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ORIGINAL RESEARCH PAPER

MATHEMATICAL MODELLING OF PASTA DOUGH DYNAMIC VISCOSITY, THERMAL CONDUCTIVITY AND DIFFUSIVITY

ANDREI IONUȚ SIMION, CRISTINA-GABRIELA GRIGORAȘ*, ALINA MOROI, NICOLETA VARTOLOMEI
"Vasile Alecsandri" University of Bacău, Department of Chemical and Food Engineering, 157 Calea Mărăşeşti, 600115 Bacău, Romania
*Corresponding author: cristina.grigoras@ub.ro

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This work aimed to study the mathematical variation of three main thermodynamic properties (dynamic viscosity, thermal conductivity and thermal diffusivity) of pasta dough obtained by mixing wheat semolina and water with dough humidity and deformation speed (for dynamic viscosity), respectively with dough humidity and temperature (for thermal diffusivity and conductivity). The realized regression analysis of existing graphical data led to the development of mathematical models with a high degree of accuracy. The employed statistical tests (least squares, relative error and analysis of variance) revealed that the obtained equations are able to describe and predict the tendency of the dough thermodynamic properties.
Keywords: pasta dough, dynamic viscosity, thermal diffusivity, thermal conductivity, mathematical modelling

Introduction

Foods thermodynamic properties such as density, specific heat capacity, viscosity etc. are related to the water amount due to its partial pressure in the gas phase at equilibrium, which is fundamental in the analysis of the heat and mass transport phenomena (Al-Muhtaseb *et al.*, 2004, Amos *et al.*, 2008, Carvalho Lago *et al.*, 2013).

As stated in different previous papers (Simion, 2009, 2012, 2014), for an efficient design and selection of processes and equipments implied by foods production and also for an adequate estimation of cooking time and control of operating costs, it is essential, among others, to understand and manage the behavior of thermodynamic properties. In the case of pasta dough, these properties strongly depend especially on temperature and water content (Matuda *et al.*, 2011). Even though it is generally accepted that the values of these properties influence the pasta fabrication, only few publications were conducted for their study.

To the best of our knowledge, scattered and insufficient information are available nowadays. As an example, Baird and Reed (1989) mention that pasta dough heat capacity (Cp) can be estimated using the equation 1, while according to Manthey and Twombly (2006), the apparent viscosity (μ_a) of a pasta dough system can be described by the equation 2.

$$Cp = 1.44 + 2.74 \cdot X_w \quad \text{(Equation 1)}$$

where X_w is the dough moisture content.

$$\mu_a = m \cdot v^{n-1} \quad \text{(Equation 2)}$$

where *m* is dough consistency, *v* is the shear rate and *n* is the flow index.

In terms of thermal diffusivity known as the parameter that defines how fast heat propagates or diffuses through a material, its calculation can be made using equation 3 which relates thermal conductivity (λ), specific heat (Cp) and density (ρ) (Erdogdu, 2007).

$$a = \frac{\lambda}{Cp \cdot \rho} \quad \text{(Equation 3)}$$

As a consequence, in this work we have focused on the study of the evolution of three main thermodynamic properties of pasta dough namely dynamic viscosity, thermal conductivity and thermal diffusivity. Values recovered from existing literature data were introduced in adequate software in order to obtain reliable and easy to use mathematical models able to describe and predict the mentioned dough properties.

Materials and methods

Data published by Macovei (2000) (Tables 1, 2 and 3), in graphical form, concerning the dynamic viscosity (η, Pa·s), thermal diffusivity (a, $m^2 \cdot s^{-1}$) and conductivity (λ, $W \cdot m^{-1} \cdot K^{-1}$) variations of pasta dough with dough humidity ($H_{\%}$, w/w) and deformation speed (γ, s^{-1}), respectively dough humidity and temperature (t, °C) were used as data for the regression analysis.

Table 1. Digital recovered values for dough thermal diffusivity, $a \cdot 10^8$ [$m^2 \cdot s^{-1}$]

No.	Humidity, $H_{\%}$ [%, w/w]							
	21.1%		24.8%		26.6%		29.0%	
	t^*, [°C]	$a \cdot 10^8$, [$m^2 \cdot s^{-1}$]	t, [°C]	$a \cdot 10^8$, [$m^2 \cdot s^{-1}$]	t, [°C]	$a \cdot 10^8$, [$m^2 \cdot s^{-1}$]	t, [°C]	$a \cdot 10^8$, [$m^2 \cdot s^{-1}$]
1	26.46	7.90	26.20	8.90	24.53	9.91	25.92	11.26
2	29.75	8.13	29.14	9.20	28.41	10.25	28.04	11.42
3	39.99	8.63	39.50	9.91	39.12	10.89	38.75	11.99
4	50.48	8.88	50.92	10.30	51.14	11.19	51.59	12.28
5	60.96	8.99	61.06	10.29	61.16	10.99	61.37	12.17
6	70.39	8.86	70.37	10.07	70.48	10.58	70.45	11.92
7	81.36	8.54	81.58	9.51	81.93	9.74	80.48	11.45
8	85.37	8.37	-	-	85.11	9.41	-	-

No.	Humidity, $H_{\%}$ [%, w/w]					
	33.6%		42.5%		45.8%	
	t, [°C]	$a \cdot 10^8$, [$m^2 \cdot s^{-1}$]	t, [°C]	$a \cdot 10^8$, [$m^2 \cdot s^{-1}$]	t, [°C]	$a \cdot 10^8$, [$m^2 \cdot s^{-1}$]
1	26.27	11.55	26.73	11.92	26.02	12.03
2	27.91	11.64	27.91	12.00	27.67	12.14
3	38.63	12.20	38.74	12.51	38.50	12.77
4	51.82	12.44	51.58	12.66	51.81	13.07
5	61.37	12.29	61.37	12.46	61.36	12.85
6	70.45	12.02	70.33	12.14	70.44	12.42

7	78.12	11.66	80.59	11.61	78.11	11.79
8	83.90	11.27	89.56	10.95	86.26	10.86

* t – temperature

Table 2. Digital recovered values for dough thermal conductivity, λ [W·m^{-1}·K^{-1}]

No.	Humidity, $H_{\%}$ [%, w/w]							
	21.1%		24.8%		26.6%		29.0%	
	t^*, [°C]	λ, [W·m^{-1}·K^{-1}]	t, [°C]	λ, [W·m^{-1}·K^{-1}]	t, [°C]	λ, [W·m^{-1}·K^{-1}]	t, [°C]	λ, [W·m^{-1}·K^{-1}]
1	25.58	0.16	25.39	0.21	25.45	0.25	25.15	0.30
2	30.38	0.17	29.27	0.22	28.93	0.26	28.75	0.31
3	40.50	0.20	39.66	0.25	39.59	0.28	39.15	0.34
4	50.35	0.21	50.18	0.27	49.97	0.30	50.46	0.36
5	60.19	0.22	60.02	0.28	59.94	0.31	60.17	0.37
6	70.01	0.22	71.17	0.28	71.23	0.31	71.32	0.37
7	80.48	0.21	80.45	0.27	80.51	0.31	80.72	0.36
8	83.79	0.20	84.16	0.27	83.56	0.30	84.83	0.35

No.	Humidity, $H_{\%}$ [%, w/w]					
	33.6%		42.5%		45.8%	
	t, [°C]	λ, [W·m^{-1}·K^{-1}]	t, [°C]	λ, [W·m^{-1}·K^{-1}]	t, [°C]	λ, [W·m^{-1}·K^{-1}]
1	25.04	0.32	26.01	0.34	24.97	0.35
2	28.37	0.33	27.88	0.35	27.64	0.36
3	38.91	0.36	38.55	0.38	38.32	0.40
4	50.23	0.38	50.26	0.40	50.04	0.42
5	60.07	0.39	59.96	0.41	59.74	0.43
6	71.22	0.39	71.24	0.40	71.28	0.42
7	80.75	0.38	80.77	0.38	80.93	0.40
8	84.59	0.37	84.48	0.38	84.23	0.39

Table 3. Digital recovered values for dough dynamic viscosity, $\eta \cdot 10^{-5}$ [Pa·s]

No.	Humidity, $H_{\%}$ [%, w/w]					
	33%		32%		31%	
	γ^*, [s^{-1}]	$\eta \cdot 10^{-5}$ [Pa·s]	γ, [s^{-1}]	$\eta \cdot 10^{-5}$ [Pa·s]	γ, [s^{-1}]	$\eta \cdot 10^{-5}$ [Pa·s]
1	0.207	9.594	0.221	7.466	0.230	7.528
2	0.210	8.298	0.225	6.448	0.237	6.479
3	0.209	7.404	0.228	5.430	0.240	5.492
4	0.213	6.448	0.248	4.166	0.268	4.136
5	0.216	5.399	0.315	2.070	0.393	2.072
6	0.235	4.135	0.567	0.905	0.572	1.460
7	0.275	2.069	0.934	0.761	0.930	1.069
8	0.567	0.596	1.305	0.679	1.305	0.956
9	0.934	0.452	1.679	0.566	1.679	0.844
10	1.304	0.432	2.034	0.576	2.034	0.761
11	1.679	0.350	2.396	0.463	2.347	0.646
12	2.038	0.329	-	-	-	-
13	2.396	0.308	-	-	-	-

No.	Humidity, $H_{\%}$ [%, w/w]			
	30%		28.5%	
	γ, [s^{-1}]	$\eta \cdot 10^{-5}$ [Pa·s]	γ, [s^{-1}]	$\eta \cdot 10^{-5}$ [Pa·s]
1	0.224	11.445	0.241	12.032
2	0.231	10.366	0.260	10.336
3	0.242	8.299	0.275	8.300
4	0.261	6.171	0.322	6.172
5	0.321	4.137	0.435	4.140
6	0.568	2.077	0.573	3.157
7	0.934	1.440	0.939	2.118
8	1.309	1.234	1.309	1.635
9	1.680	1.090	1.684	1.399
10	2.034	0.915	1.989	1.346
11	2.274	0.829	-	-

* γ - dough deformation speed

xyExtract Graph Digitizer.v2.3 software was employed to extract numerical data from graphic representations. Various models were generated and statistically evaluated in software as Microsoft Excel™ 2013, CurveExpert® software, TableCurve 3D® v.4 and XLSTAT-Pro v.7.5.

Thermodynamic Property vs. Deformation Speed or Temperature and *Thermodynamic Property vs. Dough Humidity* were plotted. Different types of tests such as least squares method, equation relative error ε (equation 4) and ANOVA were used in regression fitting analysis.

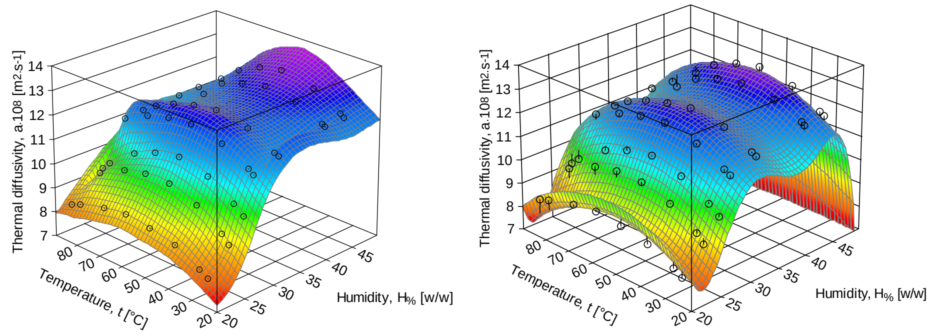
$$\varepsilon_{\%} = \left| \frac{Data_{experimental} - Data_{calculated}}{Data_{experimental}} \right| \cdot 100\ [\%] \qquad \text{(Equation 4)}$$

Results and discussion

An illustrated perspective of dough thermal diffusivity, thermal conductivity and dynamic viscosity variations with the imposed parameters were obtained by plotting the available data as 3D graphics (Figures 1a, 2a and 3a).

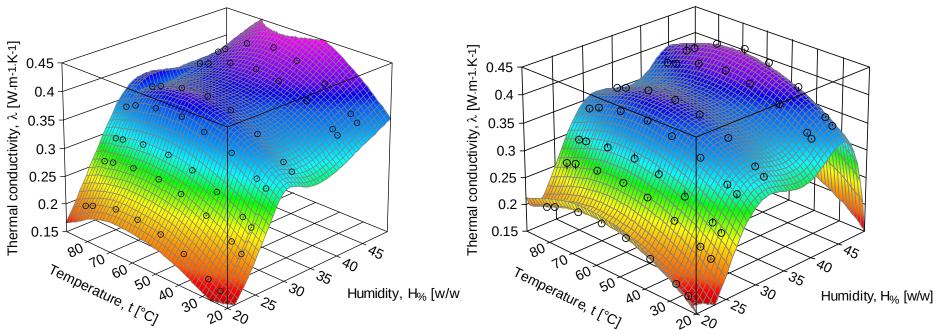
It could be noted that these variations, with a single or both state parameters in the same time, is rather difficult to be described by simple mathematical models. Moreover, the equation generator, part of the employed software, often fitted (Table 4) only the plotted points, not taking in consideration the general tendency (Figures 1b, 2b and 3b).

Due to these considerations, another approach was necessary in order to generate adequate mathematical models.



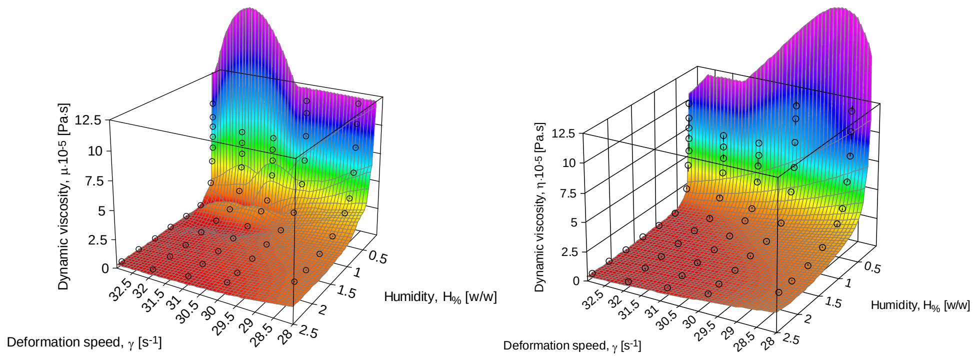
(a) (b)

Figure 1. Dough thermal diffusivity $a{\cdot}10^8$ [$m^2{\cdot}s^{-1}$] variation with dough humidity $H_{\%}$, [%, w/w] and temperature t [°C] (a) graphic representation of literature data (Macovei, 2000), (b) software generated graphic



(a) (b)

Figure 2. Dough thermal conductivity λ [$W{\cdot}m^{-1}{\cdot}K^{-1}$] variation with dough humidity $H_{\%}$, [%, w/w] and temperature t [°C] (a) graphic representation of literature data (Macovei, 2000), (b) software generated graphic



(a) (b)

Figure 3. Dough dynamic viscosity, $\eta{\cdot}10^{-5}$ [Pa·s] variation with dough humidity $H_{\%}$, [%, w/w] and dough deformation speed γ, [s^{-1}] (a) graphic representation of literature data (Macovei, 2000), (b) software generated graphic

Table 4. Dough thermodynamic properties equations generated by employed software

Thermodynamic property equation form	R^2
Thermal diffusivity, a [$m^2 \cdot s^{-1}$]	
$a = a' + b' \cdot H_{\%} + c' \cdot H_{\%}^2 + d' \cdot H_{\%}^3 + e' \cdot H_{\%}^4 + f' \cdot H_{\%}^5 + g' \cdot t + h' \cdot t^2 + i' \cdot t^3 + j' \cdot t^4 + k' \cdot t^5$	0.9774
$ln(a) = a' + \frac{b'}{H_{\%}^{0.5}} + \frac{c' \cdot ln(H_{\%})}{H_{\%}} + \frac{d'}{H_{\%}} + \frac{e'}{H_{\%}^{1.5}} + \frac{f'}{H_{\%}^2} + g' \cdot t^{1.5} + h' \cdot t^{2.5} + i' \cdot e^{\frac{t}{w.t}}$	0.9766
$a^{-1} = a' + b' \cdot ln(H_{\%}) + \frac{c'}{H_{\%}^{0.5}} + \frac{d'}{H_{\%}} + \frac{e'}{H_{\%}^{1.5}} + \frac{f'}{H_{\%}^2} + g' \cdot t^{1.5} + h' \cdot t^3 + i' \cdot e^{\frac{t}{w.t}}$	0.9762
Thermal conductivity, λ [$W \cdot m^{-1} \cdot K^{-1}$]	
$\lambda = a' + b' \cdot H_{\%} + c' \cdot H_{\%}^2 + d' \cdot H_{\%}^3 + e' \cdot H_{\%}^4 + f' \cdot H_{\%}^5 + g' \cdot t + h' \cdot t^2 + i' \cdot t^3 + j' \cdot t^4 + k' \cdot t^5$	0.9951
$\lambda = a' + b' \cdot \ln(H_{\%}) + c' \cdot \ln(H_{\%})^2 + d' \cdot \ln(H_{\%})^3 + e' \cdot \ln(H_{\%})^4 + f' \cdot \ln(H_{\%})^5 + g' \cdot \ln(t) + h' \cdot \ln(t)^2 + i' \cdot \ln(t)^3 + j' \cdot \ln(t)^4 + k' \cdot \ln(t)^5$	0.9940
$\lambda = a' + \frac{b'}{H_{\%}} + \frac{c'}{H_{\%}^2} + \frac{d'}{H_{\%}^3} + \frac{e'}{H_{\%}^4} + \frac{f'}{H_{\%}^5} + g' \cdot t + h' \cdot t^2 + i' \cdot t^3 + j' \cdot t^4 + k' \cdot t^5$	0.9935
Dynamic viscosity, η [$Pa \cdot s$]	
$ln(\eta) = a' + \frac{b'}{ln(H_{\%})} + \frac{c'}{H_{\%}^{1.5}} + \frac{d'}{\gamma^{0.5}} + \frac{e' \cdot ln(\gamma)}{\gamma} + \frac{f'}{\gamma} + \frac{g'}{\gamma^{1.5}} + \frac{h' \cdot ln(\gamma)}{\gamma^2} + \frac{i'}{\gamma^2}$	0.9932
$ln(\eta) = a' + \frac{b'}{ln(H_{\%})} + \frac{c'}{\gamma} + \frac{d'}{\gamma^{1.5}} + \frac{e' \cdot ln(\gamma)}{\gamma^2} + \frac{f'}{\gamma^2}$	0.9910

Thermal diffusivity

Multiple correlations between temperature, t [°C], and thermal diffusivity, a [$m^2 \cdot s^{-1}$], at constant dough humidity, $H_{\%}$ [%, w/w] have been established using Microsoft Excel™ 2013 spreadsheets and CurveExpert® software.

The equations that fitted (with a correlation coefficient higher than 0.990) all 7 humidity variations were: polynomial of second and third degree, rational function, Hoerl and Gaussian models and sinusoidal fit. A rational function was chosen (Equation 5).

$$a = \frac{a' + b' \cdot t}{1 + c' \cdot t + d' \cdot t^2} \quad \text{(Equation 5)}$$

a', b', c' and d' values are presented in Table 5.

Each equation regression coefficients R^2 are greater than 0.999, indicating a good correlation of variables.

Table 5. Rational function coefficients

Humidity, $H_{\%}$ [%, w/w]	Equation 5 coefficients			
	a'	*b'*	*c'*	*d'*
21.20	5.783651	0.045610	-0.007145	0.000105
24.80	6.042542	0.057121	-0.009392	0.000134
26.60	7.173970	0.034688	-0.011129	0.000141
29.00	8.702861	0.062007	-0.005962	0.000104
33.60	8.976755	0.057935	-0.006195	0.000106
42.50	9.221809	0.067808	-0.005773	0.000113
45.80	8.798037	0.039664	-0.010378	0.000136

Table 6. Rational function coefficients

Equation 5 coefficients	Equations 6 and 7 coefficients		
	x	*y*	*z*
Hoerl model			
a'	0.000603259	0.911091	3.6242695
b'	2.45E-05	0.915261	3.058702
d'	0.000443265	1.019422	-0.56096943
Quadratic fit			
c'	-0.022412237	0.000899	-1.31E-05

In order to correlate the coefficients with dough humidity, $H_{\%}$ [%, w/w], different models were used in CurveExpert® software (polynomial fit, rational function, Hoerl and Gaussian model, “vapor pressure” model etc.). Considering their best fit and simplicity in formulation, the Hoerl model (HM) (Equation 6) and the quadratic fit (QF) (Equation 7) were selected. Their coefficients are presented in Table 6.

$$HM_{Coefficient} = x \cdot \left(y^{H_{\%}}\right) \cdot \left(H_{\%}^{z}\right) \qquad \text{(Equation 6)}$$

$$QF_{Coefficient} = x + y \cdot H_{\%} + z \cdot H_{\%}^{2} \qquad \text{(Equation 7)}$$

The combination of equations 5, 6 and 7 led to the final form of the proposed mathematical model (Equation 8). Even though its regression coefficient R^2 was determined as being only 0.961, the graphical representation respects the real tendency of thermodynamic properties variation according with state parameters.

$$a = \frac{HM_{a'} + HM_{b'} \cdot t}{1 + QF_{c'} \cdot t + HM_{d'} \cdot t^{2}} \qquad \text{(Equation 8)}$$

A comparison between scientific existing data and values obtained with the developed mathematical model was realized using the relative error equation. A final overall average of -0.073% (1.95% in absolute value) was obtained (Table 7). The ANOVA analysis showed that the sample *P*-value is 0.9890 greater than the targeted alpha 0.05 indicating that there is not a statistical difference between tabular and calculated data.

Table 7. Pasta dough thermal diffusivity calculated values and the relative errors

No.	Humidity, $H_{\%}$ [%, w/w]							
	21.2%		24.8%		26.6%		29.0%	
	$a \cdot 10^8$, [$m^2 \cdot s^{-1}$]	$\varepsilon_{\%}$	$a \cdot 10^8$, [$m^2 \cdot s^{-1}$]	$\varepsilon_{\%}$	$a \cdot 10^8$, [$m^2 \cdot s^{-1}$]	$\varepsilon_{\%}$	$a \cdot 10^8$, [$m^2 \cdot s^{-1}$]	$\varepsilon_{\%}$
1	7.74	2.00	9.34	-4.96	9.87	0.31	10.73	4.69
2	7.99	1.72	9.56	-3.90	10.18	0.72	10.89	4.62
3	8.60	0.38	10.17	-2.66	10.80	0.81	11.50	4.13
4	8.96	-0.87	10.49	-1.83	11.10	0.85	11.75	4.30
5	9.02	-0.29	10.46	-1.57	11.02	-0.27	11.62	4.54
6	8.86	0.03	10.21	-1.43	10.73	-1.46	11.29	5.25
7	8.47	0.87	9.71	-2.12	10.18	-4.54	10.77	5.98
8	8.29	1.04	-	-	10.00	-6.23	-	-

No.	Humidity, $H_{\%}$ [%, w/w]					
	33.6%		42.5%		45.8%	
	$a \cdot 10^8$, [$m^2 \cdot s^{-1}$]	$\varepsilon_{\%}$	$a \cdot 10^8$, [$m^2 \cdot s^{-1}$]	$\varepsilon_{\%}$	$a \cdot 10^8$, [$m^2 \cdot s^{-1}$]	$\varepsilon_{\%}$
1	11.71	-1.39	12.13	-1.76	11.87	1.36
2	11.83	-1.61	12.22	-1.84	12.01	1.05
3	12.40	-1.58	12.84	-2.64	12.72	0.35
4	12.57	-1.05	13.01	-2.78	12.99	0.63
5	12.36	-0.61	12.78	-2.53	12.78	0.58
6	11.97	0.44	12.34	-1.61	12.34	0.62
7	11.53	1.16	11.66	-0.38	11.84	-0.35
8	11.15	1.12	10.97	-0.22	11.22	-3.32

Dynamic viscosity

Reciprocal logarithmic correlations between dough deformation speed γ, [s^{-1}], and dynamic viscosity, $\eta \cdot 10^{-5}$ [Pa·s], at constant dough humidity $H_{\%}$, [%, w/w] have been established. The correlation coefficient of all equations is over 0.990.

$$\eta = \frac{1}{a' + b' \cdot \ln(\gamma)} \qquad \text{(Equation 9)}$$

a and *b* coefficients values were quadratic fitted with dough humidity $H_{\%}$, [%, w/w] (Equation 7) generating the final form of the mathematical model (Equation 10) with the coefficients presented in Table 8.

$$\eta = \frac{1}{QF_{a'} + QF_{b'} \cdot \ln(\gamma)} \qquad \text{(Equation 10)}$$

The equation relative error in overall average is 2.52% and R^2 is 0.996. The ANOVA analysis revealed that the sample *P*-value is 0.9468 (greater than the targeted alpha 0.05), indicating no statistical difference between tabular and calculated data.

Table 8. Equation 9 coefficients

Equation 9 coefficients	Equation 10 coefficients		
	x	*y*	*z*
Quadratic fit (*QF*)			
a'	61.290167	-4.3245114	0.076868843
b'	37.504172	-2.6469969	0.047046171

Table 9. Pasta dough dynamic viscosity calculated values and the relative errors

No.	Humidity, $H_{\%}$ [%, w/w]					
	33%		32%		31%	
	$\eta\cdot10^{-5}$, [Pa·s]	$\varepsilon_{\%}$	$\eta\cdot10^{-5}$, [Pa·s]	$\varepsilon_{\%}$	$\eta\cdot10^{-5}$, [Pa·s]	$\varepsilon_{\%}$
1	9.58	0.09	6.74	9.76	7.57	-0.63
2	7.96	4.07	6.13	4.98	6.54	-0.87
3	8.22	-11.08	5.62	-3.58	6.16	-12.11
4	6.94	-7.62	3.88	6.91	4.27	-3.34
5	6.03	-11.66	2.02	2.22	2.06	0.70
6	3.50	15.39	0.94	-3.62	1.36	6.58
7	2.00	3.36	0.64	15.36	0.95	11.23
8	0.66	-11.41	0.53	21.59	0.78	18.09
9	0.46	-0.63	0.47	16.85	0.69	17.83
10	0.38	12.97	0.43	24.89	0.64	16.23
11	0.33	5.07	0.40	12.56	0.60	6.98
12	0.31	7.33	-	-	-	-
13	0.29	7.43	-	-	-	-

No.	Humidity, $H_{\%}$ [%, w/w]			
	30%		28.5%	
	$\eta\cdot10^{-5}$, [Pa·s]	$\varepsilon_{\%}$	$\eta\cdot10^{-5}$, [Pa·s]	$\varepsilon_{\%}$
1	11.78	-2.93	12.13	-0.82
2	10.09	2.63	9.64	6.78
3	8.40	-1.27	8.38	-0.99
4	6.59	-6.78	6.11	0.93
5	4.14	-0.09	4.05	2.18
6	2.04	1.82	3.09	2.07
7	1.41	1.77	2.17	-2.45
8	1.17	5.14	1.81	-10.54
9	1.04	4.74	1.60	-14.70
10	0.96	-4.44	1.49	-10.98
11	0.91	-10.12	-	-

Thermal conductivity

The same software was used also for studying the variation of dough pasta thermal conductivity. Quadratic correlations between temperature, *t* [°C], and thermal conductivity, λ [W·m^{-1}·K^{-1}], at constant dough humidity $H_{\%}$, [%, w/w] have been established. All the equations are characterized by a R^2 coefficient over 0.990.

$$\lambda = a' + b' \cdot t + c' \cdot t^2 \quad \text{(Equation 11)}$$

a', *b'* and *c'* values were quadratic fitted with dough humidity $H_{\%}$, [%, w/w] in CurveExpert® software (Equation 12) generating the final form of the mathematical model (Equation 13). Coefficients for equation 12 are presented in Table 10.

$$QF_{Coefficient} = x + y \cdot H_{\%} + z \cdot H_{\%}^2 \quad \text{(Equation 12)}$$

$$\lambda = QF_{a'} + \frac{1}{QF_{b'}} \cdot t + \frac{1}{QF_{c'}} \cdot t^2 \quad \text{(Equation 13)}$$

Table 10. Equation 11 coefficients

Equation 11 coefficients	Equations 12 and 14 coefficients		
	x	*y*	*z*
Quadratic fit (*QF*)			
a'	-0.5862615300	0.0414755830	-0.0005325348
b'	182.18744	1.8673239	-0.064017325
c'	-20291.298	-545.5306	14.397444
Logistic model (*LM*)			
a'	0.20536934	11999.425	0.38730964

Table 11. Pasta dough thermal conductivity calculated values and the relative errors

No.	Humidity, $H_{\%}$ [%, w/w]							
	21.2%		24.8%		26.6%		29.0%	
	λ, [W·m^{-1}·K^{-1}]	$\varepsilon_{\%}$	λ, [W·m^{-1}·K^{-1}]	$\varepsilon_{\%}$	λ, [W·m^{-1}·K^{-1}]	$\varepsilon_{\%}$	λ, [W·m^{-1}·K^{-1}]	$\varepsilon_{\%}$
1	0.15	4.78	0.22	-5.20	0.26	-3.55	0.29	5.31
2	0.17	2.78	0.23	-4.26	0.27	-2.95	0.30	4.63
3	0.19	2.01	0.26	-4.18	0.29	-3.92	0.33	4.02
4	0.21	1.91	0.28	-3.34	0.31	-3.22	0.35	3.29
5	0.22	1.35	0.29	-3.56	0.32	-3.08	0.36	3.74
6	0.22	1.52	0.29	-3.38	0.32	-3.14	0.36	3.07
7	0.21	-0.41	0.28	-2.92	0.31	-2.91	0.35	2.43
8	0.21	-1.75	0.27	-2.74	0.31	-2.57	0.34	1.77

No.	Humidity, $H_{\%}$ [%, w/w]					
	33.6%		42.5%		45.8%	
	λ, [W·m^{-1}·K^{-1}]	$\varepsilon_{\%}$	λ, [W·m^{-1}·K^{-1}]	$\varepsilon_{\%}$	λ, [W·m^{-1}·K^{-1}]	$\varepsilon_{\%}$
1	0.32	-0.61	0.34	-0.93	0.35	0.58
2	0.33	-0.58	0.35	-0.62	0.36	0.76
3	0.36	-0.12	0.38	-0.85	0.40	1.25
4	0.38	0.28	0.41	-1.54	0.41	2.16
5	0.39	0.43	0.41	-1.25	0.42	3.88
6	0.39	0.17	0.40	-0.73	0.40	5.17
7	0.38	-0.29	0.39	-0.19	0.38	5.86
8	0.37	-0.52	0.38	1.06	0.37	6.29

The equation`s relative error in overall average is 0.026% (3.20% in absolute value) with a R^2 of 0.965. If the coefficient *a'* is described by a Logistic model (Equation 14) and the thermal conductivity mathematical form becomes equation 15, the generated model has an improved precision with a regression coefficient of 0.982 and a relative error of 2.42% in absolute value (Table 11).

$$LM_{a'} = \frac{x}{1 + y \cdot e^{-z \cdot H_{\%}}} \qquad \text{(Equation 14)}$$

$$\lambda = LM_{a'} + \frac{1}{QF_{b'}} \cdot t + \frac{1}{QF_{c'}} \cdot t^2 \qquad \text{(Equation 15)}$$

The ANOVA analysis for the equation 11 indicated a *P*-value of 0.807 and for the equation 10 a *P*-value of 0.840.

Conclusions

Different software often employed to generate mathematical models were tested in order to establish equations able to describe the evolution of pasta dough dynamic density, thermal conductivity and thermal diffusivity with different state parameters. Even though the resulted models registered high correlation coefficients, there were cases where they fitted data without reflecting their real tendency. Better results were obtained by using Microsoft Excel™ 2013 and XLSTAT-Pro v.7.5 software. Least squares, relative error and ANOVA test values showed that the proposed mathematical models are appropriate to accurately describe and predict the tendency of dough thermodynamic properties.

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