uncertainties to the functions $f(x)$ and $g(x)$. The following assumptions are considered:

A.3.1. The functions f, g, h are all of C^n-class functions;

A.3.2. The transformation, introduced by (4), is a global diffeomorphism

A. 3.3. The relative degree of the system, defined by (13), is $\rho < n$;

A.3.4. The uncertainties in the functions $f(x)$ and $g(x)$, defined as

$$(15)\quad \begin{aligned} &\Delta_j = \left(L_g L_f^j h(x) - L_{\hat{g}} L_{\hat{f}}^j h(\hat{x})\right) u(\hat{x}, t) \\ &j = \rho - 1, \cdots, n-2 \\ &\Delta_n(x, \hat{x}, t) = L_f^n h(x) - L_{\hat{f}}^n h(\hat{x}) \\ &+ L_g L_f^{n-1} h(x) u(\hat{x}, t) - L_{\hat{g}} L_{\hat{f}}^{n-1} h(\hat{x}) u(\hat{x}, t) \end{aligned}$$

fulfill the following conditions

$$(16)\quad \left\| \begin{bmatrix} 0 & \cdots & 0 & \Delta_\rho & \cdots & \Delta_{n-1} & 0 \end{bmatrix}^T \right\| < \alpha, \quad \forall t \ge 0$$

$$(17)\quad \left| \Delta_n(x, \hat{x}, t) \right| < \varepsilon \left\| \hat{T}(\hat{x}) - T(x) \right\| + \varphi, \quad \forall t \ge 0$$

where the transformation $\hat{T}(x)$ is known diffeomorphism which allow the inverse and $\alpha, \varepsilon, \varphi$ are positive constants.

A.3.5. The vector function, $\beta(t) = \beta(y, u, t)$ is uniformly bounded for every (y, u) bounded.

Theorem 3.1. (Sliding observer convergence). It considers the systems (1), (12) or (7), the last one in adaptive observer form, $\hat{f}, \hat{g}$ available estimates of function f, g and uncertainties verifying (16), (17), then can be designed the adaptive sliding observer

$$(18)\quad \begin{aligned} &\dot{\hat{z}}_i = \hat{z}_{i+1} - \gamma_i(\hat{z}_1 - z_1) - \theta_i \tanh[k_o(\hat{z}_1 - z_1)] \\ &+ b_i \beta^T \pi, \quad i = 1, \ldots, \rho - 1 \\ &\dot{\hat{z}}_j = \hat{z}_{j+1} - \gamma_j(\hat{z}_1 - z_1) - \theta_j \tanh[k_o(\hat{z}_1 - z_1)] \\ &+ L_{\hat{g}} L_{\hat{f}}^{j-1} h\left(T^{-1}(\hat{z})\right) u + b_j \beta^T \pi, \quad j = \rho, \ldots, n-1 \\ &\dot{\hat{z}}_n = -\gamma_n(\hat{z}_1 - z_1) - \theta_n \tanh[k_o(\hat{z}_1 - z_1)] \\ &+ L_{\hat{f}}^n h\left(T^{-1}(\hat{z})\right) + L_{\hat{g}} L_{\hat{f}}^{n-1} h\left(T^{-1}(\hat{z})\right) u + b_n \beta^T \pi \end{aligned}$$

which has the evolution on the sliding surface

$$(19)\quad \tilde{z}_1 = T_1(x) - \hat{T}_1(\hat{x}) = 0$$

where k_o is a positive constant. Also, can be computed the vector gain $\Gamma = [\gamma_1, \ \cdots, \ \gamma_n]^T$ with the expression

$$(20)\quad \Gamma = -A_c b - \sigma b, \quad \sigma \in \Re_+,$$

such as $A_c + \Gamma c_c^T$ is a stable matrix and can be chosen the vector gain $\Theta = [\theta_1, \ \cdots, \ \theta_n]^T$ such that the error in the first ρ transformed state estimates is ultimately bounded by an arbitrarily small constant δ, the error of the other $n-\rho$ states is bounded.

Proof: The dynamics of the state estimation error is

$$(21)\quad \begin{aligned} &\dot{\tilde{z}}_i = \tilde{z}_{i+1} - \gamma_i \tilde{z}_1 - \theta_i \tanh(k_o \tilde{z}_1), \quad i = 1, \ldots, \rho - 1 \\ &\dot{\tilde{z}}_j = \tilde{z}_{j+1} - \gamma_j \tilde{z}_1 - \theta_j \tanh(k_o \tilde{z}_1) - L_g L_f^{j-1} h(x) u \\ &+ L_{\hat{g}} L_{\hat{f}}^{j-1} h(\hat{x}) u, \quad j = \rho, \ldots, n-1 \\ &\dot{\tilde{z}}_n = -\gamma_n \tilde{z}_1 - \theta_n \tanh(k_o \tilde{z}_1) - L_f^n h(x) \\ &- L_g L_f^{n-1} h(x) u + L_{\hat{f}}^n h(\hat{x}) + L_{\hat{g}} L_{\hat{f}}^{n-1} h(\hat{x}) u \end{aligned}$$

With the gain Γ computed as in (20), it obtains the polynomial identity

$$(22)\quad \begin{aligned} &(s + \sigma)\left(s^{n-1} + b_1 s^{n-2} + \cdots + b_n\right) \\ &= s^n + \gamma_1 s^{n-1} + \cdots + \gamma_n \end{aligned}$$

which, due to the n-1 zero-pole cancellations, leads to the first order strictly real positive transfer function

$$(23)\quad c_c^T \left[sI - \left(A_c + \gamma c_c^T\right) \right]^{-1} b = (s + \sigma)^{-1}$$

Because $A_c + \Gamma c_c^T$ is a stable matrix and satisfying the strictly positive real condition (condition B.1.2 from [5]), it may be applied Meyer-Kalman-Yacubovich. Lemma B.2 2 and the Theorem B.2.2, both of them from [5]. Consequently, the linear part of the state estimation dynamics, from (18), is globally asymptotically stable, i.e.

$$(24)\quad \lim_{t \to \infty} \left\| z(t) - \hat{z}(t) \right\| = 0, \quad \forall z_o \in \Re^n, \quad \pi \in \Re^p$$

The value of the gain θ_1 has to be chosen such that on the surface $S_o = z_1 - \hat{z}_1 = 0$ the sliding condition is fulfilled (the attractiveness condition)

$$(25)\quad S_o \dot{S}_o < 0$$

This above condition leads to

$$(26)\quad \theta_1 + \gamma_1 |z_1 - \hat{z}_1| > |z_2 - \hat{z}_2|, \quad \forall t \geq 0$$

and, by choosing the gain θ_1 such that

$$(27)\quad \theta_1 > |\hat{z}_2 - z_2| = \left|\hat{T}_2(\hat{x}) - T_2(x)\right| = |\tilde{z}_2|, \quad \forall t \geq 0$$

the inequality (26) is satisfied with the equivalent state error dynamics

$$\dot{\tilde{z}}_i = \tilde{z}_{i+1} - \frac{\theta_i}{\theta_1}\tilde{z}_2, \quad i = 1,\ldots,\rho-1$$

$$\dot{\tilde{z}}_j = \tilde{z}_{j+1} - \frac{\theta_j}{\theta_1}\tilde{z}_2 - L_g L_f^{j-1} h(x) u(\hat{x},t)$$

$$(28)\quad + L_{\hat{g}} L_{\hat{f}}^{j-1} h(\hat{x}) u(\hat{x},t), \quad j = \rho,\ldots,n-1$$

$$\dot{\tilde{z}}_n = -\frac{\theta_n}{\theta_1}\tilde{z}_2 - L_f^n h(x) - L_g L_f^{n-1} h(x) u(\hat{x},t)$$

$$+ L_{\hat{f}}^n h(\hat{x}) + L_{\hat{g}} L_{\hat{f}}^{n-1} h(\hat{x}) u(\hat{x},t)$$

The gains θ_i can be chosen such that the following polynomial identity holds

$$(29)\quad s^{n-1} + \frac{\theta_2}{\theta_1}s^{n-2} + \ldots + \frac{\theta n}{\theta_1} = (s+\theta)^{n-1}$$

where θ is positive. Making the change of variables

$$(30)\quad \begin{aligned}\tilde{z}_i &= [\tilde{z}_2 \;\; \cdots \;\; \tilde{z}_n]^T \\ &= \left[\nu_2 \;\; \cdots \theta^{i-2}\nu_i \cdots \;\; \theta^{n-2}\nu_n\right]^T, \quad i = 2,\ldots,n\end{aligned}$$

taking into account (16), the last $n-1$ error equations from (21) become

$$(31)\quad \dot{\nu} = \theta D \nu + \begin{bmatrix} 0 & \cdots & 0 & \dfrac{\Delta_\rho}{\theta^{\rho-2}} & \cdots & \dfrac{\Delta_n}{\theta^{n-2}} \end{bmatrix}^T$$

$$(32)\quad D = \begin{bmatrix} -C_{n-1}^1 & 1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -C_{n-1}^{n-2} & 0 & \ddots & 1 \\ -C_{n-1}^{n-1} & 0 & \ldots & 0 \end{bmatrix} \in \Re^{(n-1)x(n-1)}$$

It defines the Lyapunov function

$$(33)\quad V = \nu^T P \nu$$

with the positive definite matrix $P \in \Re^{(n-1)x(n-1)}$ defined as the solution of

$$(34)\quad PD + D^T P = -I$$

Its derivative can be obtained as

$$(35)\quad \begin{aligned}\dot{V} &= -\theta\|\nu\|^2 + 2\nu^T P [0 \;\; \ldots \;\; 0 \;\; 1]^T \frac{\Delta_n}{\theta^{n-2}} \\ &+ 2\nu^T P \begin{bmatrix} 0 & \ldots & 0 & \dfrac{\Delta_\rho}{\theta^{\rho-2}} & \cdots & \dfrac{\Delta_{n-1}}{\theta^{n-3}} & 0 \end{bmatrix}^T\end{aligned}$$

which, by the using (16), (17), the definition of ν and the inequality $\nu^T P \nu \leq \lambda_{\max P}\|\nu\|$, can be bounded by

$$\dot{V} \leq -\theta\|\nu\|^2 + 2\|\nu\|\|P_n\|\left(\frac{\varphi}{\theta^{n-2}} + \varepsilon\|\nu\|\right) + 2\|\nu\|\frac{\lambda_{\max P}\alpha}{\theta^{\rho-2}}$$

where P_n is the last column of matrix P. By the addition of the both sides of (36) the term $\mu\|\nu\|^2$, $\mu > 0$, it obtains the second order inequality

$$(37)\quad \begin{aligned}\dot{V} + \mu\|\nu\|^2 &\leq -(\theta - 2\varepsilon\|P_n\| - \mu)\|\nu\|^2 \\ &+ 2\frac{1}{\theta^{\rho-2}}\left(\varphi\frac{\|P_n\|}{\theta^{n-2}} + \lambda_{\max P}\alpha\right)\|\nu\|\end{aligned}$$

which is satisfied for

$$(38)\quad \|\nu\| \geq \frac{2}{\theta^{\rho-2}} \frac{\varphi\|P_n\| + \lambda_{\max P}\alpha\theta^{n-\rho}}{(\theta - 2\varepsilon\|P_n\| - \mu)\theta^{n-\rho}}$$

Due to the Corollary 5.3 of Theorem 5.1 from [3], there exists a finite time t_1 such that

$$(39)\quad \begin{aligned}\|\nu(t)\| &\leq \frac{2}{\theta^{\rho-2}}\sqrt{\frac{\lambda_{\max P}}{\lambda_{\min P}}}\frac{\varphi\|P_n\| + \lambda_{\max P}\alpha\theta^{n-\rho}}{(\theta - 2\varepsilon\|P_n\| - \mu)\theta^{n-\rho}} \\ &\forall t \geq t_1\end{aligned}$$

The definition of the vector ν, from (30), and the above inequality lead to the conclusion that

$$(40)\quad \begin{aligned}|\tilde{z}_i(t)| &= \left|\hat{T}_i(\hat{x}) - T_i(x)\right| \leq \theta^{i-2}\|\nu\| \leq \\ &\leq 2\frac{\theta^{i-2}}{\theta^{\rho-2}}\sqrt{\frac{\lambda_{\max P}}{\lambda_{\min P}}}\frac{\varphi\|P_n\| + \lambda_{\max P}\alpha\theta^{n-\rho}}{(\theta - 2\varepsilon\|P_n\| - \mu)\theta^{n-\rho}}, \\ &\forall t \geq t_1, \quad i = 2,\ldots,n\end{aligned}$$

Therefore, all the state observation errors in the transformed space converge to a bounded region, and the first ρ errors could be arbitrarily small for sufficiently large θ. The errors $|\tilde{z}_i|$, $i = 1,\ldots,\rho$ could be made arbitrarily small respecting a value depending on a positive constant δ

$$(41)\ \delta \geq \frac{2}{\theta^{\rho-2}} \sqrt{\frac{\lambda_{maxP}}{\lambda_{minP}}} \frac{\varphi\|P_n\| + \lambda_{maxP}\alpha\theta^{n-\rho}}{(\theta - 2\varepsilon\|P_n\| - \mu)\theta^{n-\rho}}, \quad \forall t \geq t_1$$

The upper bounds for the state estimation errors, given by (40) allow the using of the adaptive gain in sliding observer (18). As in [2], the gains $\theta_i, i = 1, \cdots, n$ become time depending, including λ-modification

$$(42)\ \dot{\theta}_i(t) = -\lambda_{oi}\theta_i(t) - \vartheta_{oi}|\tilde{z}_1(t)|$$

where $\lambda_{oi}, \vartheta_{oi}$ are positive constants and $\theta_i(t_o)$ are chosen respecting polynomial identity (29).

Remark 3.1. If are used adaptive gains, then, in (18), the variable structure term is introduced with changed sign.

4. ADAPTIVE GAIN SMOOTH SLIDING CONTROLLER

Generalizing the sliding controller design, presented [8] and [9], to nonlinear affine systems with the relative degree strictly less than the state dimension, it defines the controller sliding surface

$$(43)\ \hat{S}_c = \sum_{i=1}^{\rho} \xi_i \left(\hat{z}_i - y_r^{(i-1)}\right)$$

with $\xi_\rho = 1$, $\xi_i, i = 1, \cdots, \rho - 1$ Hurwitz coefficients and y_r the reference to be tracked, assumed to be a C^n function. The expression of sliding controller

$$(44)\ u = \left(L_{\hat{g}} L_{\hat{f}}^{\rho-1} h(\hat{x})\right)^{-1} \begin{bmatrix} -L_{\hat{f}}^{\rho} h(\hat{x}) + y_r^{(\rho)} \\ -\sum_{i=1}^{\rho-1} \xi_i \left(\hat{z}_{i+1} - y_r^{(i)}\right) \\ -\phi \hat{S}_c - \eta \tanh\left(k_c \hat{S}_c\right) \end{bmatrix}$$

is derived from the expression of feedback linearization controller where the gain η, of the variable structure term, has to be computed to fulfill the attractiveness of the sliding surface and k_c is a positive constant. The derivative of the controller sliding surface (43) is

$$(45)\ \dot{\hat{S}}_c = -\tilde{z}_2 \sum_{i=1}^{\rho} \xi_i \frac{\theta_i}{\theta_1} - \phi \hat{S}_c - \eta \tanh\left(k_c \hat{S}_c\right)$$

where gain ϕ is chosen in order to maintain $\hat{S}_c$ bounded during the observer transient. After convergence of the observer, applying (40), and assuming the gains θ_i are chosen as (29), the controller attractiveness condition

$$(46)\ \hat{S}_c \dot{\hat{S}}_c < 0$$

is satisfied if

$$(47)\ \eta > \frac{2}{\theta^{\rho-2}} \sqrt{\frac{\lambda_{max P}}{\lambda_{min P}}} \frac{\varphi\|P_n\| + \lambda_{max P}\alpha\theta^{n-\rho}}{(\theta - 2\varepsilon\|P_n\| - \mu)\theta^{n-\rho}} \left|\sum_{i=1}^{\rho} \xi_i C_{n-1}^{i-1} \theta^{i-1}\right|$$

The equivalent dynamics during sliding is

$$(48)\ \hat{S}_c = \sum_{i=1}^{\rho} \xi_i \left(\hat{z}_i - y_r^{(i-1)}\right) = 0$$

which can rewritten as

$$(49)\ \sum_{i=1}^{\rho} \xi_i \left(y^{(i-1)} - y_r^{(i-1)}\right) = -\sum_{i=1}^{\rho} \xi_i \tilde{z}_i$$

Assuming the gains ξ_i are the coefficients of a polynomial with all stable and real roots, and using the bounds on the state errors (40), the tracking error satisfies

$$(50)\ |y(t) - y_r(t)| \leq \frac{2}{\xi_1 \theta^{\rho}} \sqrt{\frac{\lambda_{max P}}{\lambda_{min P}}} \frac{\varphi\|P_n\| + \lambda_{max P}\alpha\theta^{n-\rho}}{(\theta - 2\varepsilon\|P_n\| - \mu)\theta^{n-\rho}} \sum_{i=1}^{\rho} \xi_i \theta^i$$

for t sufficiently large (especially after the observer transient).

Remark 4.1. If the variable structure term, in (44), has the gain adaptively updated by λ-modification

$$(51)\ \dot{\eta}(t) = -\lambda_c \eta(t) - \vartheta_c |S_c|$$

with λ_c, ϑ_c positive constants, then the tracking error will decreasing asymptotically, after observer and controller transient.

5. APPLICATION TO A FLEXIBLE JOINT ROBOTIC MANIPULATOR

The dynamic equations of a single link robot arm with a revolute elastic joint (robot with flexible joints) rotating in a vertical plane are given by

$$J_a\ddot{q}_1 + F_a\dot{q}_1 + k(q_1 - q_2) + Mgl\sin q_1 = 0$$

$$(52)\quad J_m\ddot{q}_2 + F_m\dot{q}_2 - k(q_1 - q_2) = u$$

$$y = q_2$$

in which q_1 and q_2 are the link displacement angle and the rotor (motor shaft) displacement angle, respectively. The link inertia J_a, the motor rotor inertia J_m, the elastic constant k, the mass link M, the gravity constant g, the centre of mass l and the viscous friction coefficients F_a, F_m are positive constant parameters.

The control u is the torque delivered by the motor. Choosing as state variables $x_1 = q_1$, $x_2 = \dot{q}_1$, $x_3 = q_2$, neglecting the viscous friction of the arm, considering as measured output the position of the motor shaft, and introducing the vector term

$$(53)\quad b\beta^T(y,u,t)\pi = b[y \quad u][1 \quad 1]^T$$

then can be written the system steady state space equations

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix} x_2 \\ -(Mgl/J_a)\sin(x_1) - (k/J_a)(x_1 - x_3) \\ x_4 \\ -(F_m/J_m)x_4 + (k/J_m)(x_1 - x_3)\end{bmatrix}$$

$$(54)\quad + \begin{bmatrix}0\\0\\0\\(1/J_m)\end{bmatrix} u + \begin{bmatrix}u+y\\3(u+y)\\3(u+y)\\u+y\end{bmatrix} = f(x) + g(x)u + b\beta^T\pi$$

$$y = x_3$$

The following parameters and uncertainties are considered (note that the matching conditions are not fulfilled):

$M = 5$, $g = 10$, $l = 0.5$, $k = 200$, $J_a = 1$, $J_m = 0.05$, $F_m = 0.1$, $K_a = 200$, $K_m = 4500$, $B_m = 2$, $\hat{J}_m = 0.06$, $\hat{M}_a = 30$, $\hat{K}_a = 300$, $\hat{K}_m = 4500$, $\hat{B}_m = 1.5$,

where: $M_a = \dfrac{Mgl}{J_a}$, $K_a = \dfrac{k}{J_a}$, $K_m = \dfrac{k}{J_m}$, $B_m = \dfrac{F_m}{J_m}$.

With these notations the robot state equations (54) can be rewritten as

$$\dot{x}_1 = x_2 + u + y$$

$$\dot{x}_2 = -M_a\sin(x_1) - K_a(x_1 - x_3) + 3(u+y)$$

$$(55)\quad \dot{x}_3 = x_4 + 3(u+y)$$

$$\dot{x}_4 = -B_m x_4 + K_m(x_1 - x_3) + \frac{1}{J_m}u + u + y$$

$$y = x_3$$

The Lie derivatives, $L_f^i h(x), i = 0, \cdots, n$, are:
$h(x) = x_3$, $L_f h(x) = x_4$,

$$L_f^2 h(x) = -B_m x_4 + K_m(x_1 - x_3),$$

$$L_f^3 h(x) = (B_m^2 - K_m)x_4 - B_m K_m(x_1 - x_3) + K_m x_2$$

$$L_f^4 h(x) = (2B_m K_m - B_m^3)x_4 - M_a K_m \sin(x_1) + K_m(B_m^2 - K_m - K_a)(x_1 - x_3) - B_m K_m x_2$$

The Lie derivatives, $L_g L_f^i h(x), i = 0, \cdots, n-1$, are:

$L_g h(x) = 0$, $L_g L_f h(x) = \dfrac{1}{J_m}$, $L_g L_f^2 h(x) = -\dfrac{B_m}{J_m}$,

$$L_g L_f^3 h(x) = \frac{B_m^2 - K_m}{J_m}$$

It is easy to observe that the system is of order 4 (n=4) and of relative degree two ($\rho = 2$). The state transformation, defined in (4), is

$$(56)\quad z = \begin{bmatrix} x_3 \\ x_4 \\ -B_m x_4 + K_m(x_1 - x_3) \\ (B_m^2 - K_m)x_4 - B_m K_m(x_1 - x_3) + K_m x_2\end{bmatrix}$$

which has the following inverse transform

$$(57)\quad x = T^{-1}(z) = \begin{bmatrix} \dfrac{z_3 + B_m z_2 + K_m z_1}{K_m} \\ \dfrac{z_4 + B_m z_3 + K_m z_2}{K_m} \\ z_1 \\ z_2 \end{bmatrix}$$

The transformed state equations are

$$\dot{z}_1 = z_2 + u + y$$

$$\dot{z}_2 = z_3 + \frac{1}{J_m} u + 3(u + y)$$

$$\dot{z}_3 = z_4 - \frac{B_m}{J_m} u + 3(u + y)$$

(58)

$$\dot{z}_4 = -M_a K_2 \sin\left(\frac{z_3 + B_m z_2 + K_m z_1}{K_m}\right)$$

$$- B_m K_a z_2 - (K_a + K_m) z_3 - B_1 z_4$$

$$+ \frac{B_m^2 - K_m}{J_m} u + (u + y)$$

In order to alleviate the chattering in the state estimates and in control input, will be used a parameterized tangent hyperbolic as switching function and gain adaptively updated to the observer, as (42), and to the controller as (51). Choosing $b = [1 \quad 3 \quad 3 \quad 1]^T$ and $\sigma = 10$, the relationship (20) yields the observer vector gain $\Gamma = [-13 \quad -33 \quad -31 \quad -10]^T$. With the values: $\theta = 50$, $\theta_1 = 1$, from the polynomial identity (29) can be obtained the other observer variable structure gains: $\theta_2 = 150$, $\theta_3 = 7500$ and $\theta_4 = 125000$.

$$\dot{\hat{z}}_1 = \hat{z}_2 - \gamma_1 \tilde{z}_1 + \theta_1(t)\tanh(k_o \tilde{z}_1) + u + y$$

$$\dot{\hat{z}}_2 = \hat{z}_3 - \gamma_2 \tilde{z}_1 + \theta_2(t)\tanh(k_o \tilde{z}_1)$$
$$+ \frac{1}{\hat{J}_m} u + 3(u + y)$$

$$\dot{\hat{z}}_3 = \hat{z}_4 - \gamma_3 \tilde{z}_1 + \theta_3(t)\tanh(k_o \tilde{z}_1)$$
$$- \frac{\hat{B}_m}{\hat{J}_m} u + 3(u + y)$$

$$\dot{\hat{z}}_4 = -\hat{M}_a \hat{K}_m \sin\left(\frac{\hat{z}_3 + \hat{B}_m \hat{z}_2 + \hat{K}_m \hat{z}_1}{\hat{K}_m}\right)$$
$$- \gamma_4 \tilde{z}_1 + \theta_4(t)\tanh(k_o \tilde{z}_1) - \hat{B}_m \hat{K}_a \hat{z}_2$$

$$- (\hat{K}_a + \hat{K}_m)\hat{z}_3 - \hat{B}_m \hat{z}_4$$

(59) $$+ \frac{\hat{B}_m^2 - \hat{K}_m}{\hat{J}_m} u + u + y$$

Note that, if is used adaptive gain, including λ-modification the observer variable structure gains become negative initial values for the adaptation law (42).

Accordingly with (43), the controller sliding surface is defined as

$$(60)\quad \hat{S}_c = \hat{z}_2 - \dot{y}_r + \xi(\hat{z}_1 - y_r)$$

with $\xi = 10$. The corresponding sliding control input, can be expressed as

$$(61)\quad u = \hat{J}\left[-\hat{z}_3 + \ddot{y}_r - \xi(\hat{z}_2 - \dot{y}_r) - \phi\hat{S} + \eta(t)\tanh(k_c \hat{S}_c)\right]$$

where, to fulfill the attractiveness condition (47), the initial values to the updated law (51) has been chosen $\eta(0) = -50$. In order to increase the sliding observer convergence and to feed to the sliding controller state estimates closer to the true ones, the parameter k_o has to be chosen greater than k_c in their corresponding switching function. Therefore, the gain of the tangent hyperbolic switching function is greater around the origin.

The trajectory to be tracked is assumed to be sinusoidal as $y_r(t) = 1 + \cos(2t)$.

In the figure 1 can be observed the response without chattering, due to an appropriate choice of the parameters in switching functions (the convergence speed of the observer is greater then of the controller one). The chattering could appear due to uncertainties in the functions f, g and to the supplementary term $b\beta^T\pi$. The response from the figure 2 exhibits a chattering during the transient time of the observer, that has a convergence rate comparable of the controller one. Limitation of the controller amplitude has been introduced. The above values of θ_i, obtained by respecting the polynomial identity (29) have been used as negative initial values in the update law (42).

6. CONCLUSIONS

A sliding observer and sliding controller with gains of the modulation functions adaptively updated are proposed for controlling of nonlinear systems with relative degree smaller than state dimension. The switching function has been chosen a parameterised tangent hyperbolic function. The state dynamics of the controlled system include an extra parameter term, further adaptively updated, in order to obtain useful information despite fault detection. The parameterised tangent hyperbolic switching function assures the alleviation or completely elimination of chattering, by appropriate choice of parameters in the switching functions of the controller and observer Adaptive gains, starting from appropriate initial values lead to the better output tracking and to the augmented robustness. Convergence rates, both for the observer and controller have been established. An application to flexible joint one rigid link robot is presented. Closed loop response, obtained via simulation, confirms the theoretical results.

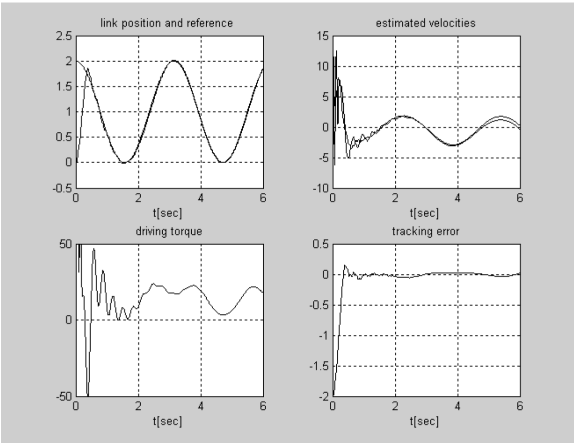

Fig.1. Closed loop robot response, smooth sliding observer and controller, parameterized tangent hyperbolic switching function $k_o = 0.5$, $k_c = 0.25$, adaptive gains with λ-modification: $\lambda_o = 1, \vartheta_o = 1$ $\lambda_c = 1$, $\vartheta_c = 1$

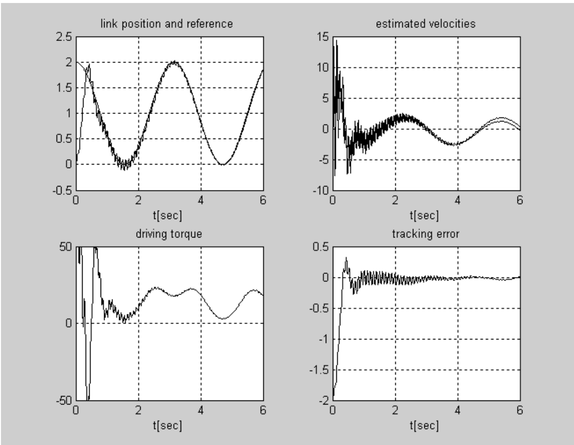

Fig.2. Closed loop robot response, smooth sliding observer and controller, parameterized tangent hyperbolic switching function $k_o = 50$, $k_c = 50$, adaptive gains with λ-modification: $\lambda_o = 1, \vartheta_o = 1$ $\lambda_c = 1$, $\vartheta_c = 1$.

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