

SOME ASPECTS CONCERNING THE ANALYSIS OF THE PLANAR LINKAGES USING GEOGEBRA

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ABSTRACT

The goal of this paper is to highlight the great opportunity to analyze the planar linkages using the Graphic Statics methods and the capabilities of the interactive mathematics software GeoGebra.

The introductory part of the article is dedicated to highlighting the present revival witnessed by the Graphic Statics methods, in the general context of the creation and development of the interactive mathematical environments.

Next, are presented the main functionalities of GeoGebra environment which are applicable to planar linkages analysis.

In order to justify the applicability of GeoGebra environment to planar linkages analysis, the paper contains a case study for which a GeoGebra application was achieved. This application is described in terms of its architecture and functionality.

At the end of the article are listed some remarks of the author.

KEYWORDS: GeoGebra, Graphic Statics, planar linkage, coupler curve

1. Preliminary

Undoubtedly, for the mechanical engineer, geometric interpretation is one of the most powerful mathematical tools whenever mathematical model of the problem to be solved is suitable to accept such a geometric interpretation or whenever solving the mathematical model allows for the use of the graphic-analytical calculus method.

Graphic Statics, a branch of mechanics, provides a collection of procedures for the design and analysis of two-dimensional structural systems, involving only geometric operations on two-dimensional form and force diagrams.

In recent decades, due to an exponential increase of the computer calculus power, Graphic Statics was almost abandoned because it quasi-generalized the using of the analytical calculus methods and their associated numeric calculus models.

Starting from the idea of graphical representations made by humans, the main disadvantage of the graphical calculus methods is related to the accuracy of the results which is dependent

on the precision and accuracy of graphical representations on paper and of the measurements accuracy. Thus it came the idea that graphical calculus methods provided by Graphic Statics are outdated.

However, it is important to be noted that graphical calculus methods have two major advantages over analytical calculus methods:

- Graphical calculus methods allow for a full coverage of the problem in a single graphical representation and they make the results to be suggestive and easily interpretable.
- Using graphical calculus methods it is very easy to find possible errors that were made during of the problem solving.

The development of the interactive mathematics software packages, especially the development of their geometric part, may lead to a revival of the graphical methods, especially because the main disadvantage which has been above presented is eliminated. In this context, the GeoGebra package which is a dynamic and interactive mathematics environment, having a lot powerful geometric instruments, is an

excellent example.

2. GeoGebra functionality vs. planar linkages analysis

In accordance with one of the most common definition, GeoGebra is an interactive mathematics software program for learning and teaching mathematics and science from primary school up to university level.

GeoGebra environment is governed by more than 500 orders, which are structured into several categories, such as: *3D Commands*, *Algebra Commands*, *Geometry Commands*, *Logical Commands*, *Scripting Commands*, *Transformation Commands*, etc.

In GeoGebra environment, geometric constructions can be made with points, lines, rays, segments, vectors, polygons, etc.

Observing that the geometric primitives listed above are used in the graphic representation of planar linkages, it is found that interactive geometric environment *GeoGebra has the tools necessary to generate the graphical representation of these mechanisms*.

It is important to notice that all geometric primitives can be associated to the so-called *slider* that provides a very convenient and highly illustrative method. All of them can be changed dynamically afterwards. Elements can be entered and modified directly via mouse and touch, or through the Input Bar. In addition, GeoGebra has the ability to use variables for numbers, vectors and points

Thus, the position of the points can be modified interactively in the plan of the mechanism and the orientation of lines and rays, length of segments, etc. can be interactively modified.

Using these skills of the interactive geometric environment GeoGebra, the user can view and change configuration mechanism, can verify its existence, etc.

In addition, it is possible to graphically render functions which are defined implicitly, explicitly or parametrically. Consequently, the curves associated to the positions, velocities, accelerations, variations of forces can be plotted, etc.

GeoGebra has? A *built-in spreadsheet* where the user can insert strings, numeric variables and geometric variables which change their state dynamically, in concordance with changes made in the graphic window.

A very useful GeoGebra capability for the analysis of planar linkages is the possibility of activating the animation for the adequate geometric elements. Thus, it is possible to graphically simulate the mechanism functionality.

Another very useful tool in the analysis of planar mechanisms is the command *Locus* that

allows plotting of the geometric locus of a point whose motion depends on the movement of another point or by numerical values associated with a *slider*. This command is obviously useful for plotting the coupler curves, curves much needed in the synthesis, analysis and optimization of planar linkages. Also, in some situations, GeoGebra environment can provide equation of the geometric locus.

Also, in order to view the geometric locus, environment GeoGebra offers the user the ability to draw the trace of a moving point, point that belongs to the studied geometric system.

GeoGebra supports scripts that allow programming in GeoGebra environment. All the commands necessary to achieve a script belong to the category *Scripting Commands*.

Programming in GeoGebra environment leads to the automation of the actions and can create very complex correlations between geometric elements of the system studied. Thus, the prerequisites for optimum modeling of real problems studied are created.

3. Case study: couple curves in planar four-bar linkage

Some theoretical basics

A four-bar linkage is the simplest movable linkage. It consists of four rigid bodies (which are commonly called links), each attached to two others by single joints to form a closed loop as shown in Figure 1.

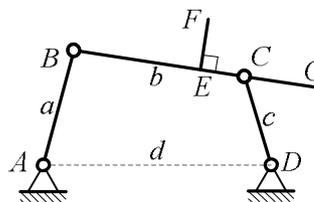


Fig. 1 – Planar four-bar linkage

The notations in Figure 1 have the following meanings:

A, B, C, D - Single joints;

a, c - Length of the side links;

b - Length of the coupler link;

d - Length of the fixed link;

E, F, G - Points on the coupler link.

In terms of mechanical action, one of the side links is the *input link*, i.e., the link to which an external torque is applied to rotate it. The second side link is called the *follower link*, since its motion is completely determined by the motion of the input link.

In this area, if we refer to links lengths, the following notations are used:

s - Length of the shortest link;

l - Length of the longest link;

p, q - Length of intermediate links.

With the previous notations and considering Figure 1, it is possible to write:

$$l = \max\{a, b, c, d\} \quad (1)$$

$$s = \min\{a, b, c, d\} \quad (2)$$

$$p, q \in \{a, b, c, d\} - \{l, s\} \quad (3)$$

Depending on the nature of the movements they perform side links which have specific names:

- *Crank* - a side link which revolves relatively to the fixed link.
- *Rocker* - a side link which does not revolve relatively to the fixed link.

Therefore, it follows that any four-bar linkage can be assigned to one of types below :

- *Crank-rocker mechanism* - the shorter side link revolves and the other one rocks.
- *Double-crank mechanism* - both of the side links revolve.
- *Double-rocker mechanism* - both of the side links rock.

The input link for crank-rocker mechanism is the crank and for double-crank mechanism and double-rocker mechanism the input link is any side link.

The type of four-bar linkage is found based on Grashof's theorem. Without a rigorous demonstration of this theorem, the results of applying this theorem to the classification of the four-bar linkages are presented below. There are two cases, depending on the relation between $s + l$ and $p + q$:

- For $s + l \leq p + q$ (see Figure 2).
- For $s + l > p + q$ (see Figure 3).

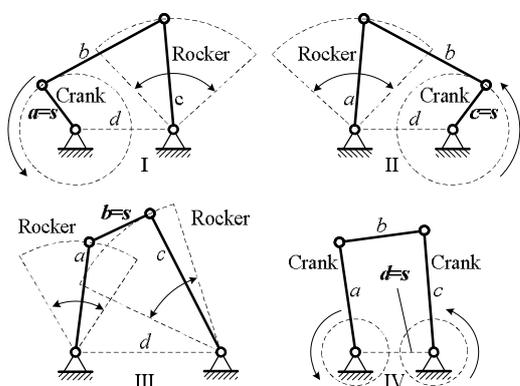


Fig. 2 – First case: $s + l \leq p + q$

As seen in Figure 2, for $s + l \leq p + q$, the type of the four-bar linkage depends on the position of the shortest link: if the shortest link is a side link (Figure

2 I, II) the four-bar linkage is a crank-rocker mechanism, if the shortest link is the coupler link (Figure 2 III), the four-bar linkage is a double-rocker mechanism and if the shortest link is the fixed link (Figure 2 IV), the four-bar linkage is a double-crank mechanism.

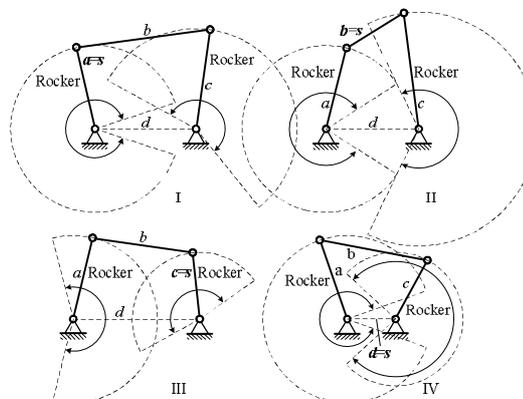


Fig. 3 – Second case: $s + l > p + q$

For $s + l > p + q$, the type of the four-bar linkage is independent of the position of the shortest link. In this case, it is possible to obtain only double-rocker mechanisms.

The curve which is generated by a point on the coupler link when the input link is moving is called *coupler curve* (see on Figure 4).

Remark: Thus defined, a coupler curve is nothing but the geometric locus on the coupler link when point B is moving on an arc centered on A .

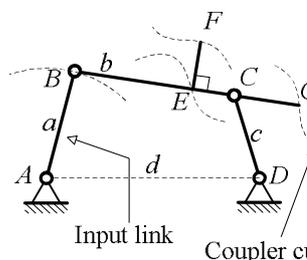


Fig. 4 – Coupler curve associated to a point on the coupler link

The coupler curves are very important in the synthesis of the linkage mechanisms because they are often imposed to obtain a corresponding planar linkage.

Coupler curves verify Roberts–Chebyshev theorem which states that *for a given coupler curve there exist three four-bar linkages, three geared five-bar linkages, and more six-bar linkages which will generate the same path.*

GeoGebra application

The GeoGebra application made by the author is considering the case of a generalized planar four-linkage (Figure 1), with the possibility of its customization through interactive changes made on

its geometric elements. Summarizing, the main aspects considered in GeoGebra application are presented below.

- Possibility to modify the length of the elements. For this purpose sliders were used.
- Possibility to modify positions of the points E , F and G on the coupler link. The user can move these points interactively.
- Displaying the length of the longest element and the length of shortest element (l and s - see above).
- Displaying the mechanism type in accordance with the relation between $s + l$ and $p + q$.
- Possibility to modify animation speed. For this purpose a slider was used.
- Possibility to select the point for which the plot of the coupler curve is desired. For this purpose, three check boxes, named E , F and G , were used.
- Start-up or shutdown of the mechanism animation. For this purpose, two buttons, named *Start*, respectively *Stop* were used.
- Possibility of resetting the graphic window. In this case, a button named *Reset View* was used.

Figure 5 shows a capture of the graphical window of the GeoGebra application. The controls associated to the aspects detailed above can be noticed, as well a crank-rocker mechanism with a certain configuration.

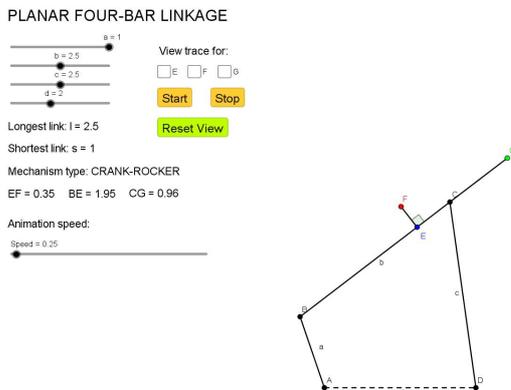


Fig. 5 – Graphical window of the GeoGebra application

Figure 6 shows the coupler curves associated to the points E , F and G .

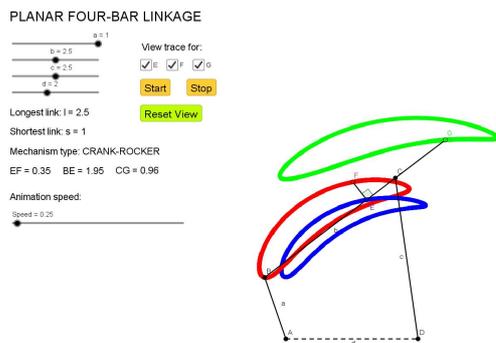


Fig.6 – Coupler curves associated to E , F and G

In Figure 7 is presented the Hoekens linkage ($a = 1, b = 2.5, c = 2.5, d = 2, CG = 2.5$) that converts the rotational motion into an approximate straight-line motion. For a better visualization of the coupler curve of point G , its color was set to red.

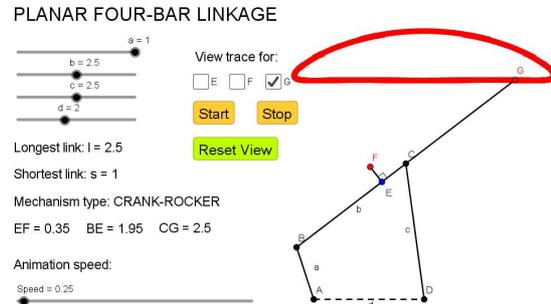


Fig. 7 – Hoekens linkage

As seen in Figure 4, the Hoekens mechanism generates a coupler curve with a portion very close to a straight line parallel with segment AD .

Remarks

- Firstly, it is to be noted that although there are numerous programs for the analysis of mechanisms, the GeoGebra environment provides all the tools needed to build advanced applications in this area.
- GeoGebra allows creating custom tools, which give this environment a great power in terms of adaptation to a particular area (for example, the area of the linkages analysis). Using this capability of the GeoGebra environment, it can be transformed into a veritable working platform dedicated to a specific area. In this context, GeoGebra environment can become not only a dynamic environment for learning and teaching, but also an environment where it is possible to build advanced technical studies in areas such as synthesis and analysis mechanisms, mechanics, statics of the strength structures, etc.
- The above presented GeoGebra application is a part of a larger development whose purpose is the design and analysis of the planar linkage mechanisms.
- Unlike other interactive mathematical environments, GeoGebra is absolutely free and benefits from an accelerated development due to a large international community of users.
- The GeoGebra application above presented is available via [pdumitrache.ugal.ro](mailto:pdumitrache@ugal.ro).

References

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