ANNALS OF "DUNAREA DE JOS" UNIVERSITY OF GALATI
FASCICLE XI – SHIPBUILDING. ISSN 1221-4620
2015

THE APPLICATION OF SIMULTANEOUS LOOP EQUATION SOLUTION TO SHIP NETWORK

Alexandru Ioan
University "Dunarea de Jos" of Galati,
Faculty of Naval Architecture, Galati, 47
Domneasca Street, 800008, Romania,
E-mail:Ioan.Alexandru@ugal.ro

ABSTRACT

The paper presents a brief description of the Simultaneous Loop Equation method developed by Epp and Fowler and its application to a drinking water network fitted on a ship. The results, flows and heads are obtained with required precision.

Keywords: solution, Newton-Raphson, network, ship, system, loop, node.

1. GENERAL CONSIDERATIONS AND THEORETICAL FRAMEWORK

The drinking water system delivers drinking water to consumption points on shipboard. The Simultaneous Loop Equation Solution approach was developed by Epp and Fowler in 1970 [1] based on the Newton-Raphson method. This solution can be used for complex networks where the Hardy-Cross method has weak convergence. It can also be adapted to programming. In this application variant with unknowns is used as flows. The system of equations is composed of mass conservation in nodes and energy loss conservation on loops and pseudo-loops. A loop is a sequence of pipes that starts and ends in the same node. A pseudo-loop is an open loop with known values of energy at the ends. The mass conservation (Fig.1) has the form:

$$\sum_{j=1...m} Q_{ij} = 0 \qquad (1)$$

where:

i - number of nodes,

j - number of pipes connected to node i.

$Q_{i1}...Q_{im}$- the flows in pipes.

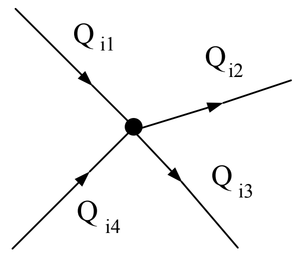

Fig.1 The flows in a node

The energy work on a loop (Fig.2) has the form:

$$-\Delta h_1 - \Delta h_2 + \Delta h_3 = 0 \qquad (2)$$

where:

Δh_1 - head loss, pipe 1,

Δh_2 - head loss, pipe 2,

Δh_3 - head loss, pipe 3.

The flows in pipes are Q_1, Q_2, Q_3, the flow that enters node A is Q_{inA}, the flows that exit nodes B and C are Q_{exB}, Q_{exC}. The convention path is clockwise. The pressure loss in a pipe has the equation:

$$\Delta p_t = \Delta p_f + \Delta p_{loc} \qquad (3)$$

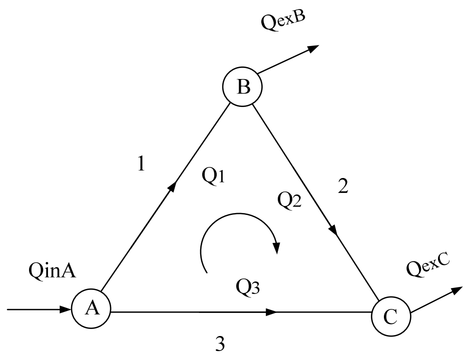

Fig. 2 A loop

Δp_f – friction pressure loss,
Δp_{loc} – local pressure loss.
The calculation of Δp_f is made with the Darcy-Weisbach equation:

$$\Delta p_f = f\frac{L}{D}\frac{v^2}{2}\rho \qquad (4)$$

where:
f - frictional coefficient,
L - pipe length,
D - inner pipe diameter,
v - flow velocity,
ρ - density.

Based on equation (4), the head loss is calculated with the equation:

$$\Delta h_t = f\cdot\frac{L_{tot}}{D}\cdot\frac{v^2}{2\cdot g}\ . \qquad (4')$$

The local pressure loss has the equation:

$$\Delta p_{loc} = \varsigma\frac{v^2}{2}\rho \qquad (5)$$

where:
ς - local loss coefficient,
v - flow velocity,
ρ - density.
An equivalent length is usually calculated for the local pressure loss.

$$l_{ech} = \varsigma\frac{D}{f} \qquad (6)$$

where:
ς - local loss coefficient,
D - pipe diameter,
f - frictional coefficient.

The equations for solution of a complex network are mass conservation in nodes and conservation energy loss on loops and pseudo-loops. The general form of the equation system is:

$$\begin{aligned} &F_1\left(x_1, x_2, ..., x_n\right) = 0 \\ &F_2\left(x_1, x_2, ..., x_n\right) = 0 \\ &\ldots\ldots\ldots\ldots\ldots\ldots \\ &F_n\left(x_1, x_2, ..., x_n\right) = 0 \end{aligned} \qquad (7)$$

The unknowns x_1, x_2, …x_n may be flows Q_1, Q_2, …Q_n, heads in nodes H_1, H_2,…H_n. or corrections of flows ΔQ_1, ΔQ_2, … ΔQn. The iterative relation Newton-Raphson to solve system (7) has the condensate form:

$$\{x\}^{(m+1)} = \{x\}^{(m)} - \left[J^{(m)}\right]^{-1}\{F\}^{(m)} \qquad (8)$$

where:
{x} - vector of unknown x,
[J] - Jacobian matrix,
{F} - vector of equations F,
m - number of iterations.

$$\{x\} = \{x_1, x_2, \ldots, x_n\}^T \qquad (9)$$

$$\{F\} = \{F_1, F_2, ..., F_n\}^T \qquad (10)$$

$$[J] = \begin{bmatrix} \frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} & \cdots & \frac{\partial F_1}{\partial x_n} \\ \frac{\partial F_2}{\partial x_1} & \frac{\partial F_2}{\partial x_2} & \cdots & \frac{\partial F_2}{\partial x_n} \\ \ldots & \ldots & \ldots & \ldots \\ \frac{\partial F_n}{\partial x_1} & \frac{\partial F_n}{\partial x_2} & \cdots & \frac{\partial F_n}{\partial x_n} \end{bmatrix} \qquad (11)$$

The relation of the first iteration m=0 is obtained from equation (8),

$$\{x\}^{(1)} = \{x\}^{(0)} - \left[J^{(0)}\right]^{-1} \{F\}^{(0)} \tag{12}$$

In this application the unknowns are flows Q_1, Q_2, …Q_n. For the first iteration flows Q_1^0, Q_2^0, …Q_n^0 are adopted to meet the mass conservation.

The vectors $\{x^0\}$ (equation 9) and $\{F^0\}$ (equation 10) have the form:

$$\{x\}^{(0)} = \left\{Q_1{}^0, Q_2{}^0, \ldots, Q_n{}^0\right\}^T \tag{13}$$

$$\{F\}^{(0)} = \left\{F_1{}^0, F_2{}^0, \ldots, F_n{}^0\right\}^T \tag{14}$$

$$F_i^0 = F_i\left(Q_1^0, Q_2^0, \ldots, Q_n^0\right) \; i = 1 \ldots n \tag{15}$$

The Jacobian matrix $[J^{(0)}]$ (equation 11) has the form:

$$\left[J^{(0)}\right] = \begin{bmatrix} \frac{\partial F_1\left(Q_1^0\right)}{\partial Q_1} & \frac{\partial F_1\left(Q_2^0\right)}{\partial Q_2} & \ldots & \frac{\partial F_1\left(Q_n^0\right)}{\partial Q_n} \\ \frac{\partial F_2\left(Q_1^0\right)}{\partial Q_1} & \frac{\partial F_2\left(Q_2^0\right)}{\partial Q_2} & \ldots & \frac{\partial F_2\left(Q_n^0\right)}{\partial Q_n} \\ \ldots & \ldots & \ldots & \ldots \\ \frac{\partial F_n\left(Q_1^0\right)}{\partial Q_1} & \frac{\partial F_n\left(Q_2^0\right)}{\partial Q_2} & \ldots & \frac{\partial F_n\left(Q_n^0\right)}{\partial Q_n} \end{bmatrix} \tag{16}$$

The right term of equation (12) results from relations (13), (14) and (16). Having vector $\{x\}^{(1)} = \left\{Q_1{}^1, Q_2{}^1, \ldots, Q_n{}^1\right\}^T$ the iteration number 2 can be calculated. The iteration process continues till convergence is achieved .

$$\left|Q_i^{(k+1)} - Q_i^{(k)}\right| \langle \varepsilon \tag{17}$$

$Q_i^{(k+1)}$- the flow at iteration (k+1),

$Q_i^{(k)}$-t he flow at iteration (k),

E - the value depends on the degree of required precision.

2. APPLICATION

The method presented above is applied to a system fitted on a ship (Fig.3). The notations used in (Fig.3) are:

D_{c1} - deck connection 1,
D_{c2} - deck connection2,
V_{tw} - 3-way valve,
V- valve, Ve - empty valve,
P - pump, V_{nr} - non return valve,
F - bactericide filter,
H - hydrophore,
T_1, T_2, T_3- drinking water tanks,
A,B,C…T - nodes.

The model of the system has 23 nodes and 3 loops corresponding to the 3 decks of the ship. The loops are connected with pipes 1,8 and 18. The ship water reserve is calculated with the relation:

$$R_{as} = c_{as} \cdot n_{as} \cdot A = 18\left[m^3\right] \tag{18}$$

where:
$c_{as} = 50[l]$, daily norm of drinking water for a person,
$n_{as} = 24$, number of crew members,
A=15 days, autonomy.
The water is stored in 3 tanks, T_1, T_2, T_3 (Fig.3). The flow of the delivery pump is calculated with the relation:

$$Q = \sum_i n_i \cdot \alpha \cdot q_i = 6\left[m^3 / h\right] \tag{19}$$

where:
n_i - number of consumers,
α - simultaneity coefficient,
q_i- specific consumption.

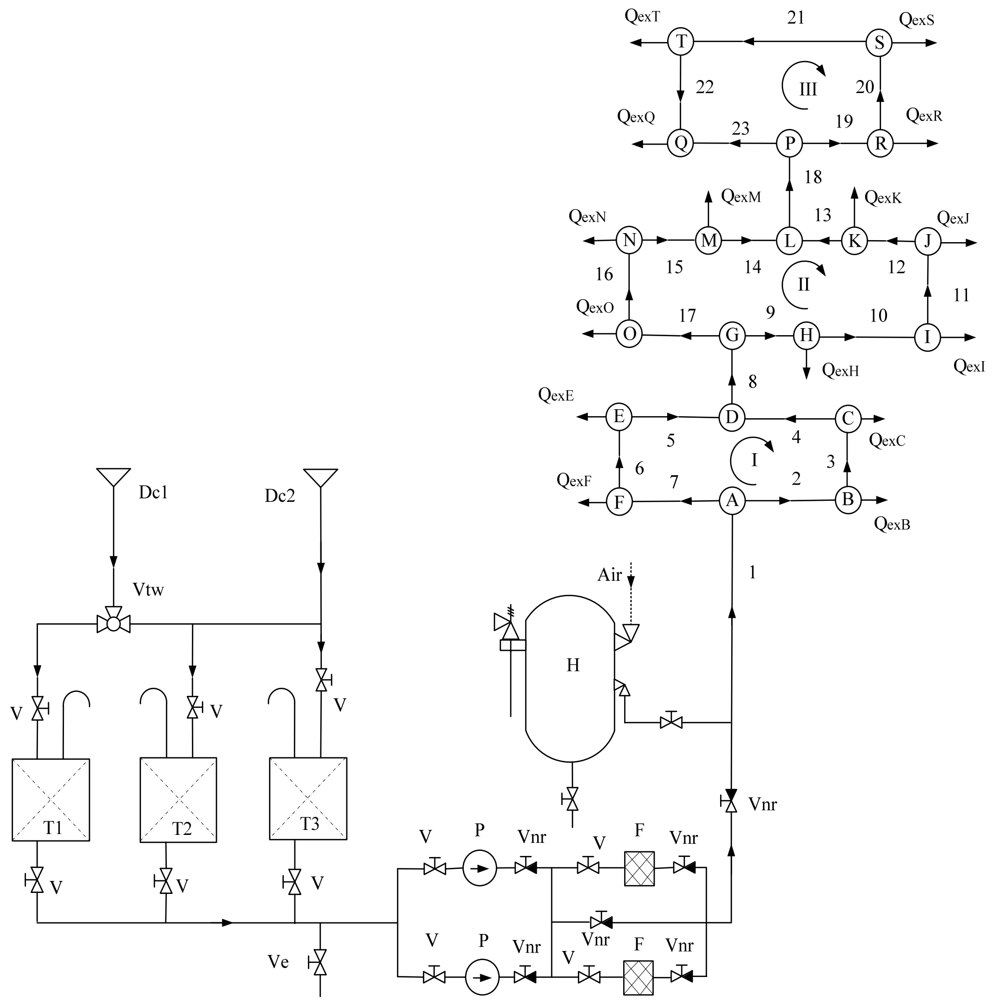

Fig.3 The system fitted on the ship

Tab.1 contains the values of α and q_i.

Tab.1 The values of α and q_i

No.	Type of consumer	Pieces	q_i [l/s]	α
1	Basin	20	0,07	0,5
2	Shower	13	0,13	0,5
3	Washer	14	0,06	0,5

The flows requested by consumers are presented in Tab.2:

Tab.2 The requested flows (see Fig.3)

Node	Flow	Value	Node	Flow	Value
B	Q_{exB}	0,126	I	Q_{exI}	0,72
C	Q_{exC}	0,252	J	Q_{exJ}	0,126
F	Q_{exF}	0,126	K	Q_{exK}	0,324
H	Q_{exH}	0,72	M	Q_{exM}	0,72
N	Q_{exN}	0,36	S	Q_{exS}	0,46
O	Q_{exO}	0,72	T	Q_{exT}	0,36
R	Q_{exR}	0,46	Q	Q_{exQ}	0,36

The adopted initial velocity of water is 4 [m^3/s]. The pipe diameter is calculated with the relation:

$$D = \sqrt{\frac{4 \cdot Q}{\pi \cdot v}} \, [m] \quad (20)$$

where:

Q - pipe flow,

v - water velocity.

The adopted pipe flows for the first iteration presented in Tab.3 are calculated based on consumer's flow, mentioned in Tab.2.

Tab.3 The adopted pipe flow (see Fig.3)

Pipe flow	m^3/h	Pipe flow	m^3/h
Q_1	5,994	Q_{13}	0,632
Q_2	2,997	Q_{14}	0,72
Q_3	2,871	Q_{15}	1,44
Q_4	2,619	Q_{16}	1,8
Q_5	2,54	Q_{17}	2,52
Q_6	2,871	Q_{18}	1,44
Q_7	2,997	Q_{19}	0,9
Q_8	5,13	Q_{20}	0,54
Q_9	2,61	Q_{21}	0,18
Q_{10}	1,89	Q_{22}	0,18
Q_{11}	1,17	Q_{23}	0,34
Q_{12}	1,044		

In Table 4 there are the catalogue pipe diameters selected based on the calculations with Eq.20.

In this application the material of the pipe is PVC whose roughness is k=0,007 [mm]. The value of the Reynolds number is calculated with Eq. 21 and the results are presented in Tab.5.

$$\text{Re} = \frac{v \cdot D}{\upsilon} \quad (21)$$

where:

D - pipe inner diameter,

ν - cinematic viscosity of water,

Re - Reynolds number,

v - water velocity.

Tab.4 Pipe diameter

No.Pipe	ODxt [mm]	No.Pipe	ODxt [mm]
1	32x3,6	13	20x3,4
2	25x4,2	14	20x3,4
3	25x4,2	15	25x4,2
4	25x4,2	16	25x4,2
5	25x4,2	17	25x4,2
6	25x4,2	18	25x4,2
7	25x4,2	19	20x3,4
8	32x3,6	20	20x3,4
9	25x4,2	21	20x3,4
10	25x4,2	22	20x3,4
11	20x3,4	23	20x3,4
12	20x3,4		

ODxt - outer diameter x thickness

Tab.5 Reynolds number

No. Pipe	Re	No. Pipe	Re
1	42083,7124	13	8335,80982
2	29924,5714	14	10046,3741
3	28645,5700	15	15297,2622
4	26087,5673	16	18291,7495
5	25986,0592	17	26260,1310
6	29640,3488	18	14617,1583
7	30919,3502	19	10965,4833
8	36017,5917	20	6369,93736
9	25813,4956	21	1774,39137
10	18504,9164	22	2821,15462
11	14080,2423	23	7416,70062
12	12471,8012		

Acording to Tab.5, the Reynolds number is situated in the domain $10^4 < \text{Re} < 10^6$ and the frictional coefficient is calculated with the Blasius relation:

$$f = 0,3164 \cdot \text{Re}^{-0,25} = 0,011 \quad (22)$$

where:

f - frictional coefficient,

Re - Reynolds number.

The pressure loss is calculated with relations (4) and (5) and the equivalent length with (6). The total length set up from pipe length and equivalent length is presented in Tab.6.

Tab.6 Total pipe length

Nr.Pipe	[m]	Nr.Pipe	[m]
L_1	5,5	L_{13}	1
L_2	8,6	L_{14}	5
L_3	10,3	L_{15}	7,9
L_4	1,6	L_{16}	7,15
L_5	1,7	L_{17}	4,46
L_6	10	L_{18}	2,8
L_7	7,75	L_{19}	3,2
L_8	2,7	L_{20}	3,25
L_9	1,83	L_{21}	9,5
L_{10}	8,95	L_{22}	3,25
L_{11}	8,4	L_{23}	7,55
L_{12}	4,3		

With velocity from equation (20), head loss from equation (4') become:

$$\Delta h_t = f \cdot \frac{L_{tot}}{D} \frac{8}{\pi^2 \cdot D^4 \cdot g} \cdot Q^2 = a \cdot Q^2 \qquad (23)$$

The pressure loss coefficient "a" is calculated from the relation:

$$a = f \cdot \frac{L_{tot}}{D} \frac{8}{\pi^2 \cdot D^4 \cdot g} \qquad (24)$$

where:

a - coefficient pressure loss,
f - frictional coefficient,
L_{tot} - total pipe length,
D - pipe diameter,
ρ - flow density,
v - flow velocity,
Q - pipe flow.

The values of the pressure loss coefficient "a" is presented in Tab.7.

Tab.7 The values of the pressure loss coefficient

Nr.	$[s]^2/[m]^5$	a_{12}	26571414
a_1	1262049	a_{13}	6834261
a_2	13575292	a_{14}	32613407
a_3	16437304	a_{15}	14747941
a_4	2613782	a_{16}	12764387
a_5	2779851	a_{17}	7273917
a_6	15822929	a_{18}	5286887
a_7	12133940	a_{19}	20420743
a_8	644135,2	a_{20}	23756260
a_9	2997417	a_{21}	95585002
a_{10}	15931584	a_{22}	29120923
a_{11}	50356467	a_{23}	53127914

The system of equation (7) has for this application the form:

$$\begin{aligned}
&Node\,A: Q_1 - Q_2 - Q_7 = 0\\
&Node\,B: Q_2 - Q_3 - Q_{exB} = 0\\
&Node\,C: Q_3 - Q_4 - Q_{exC} = 0\\
&Node\,D: Q_4 + Q_5 - Q_8 = 0\\
&Node\,E: Q_6 - Q_5 - Q_{exE} = 0\\
&Node\,F: Q_7 - Q_6 - Q_{exF} = 0\\
&Node\,G: Q_8 - Q_9 - Q_{17} = 0\\
&Node\,H: Q_9 - Q_{10} - Q_{exH} = 0\\
&Node\,I: Q_{10} - Q_{11} - Q_{exI} = 0\\
&Node\,J: Q_{11} - Q_{12} - Q_{exJ} = 0\\
&Node\,K: Q_{12} - Q_{13} - Q_{exK} = 0\\
&Node\,L: Q_{13} - Q_{14} - Q_{18} = 0\\
&Node\,M: Q_{15} - Q_{14} - Q_{exM} = 0\\
&Node\,N: Q_{16} - Q_{15} - Q_{exN} = 0 \qquad (25)\\
&Node\,O: Q_{17} - Q_{16} - Q_{exO} = 0\\
&Node\,P: Q_{19} - Q_{20} - Q_{23} = 0\\
&Node\,R: Q_{19} - Q_{20} - Q_{exR} = 0\\
&Node\,S: Q_{20} - Q_{21} - Q_{exS} = 0\\
&Node\,T: Q_{21} - Q_{22} - Q_{exT} = 0\\
&Node\,Q: Q_{23} - Q_{22} - Q_{exQ} = 0
\end{aligned}$$

$$Loop\ I: -a_2 \cdot Q_2^2 - a_3 \cdot Q_3^2 - a_4 \cdot Q_4^2 +$$
$$a_5 \cdot Q_5^2 + a_6 \cdot Q_6^2 + a_7 \cdot Q_7^2 = 0$$
$$Loop\ II: -a_9 \cdot Q_9^2 - a_{10} \cdot Q_{10}^2 - a_{11} \cdot Q_{11}^2$$
$$-a_{12} \cdot Q_{12}^2 - a_{13} \cdot Q_{13}^2 + a_{14} \cdot Q_{14}^2 + a_{15} \cdot Q_{15}^2$$
$$+a_{16} \cdot Q_{16}^2 + a_{17} \cdot Q_{17}^2 = 0$$
$$Loop\ III: -a_{19} \cdot Q_{19}^2 - a_{20} \cdot Q_{20}^2 - a_{21} \cdot Q_{21}^2$$

The first iteration has adopted the flows presented in Tab.3. It follows equation (12). The vectors and matrix are:

$$\{x\}^{(0)} = 10^{-3} \cdot \{1{,}665\ 0{,}833\ 0{,}797\ 0{,}725\ 0{,}705\ 0{,}797$$
$$0{,}832\ 1{,}425\ 0{,}725\ 0{,}525\ 0{,}325\ 0{,}290\ 0{,}175\ 0{,}2$$
$$0{,}4\ 0{,}5\ 0{,}7\ 0{,}4\ 0{,}25\ 0{,}15\ 0{,}05\ 0{,}05\ 0{,}094\ \}^T$$

$$\{F\}^{(0)} = \{0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$$
$$0\ 0\ 0\ 0\ -933{,}86\ -3191{,}41\ -29772{,}26\ \}^T$$

$$\left[J^{(0)}\right]^{-1} \cdot \{F\}^{(0)} = 10^{-5} \cdot \{1{,}21\ 1{,}21\ 1{,}21$$
$$-1{,}21 -1{,}21\ -1{,}21\ 0\ 1{,}47\ 1{,}47\ 1{,}47\ 1{,}47$$
$$1{,}47\ -1{,}47 -1{,}47\ -1{,}47\ -1{,}47\ 0\ 0{,}98$$
$$0{,}98\ 0{,}98 -0{,}98\ -0{,}98\ \}^T$$

The result of the first iteration $\{x\}^{(1)}$ is calculated with Eq. 12.

$$\{x\}^{(1)} = 10^{-3} \cdot \{1{,}665\ 0{,}82 0{,}785\ 0{,}715\ 0{,}71$$
$$0{,}81\ 0{,}845\ 1{,}425\ 0{,}71\ 0{,}51\ 0{,}31\ 0{,}275$$
$$0{,}185\ 0{,}215\ 0{,}415\ 0{,}515\ 0{,}715\ 0{,}4$$
$$0{,}24\ 0{,}14\ 0{,}04\ 0{,}06\ 0{,}16\ \}^T$$

The limit $\varepsilon=10^{-6}$ and convergence are achieved at the fourth iteration. The final flow values are presented in Tab. 8.

Having flows in every pipe we can calculate the head in the network nodes.

In the first step we calculate the head loss in the pipes with equation (22). The results are presented in Tab.6.

Tab.8 The final flows

Pipe flow	m^3/h	Pipe flow	m^3/h
Q_1	5,994	Q_{13}	0,653
Q_2	2,948	Q_{14}	0,787
Q_3	2,822	Q_{15}	1,507
Q_4	2,57	Q_{16}	1,802
Q_5	2,56	Q_{17}	2,587
Q_6	2,92	Q_{18}	1,44
Q_7	3,046	Q_{19}	0,859
Q_8	5,13	Q_{20}	0,499
Q_9	2,543	Q_{21}	0,139
Q_{10}	1,823	Q_{22}	0,221
Q_{11}	1,103	Q_{23}	0,581
Q_{12}	0,977		

Tab.9 Head loss in pipes

No. Pipe	Head loss [m]	No. Pipe	Head loss [m]
1	3,498683072	13	0,22486036
2	9,103306001	14	1,55862139
3	10,10042371	15	2,58435925
4	1,332080743	16	3,19819190
5	1,405712356	17	3,75626488
6	10,40992479	18	0,84590188
7	8,686736088	19	1,16266034
8	1,307997081	20	0,45642998
9	1,49566669	21	0,14249983
10	4,085331542	22	0,10974498
11	4,727170638	23	1,38378949
12	1,957035883		

The head at the hydrophor exit is 60 [m] H_2O. The height between the hydrophor and node A and between loops I, II, III is 3 [m] (Fig.3). The head loss is calculated from node to node around the loop (see Fig.3). For example:

$$P_A = P_{Hydrophor} - 3[m] = 60 - 3 = 57[m]H_2O$$

where:

P_A- head in node A,

$$P_F = P_A - a_7 \cdot Q_7^2 = 57 - 8,686 = 38,314 \; [m]H_2O$$

where:

P_A- head in node A,

P_F - head in node F,

$a_7 \cdot Q_7^2$ -head loss in pipe 7 (Tab.6).

$$P_E = P_F - a_6 \cdot Q_6^2 = 38,314 - 10,409 = 27,905 \; [m]H_2O$$

where:

P_A- head in node A,

P_F- head in node F,

$a_6 \cdot Q_6^2$ - head loss in pipe 6 (Tab.6).

$$P_D = P_E - a_5 \cdot Q_5^2 = 27,905 - 1,405 = 26,5 \; [m]H_2O$$

where:

P_D - head in node D,

P_E - head in node E,

$a_5 \cdot Q_5^2$ - head loss in pipe 5 (Tab.6).

$$P_B = P_A - a_2 \cdot Q_2^2 = 57 - 9,103 = 37,897 \; [m]H_2O$$

where:

P_B - head in node B,

P_A- head in node A,

$a_2 \cdot Q_2^2$ -head loss in pipe 2 (Tab.6).

$$P_C = P_B - a_3 \cdot Q_3^2 = 37,897 - 10,1 = 27,797 \; [m]H_2O$$

where:

P_C - head in node C,

P_B - head in node B,

$a_3 \cdot Q_3^2$ - head loss in pipe 3 (Tab.6).

$$P_G = P_D - d = 26,5 - 3 = 23,5 \; [m]H_2O$$

where:

P_D - head in node D,

d - deck height.

The procedure is the same for the loops II and III. The heads for all nodes are presented in Tab.10.

Tab.10 The head in the network nodes

Node	Head [m]	Node.	Head [m]
A	60	L	27,674
A	57	H	32,005
F	48,314	I	30,51
E	37,905	J	26,425
D	36,5	K	21,698
B	47,897	P	24,45
C	37,797	Q	14,35
G	33,5	T	14,347
O	30,302	R	23,605
N	27,718	S	22,443
M	27,45		

3. CONCLUSIONS

1.The method of simultaneous loop equation solution achieved convergence after a small number of equations, compared to the Hardy-Cross method.

2. The flow of consumers and the head in nodes are obtained with required precision.

3.This method has the advantage that it can be programmed.

REFERENCES

[1]. **Epp, R. and Fowler, A.**, "*Efficient code for steady-state flows in networks*", Journal of Hydraulics Division, Proceedings of the ASCE 96 (HY1), 1970.

[2]. **Jeppson, R.,** *"Steady Flow Analysis Pipe Networks",* An Instructional Manuals, Utah State Universit , 1984.

[3]. **Larock, B., Jeppson, R., Watters, G.**, "*Hydraulics of Pipeline Systems",* CRC Press, ISBN 0-8493-1906-8, pp 43-96 New York, 2000.

[4]**. Mun-Fong L.,** *"Pipe Network Analysis"*, Publication No.77, University of Florida Gainesville, 1983.

[5]. **Wood, D.J. and Charles O.A.**. "*Hydraulic network analysis using linear theory"*, Journal of Hydraulic Division, ASCE HY7, 1972.

Paper received on December 18th, 2015

