NUMERICAL TREATMENT OF SOME COMPLEX DYNAMIC PROCESSES OF SATURATED INDUCTION MACHINE

Aurel CAMPEANU, Augustin-Iulian IONESCU, Toma CAMPEANU

University of Craiova, Electromechanical Faculty, Romania

Abstract: Three variants for the mathematical models of the saturated induction machine (the exact one, the classical one and the simplified one) are used for the analysis of the reversal process combined with one short mechanical overload. The limits of the classical and simplified models are emphasised and the suitability of the exact model is justified.

Keywords: induction machine, modeling, saturated model, simulation, dynamic processes

1. INTRODUCTION

In (Campeanu 1995a, b), there are detailed analyzed nonlinear dynamic models for the induction machine. Usually, in the analyses of the saturated machines the stator and rotor currents (Câmpeanu 1993) or flux linkages as state variables are taken in account. In (Brown 1983) the method of space phasors is applied with winding currents as state variables and in (Rousseau 1996) the equations of start processes are solved using Simulink techniques. In (Levi 1994) some combinations of currents and flux linkages as state variables are also taken in account. In the present paper a unique computational algorithm is presented. One of the models is used for the analyses of a reversal process combined with a very short mechanical overload. The results of the simulation and some conclusions based on these results are also presented.

2. MAIN RESULTS

2.1 The saturated induction machine models

For the induction machine saturated on the main flux way the mathematical model (1) is considered.

$$U_{dq} = A\frac{dX_{dq}}{dt} + BX_{dq}$$

$$(1)\quad U_{dq} = \left| u_{sd} \;\; u_{sq} \;\; 0 \;\; 0 \right|^{T}$$

$$X_{dq} = \left| X_{sd} \;\; X_{sq} \;\; X_{rd} \;\; X_{rq} \right|$$

where $X_{sd}, X_{sq}, X_{rd}, X_{rq}$ are the d,q projections of the currents and flux linkages taken as state variables.

To the above equations the motion equation is added

$$(2)\quad M - M_r = \frac{J}{p}\frac{d\omega}{dt}\ ,\quad M = \frac{3}{2}p\,\mathrm{Re}\left[j\underline{\Psi}_s \underline{i}_s^*\right]$$

It is assumed that the rotor is short-circuited and all quantities are related to the stator. The notations are the usual ones.

Saturation occurs in the expression of the main flux linkage

$$\underline{\psi}_m = L_m\left(\underline{i}_s + \underline{i}_r\right) = L_m\left(i_m\right)\underline{i}_m$$

$$\underline{i}_m = i_m e^{j\varphi}$$

L_m is the magnetization inductance and $\underline{i}_m$ is the space phasor of the magnetizing current. $\underline{\psi}_m$ and $\underline{i}_m$ have the same direction because the iron losses are neglected and consequently.

For establishing a direct calculus algorithm the possible combinations of the state variables are grouped in two classes:

This paper was recommended for publication by Dumitru CĂLUEANU

1) *The winding currents – flux linkages as state variables*;
2) The current $\underline{i}_m$ – currents (flux linkages) of the windings as state variables.

In the following only the mathematical models for the first group there are deduced.

In (Levi 1994) there are deduced the mathematical models for mixed combinations currents - flux linkages associated with the introduction of a generalized space phasor $\underline{\psi}$, which has the same orientation as the phasors of the magnetizing current $\underline{i}_m$. For different combinations of state variables, $\underline{\psi}$ may or may not have a real existence in the machine For the given state variables, $\underline{\psi}$ is a linear combination of them.

In what follows it will be seen that regardless the state variables choice (combinations winding currents - flux linkages), the deduction of the mathematical models is always associated with a term of the form $\underline{\psi}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{L_m^{'}}\right)$ which may be expressed in a general form; $L'_m=\underline{\psi}/\underline{i}_m$ is an inductance associated to the saturation state of the magnetic circuit. All is needed is that $\underline{\psi}$ and L'_m be defined for each choice of the state variables. Further it is shown how to define and express in detail this term.

It is computed first $\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{L_m^{'}}\right)=-\frac{1}{L_m^{'2}}\frac{\mathrm{d}L_m^{'}}{\mathrm{d}t}$ and results

$$(3)\quad \frac{\mathrm{d}L_m^{'}}{\mathrm{d}t}=\frac{\mathrm{d}L_m}{\mathrm{d}t}=\frac{1}{i_m}\left(L_{mt}-L_m\right)\frac{\mathrm{d}i_m}{\mathrm{d}t}$$

where $L_{mt}=\frac{\mathrm{d}\psi_m}{\mathrm{d}i_m}$ is the differential magnetizing inductance associated to L_m.
Also

$$(4)\quad \frac{\mathrm{d}\underline{\psi}}{\mathrm{d}t}=\frac{\mathrm{d}}{\mathrm{d}t}\left(L_m^{'}\underline{i}_m\right)=L_{mt}^{'}e^{j\varphi}\frac{\mathrm{d}i_m}{\mathrm{d}t}+jL_m^{'}\frac{\mathrm{d}\varphi}{\mathrm{d}t}\underline{i}_m$$

where:

$$(5)\quad L_{mt}^{'}=L_{mt}+\left(L_m^{'}-L_m\right)$$

L'_{mt} is the differential inductance associated to L'_m.

For the computation of $\frac{\mathrm{d}\varphi}{\mathrm{d}t}$ it is taken into account that $\underline{\psi}$ and $\underline{\psi}_m$ are in phase and it follows

$$(6)\quad \frac{\mathrm{d}\varphi}{\mathrm{d}t}=\frac{1}{\psi^2}\left(\psi_d\frac{\mathrm{d}\psi_q}{\mathrm{d}t}-\psi_q\frac{\mathrm{d}\psi_d}{\mathrm{d}t}\right)=\frac{1}{\psi}\left(\frac{\mathrm{d}\psi_q}{\mathrm{d}t}\cos\varphi-\frac{\mathrm{d}\psi_d}{\mathrm{d}t}\sin\varphi\right),$$

Finally taking into account (4), (5), (6) and introducing

$$(7)\quad \underline{i}_m=\frac{1}{L_m^{'}}\underline{\psi}=\frac{1}{L_m^{'}}\psi e^{j\varphi}$$

the term containing saturation becomes

$$(8)\quad \begin{aligned}\frac{1}{L_m^{'2}}\underline{\psi}\frac{\mathrm{d}L_m^{'}}{\mathrm{d}t}&=\left(\frac{1}{L_m^{'}}-\frac{1}{L_{dd}}\right)\frac{\mathrm{d}\psi_d}{\mathrm{d}t}-\frac{1}{L_{dq}}\frac{\mathrm{d}\psi_q}{\mathrm{d}t}+\\&+j\left[\left(\frac{1}{L_m^{'}}-\frac{1}{L_{qq}}\right)\frac{\mathrm{d}\psi_q}{\mathrm{d}t}-\frac{1}{L_{dq}}\frac{\mathrm{d}\psi_d}{\mathrm{d}t}\right].\end{aligned}$$

The computation inductance are introduced:

$$(9)\quad \begin{aligned}&\frac{1}{L_{dd}}=\frac{\sin^2\varphi}{L_m^{'}}+\frac{\cos^2\varphi}{L_{mt}^{'}},\quad \frac{1}{L_{qq}}=\frac{\cos^2\varphi}{L_m^{'}}+\frac{\sin^2\varphi}{L_{mt}^{'}},\\&\frac{1}{L_{dq}}=\left(\frac{1}{L_m^{'}}-\frac{1}{L_{mt}^{'}}\right)\sin\varphi\cos\varphi.\end{aligned}$$

From (6) also results the transient angular speed $\omega_\psi(t)$ of the main flux likage

$$(10)\quad \omega_\psi(t)=\frac{d\varphi}{dt}+\omega_B$$

where ω_B is the angular speed of the general reference frame *d,q*.

Expressions (8), (9), (10) are valid for any combinations of the state variables discussed above.

The general flux linkage $\underline{\psi}$ and the inductance $L_m^{'}$, $L_{mt}^{'}$ for various pairs of currents and flux linkages are relatively easily computed.

Let $\underline{i}_s,\underline{\psi}_r$ be state variables. After performing of the calculus it is obtained $L_m^{'}=L_{r\sigma}+L_m=L_r$ and the general flux linkage

$$\underline{\psi}=\psi_d+j\psi_q=L_m^{'}\underline{i}_m=\underline{\psi}_r+L_{r\sigma}\underline{i}_s$$

where $\underline{\psi}$, $\underline{i}_m$, $\underline{\psi}_m$ are co-linear and $\underline{\psi}$ depends linearly of the state variables $\underline{i}_s$, $\underline{\psi}_r$.
Separating the real and imaginary components in the mathematical model (1) where

$$X_{dq}=\left|i_{sd}\quad i_{sq}\quad \psi_{rd}\quad \psi_{rq}\right|^{\mathrm{T}}$$

the matrices *A, B* obtain the form

$$A=\begin{vmatrix} L_\sigma-\frac{L_{r\sigma}^2}{L_{dd}} & -\frac{L_{r\sigma}^2}{L_{dq}} & 1-\frac{L_{r\sigma}}{L_{dd}} & -\frac{L_{r\sigma}}{L_{dq}} \\ \frac{L_{r\sigma}^2}{L_{dq}} & L_\sigma-\frac{L_{r\sigma}^2}{L_{qq}} & -\frac{L_{r\sigma}}{L_{dq}} & 1-\frac{L_{r\sigma}}{L_{qq}} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$B=\begin{vmatrix} R_s & -\omega_B\left(L_\sigma-\frac{L_{r\sigma}^2}{L_r}\right) & 0 & -\omega_B\frac{L_m}{L_r} \\ \omega_B\left(L_\sigma-\frac{L_{r\sigma}^2}{L_r}\right) & R_s & \omega_B\frac{L_m}{L_r} & 0 \\ -R_r\frac{L_m}{L_r} & 0 & \frac{R_r}{L_r} & -(\omega_B-\omega) \\ 0 & -R_r\frac{L_m}{L_r} & \omega_B-\omega & \frac{R_r}{L_r} \end{vmatrix}$$

where $L_\sigma = L_{s\sigma} + L_{r\sigma}$.

The electromagnetic torque M in the coordinates of the state variables $\underline{i}_s$, $\underline{\psi}_r$ is obtained from (2) by eliminating $\underline{\psi}_s$. The final form is

$$M=\frac{3}{2}p\frac{L_m}{L_r}\left(\psi_{rd}i_{sq}-\psi_{rq}i_{sd}\right)$$

2.2 *Simulation results*

It is analyzed the process of reversing on which is superposed a short overload for an asynchronous motor of 4KW, with the parameters:

$R_s = 1{,}16\ \Omega$, $R_r = 4.74\ \Omega$, $L_{s\sigma} = 0{,}024$ H, $L_{r\sigma} = 0{,}0034$H, $p = 2$, $J = 0{,}048$ kgm^2.

The magnetic characteristic $\psi_m(i_m)$ is approximated in the form:

$$\psi_m=\begin{vmatrix} 0{,}4i_m & i_m<0{,}75A \\ 0{,}407\cdot 0{,}921^{i_m}\cdot i_m^{0{,}81} & i_m>0{,}75A \end{vmatrix}$$

There are compared the results of simulations when considering the model with matrices A, B having the above presented format, which take into account the saturation **(**$L_{mt}(i_m)\neq L_m(i_m)$;$L_{dq}\neq 0$**)**, and those of the simplify model ($L_{mt}=L_m(i_m)$;$L_{dq}=0$). There are plotted the curves $i_A(t), i_m(t), \omega_\psi(t), \omega(t)$ etc (ω_ψ is the rotation electric speed of the main rotating magnetic field during transients). In order to ensure a high saturation level, all the simulations were performed at U=300V.

The initial value for the rotation speed is ω =314.

The short mechanical overload is presented in the figure Fig.1.

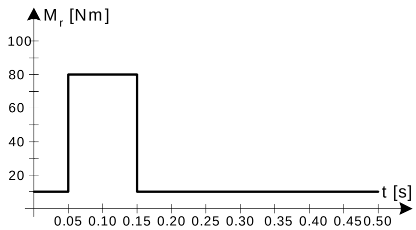

Fig.1 The short mechanical overload

In the figures Fig. 2a and Fig. 2b it is represented the current in the faze A. The difference between the two models is evident. The exact method emphasizes the existence of an oscillation of low frequency superposed on the transient evolution of the current.

In the figures Fig. 3a and Fig. 3b it is represented the current i_s(t). The taken in account of the inductance *Lmt* produces the oscillations for t>0.7s.

The same situation can be observed in the figures Fig. 4a and Fig. 4b. for the current i_m.

In all this cases the influence of the mechanical overload is not evident.

In the figures Fig. 5a and Fig. 5b there are represented the velocity $\omega_\psi(t), \omega(t)$**.** The evolution of $\omega_\psi(t)$ is practically not affected by the mechanical overload but the evolution of $\omega(t)$ is profoundly affected by this overload. The effect of the saturation on the evolution of the two velocities is not emphases by the simplified model.

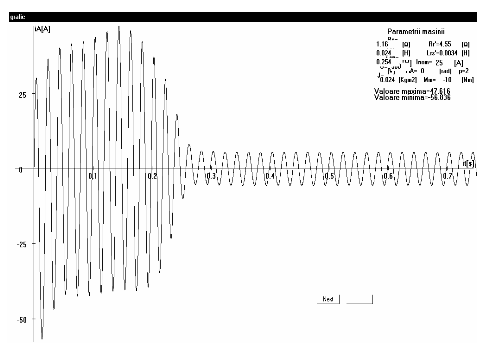

Fig. 2a

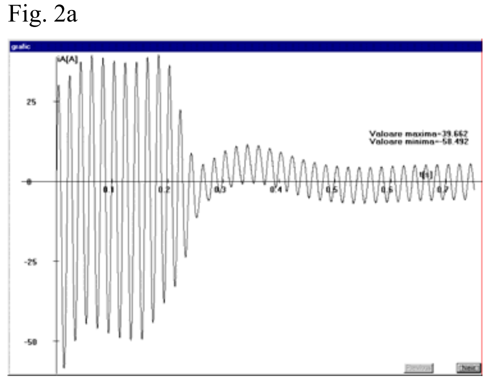

Fig. 2b

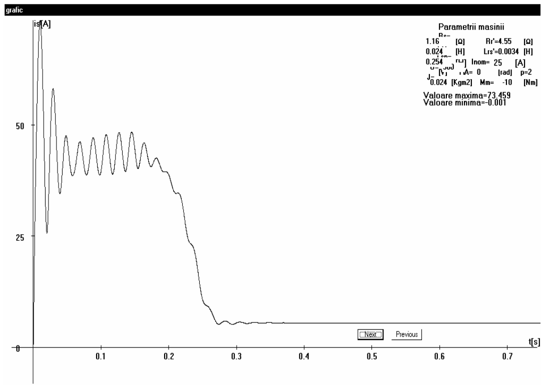

Fig. 3a

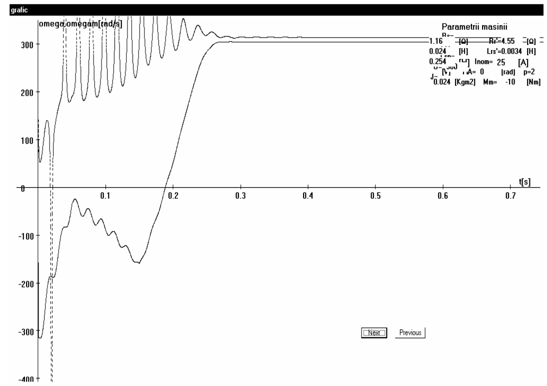

Fig. 5a

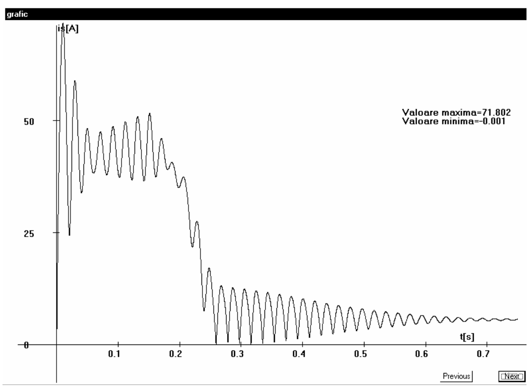

Fig. 3b

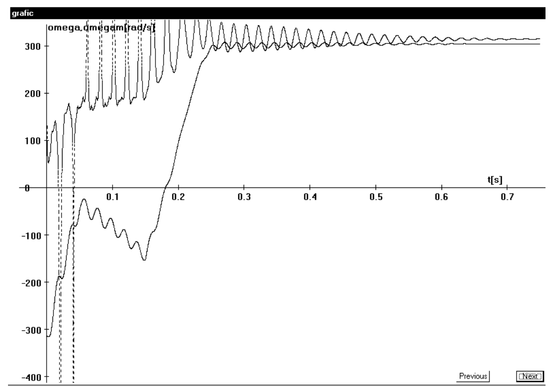

Fig. 5b

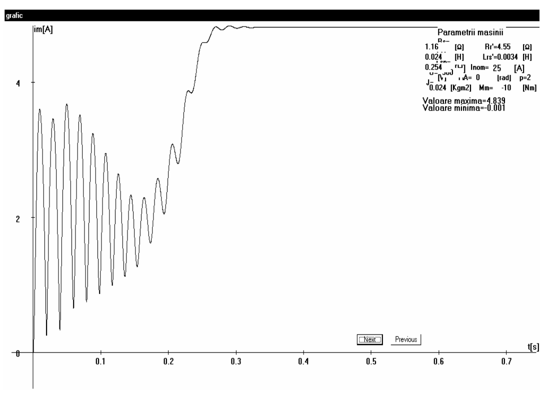

Fig. 4a

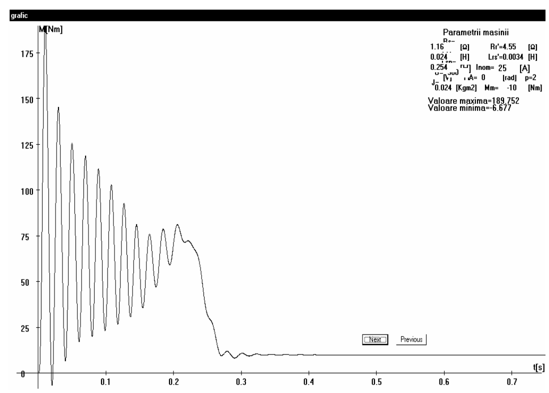

Fig. 6a

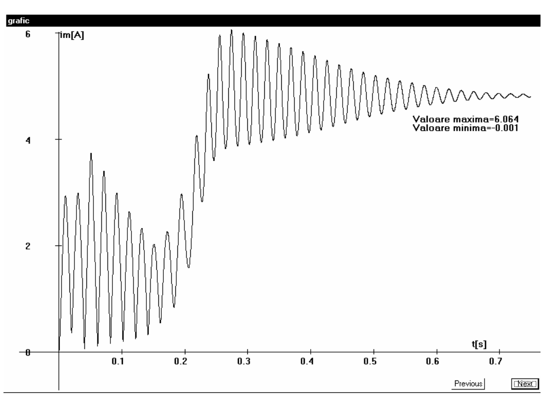

Fig. 4b

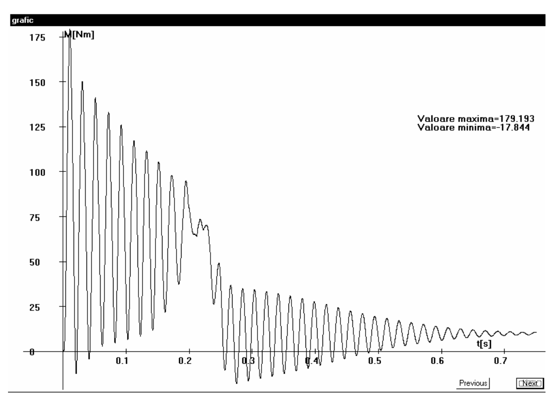

Fig. 6b

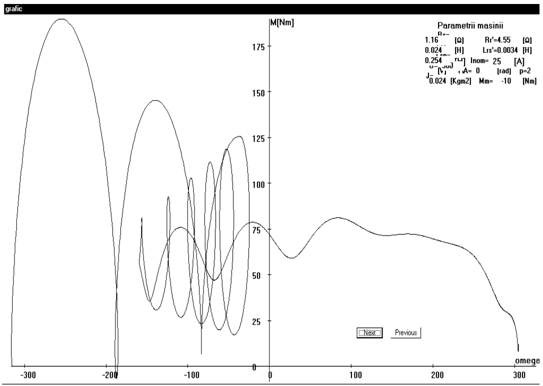

Fig. 7a

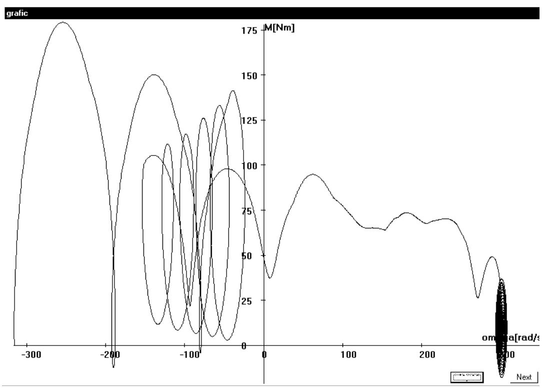

Fig. 7b

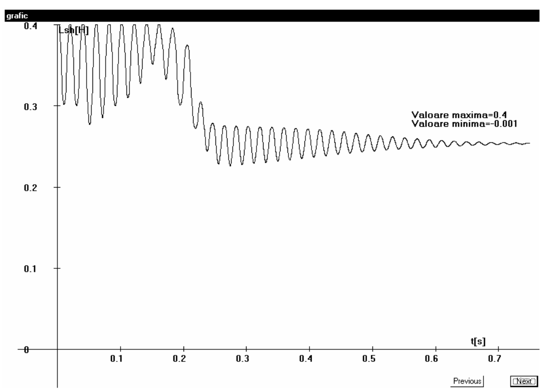

Fig. 8

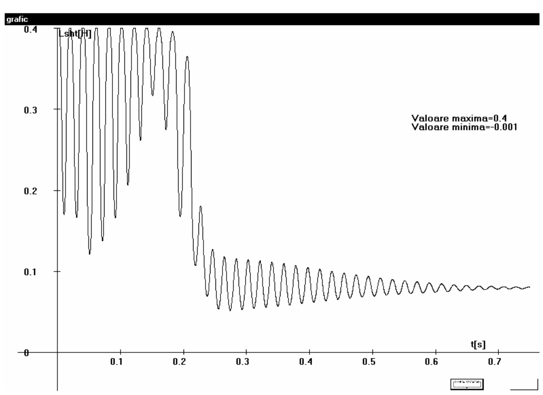

Fig. 9

In the figures Fig. 6a and Fig. 6b there is plotted the torque *M(t)*. The effect of saturation is evident but the effect of the mechanical overload is difficult to be emphasized. For the visualization of the mechanical overload on the electromagnetic torque, the graphics *M(ω)* are represented in Fig. 7a and Fig. 7b.

The figures Fig. 8 and Fig. 9 represent the evolution of the inductance L_m respectively L_{mt} . Comparing the evolution of the inductance with all the other values it is evident the influence of the saturation on the dynamic processes in the induction motor.

At the beginning of the dynamic process the effect of saturation is negligible and the graphics for the both models are very closed.

3. CONCLUSIONS

1. In the presence of saturation the simplified models do not reveal in a correct way the evolution of the dynamic electromechanical process.

2. The dynamic variation of the saturation essentially justifies the forced electromechanical oscillations.

3. At the reduced saturation, all models derive identical results.

4. REFERENCES

Brown, K. P., Vas, P.(1983) *Method of Including the Effects of Main Flux Path Saturation in the Generalized Equations of A. C. Machines*. IEEE Transactions on PAS-102, 1, 1983.

Campeanu, A..(1993): *About the Saturation Effects in the Alternative Current Machine Equations. Rev. Roumaine des Sci Techniques,Seria Electrot. Et Energ.*.1993

Campeanu, A (1995a): *Transient Performance of the Saturated Induction Machine* Electrical Engineering (A. f. E), 78, 4,.

Campeanu, A., Cimpeanu, T (1995b).: *General Case Analysis of Saturated Induction Machine*. ELECTROMOTION ’95 Cluj-Napoca.

Halenius, K, E., Vas, P., Brown, J.E. (1988): The Analysis of a Saturated Self-excited Asynchronous Generatore. IEEE Paper, 88 WM017-6

Levi, E.(1994): *Magnetic Saturation of Rotor-Flux Oriented Induction Motor Drives: Operating Regimes, Consequences and Open-Loop Compensation* ETEP 4

Rousseau, I. M., Hennuy, G.: *Numerical Treatment of the Transient Behaviour of an Induction Machine Taking into Account the Saturation Effect*. OPTIM’96, Brasov.