THE ANNALS OF "DUNAREA DE JOS" UNIVERSITY OF GALATI
FASCICLE XIV MECHANICHAL ENGINEERING, ISSN 1224-5615
2016

DYNAMIC ANALISYS OF THE PARAMETERS OF THE MECHANICAL SYSTEMS WITH STRUCTURAL DAMPING. VISCOELASTIC SLS MODEL. PART 1: AMPLITUDE FACTOR

Assoc. Prof. Dr. Eng. Nicusor DRAGAN
MECMET - The Research Center of Machines, Mechanic and Technological Equipments
Engineering and Agronomy Faculty of Braila
"Dunarea de Jos" University of Galati

ABSTRACT

The paper proposes an approach of a 1DOF (1 Degree Of Freedom) dynamic model of an elastic mechanical system with structural damping rheologically modeled as a Zener model. This system is perturbated by a harmonic force $F(t) = H \sin \omega t$, *the dynamic parameter being the amplitude of the forced steady-state vibration and the transmitted force to the base. The parametric dynamic characteristic that is drawn and analyzed is the amplitude factor* $A(\Omega, \delta)$. *This study is useful to validate and/or to assess the viscous materials with Standard Linear Solid (SLS) Model behavior.*

KEYWORDS: steady-state vibration, structural damping, SLS model, amplitude factor

1. INTRODUCTION

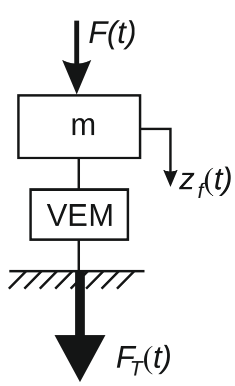
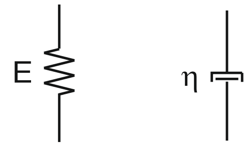


Fig. 1. Simplified scheme for 1DOF mechanical system supported by a viscoelastic element

We consider a simple 1DOF (1 Degree Of Freedom) mechanical system composed of a single mass m supported by a viscoelastic element VEM as in figure 1. The 1DOF system is perturbated by a variable force $F(t)$, the vertical forced steady-state vibration of the mass m being described by the law of motion $z_f(t)$. Through the viscous-elastic element VEM, the inertial dynamic force of the mass m is transmitted to the base. The dynamic parameters of the displacement $z_f(t)$ and transmitted force $F_T(t)$ are determined by the rheological characteristics of the VEM.

2. ZENER RHEOLOGICAL MODEL

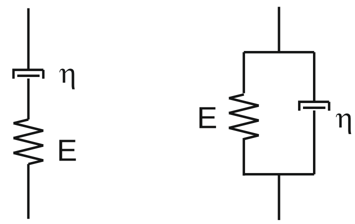
Basic rheological models and complex rheological models are described in the books on viscoelasticity [1] [2]. The rheological characteristics of different viscoelastic materials can be described by the basic conceptual models of linear spring (Hooke) and linear dashpot (Newton), see figure 2.



a) b)

Fig. 2. Basic rheological elements
a)linear massless spring (Hooke model)
b)linear viscous dashpot (Newton model)

Different rheological models can be obtained by the combination, in series and/or in parallel, of springs and dashpots. The simplest rheological viscoelastic models are Maxwell model and Kelvin-Voigt model, see figure 3.



a) b)

Fig. 3. Simple rheological elements
a)Maxwell model (spring - series - dashpot)
b)Kelvin-Voigt model (spring - parallel - dashpot)

The simple rheological models M and K-V cannot describe at the same time all the viscoelastic properties: M model don't describe creep or recovery and K-V model don't describe stress relaxation. The Standard Linear Solid (SLS) model can describe both phenomena and is the most used to accurately describe the overall behavior of a elastic mechanical system under a given set of loading conditions. The SLS model, known as well as Zener model, is composed from a Hooke model in parallel with a Maxwell model, as in figure 4.

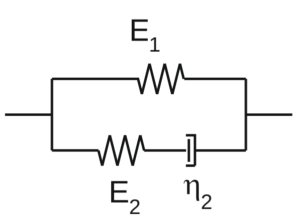


Fig. 4. Rheological Zener model
(Hooke-Maxwell model)

3. DYNAMIC MODEL OF 1DOF MECHANICAL SYSTEM WITH ZENER VISCOUS DAMPING

We consider a 1DOF mechanical system supported by a viscoelastic element as in figure 5.

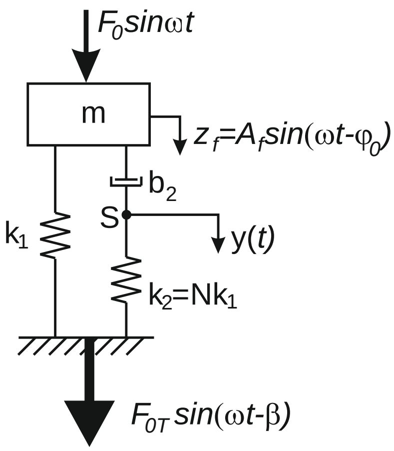


Fig. 5. Calculus scheme for the dynamic characteristics of 1DOF mechanical system

The moving equations of the steady-state forced vibration can be written as follows [3]:

$$\begin{cases} m\ddot{z} + b_2(\dot{z}_f - \dot{y}) + k_1 z_f = F(t) \\ b_2(\dot{z}_f - \dot{y}) = k_2 y \end{cases} \tag{1}$$

Considering the perturbating force as harmonic, $F(t) = F_0 \sin\omega t$, the first equation of the system (1) becomes

$$\ddot{z}_f + \frac{b_2}{m}(\dot{z}_f - \dot{y}) + \frac{k}{m} z_f = \frac{F_0}{m} \sin\omega t \tag{2}$$

or

$$\ddot{z}_f + 2n(\dot{z}_f - \dot{y}) + p^2 z_f = h \sin\omega t \ , \tag{3}$$

where:

$n = \frac{b_2}{2m}$ is the damping factor (Maxwell model)

$p = \sqrt{\frac{k_1}{m}}$ - natural (angular) frequency of the ideal elastic system (Hooke model)

$\varsigma = \frac{b_2}{b_{critical}} = \frac{b}{2\sqrt{mk}}$ - damping ratio

$\delta = \frac{b_2\omega}{k_2} = 2\varsigma\Omega$ - structural damping ratio

$\Omega = \frac{\omega}{p}$ - relative (angular) frequency

If we consider the second equation of (1), the differential moving equation can be written as follows:

$$m\ddot{z} + k_2 y + k_1 z_f = F_0 \sin\omega t \tag{4}$$

4. AMPLITUDE OF THE FORCED STEADY-STATE VIBRATION

Since 1DOF mechanical system has linear viscoelastic elements, the displacement $z_f(t)$ of the vibrating mass m and the displacement $y(t)$ of the serial point linkage **P** of Maxwell model have harmonic time variation

$$z_f(t) = A_f \sin(\omega t - \varphi_0) \tag{5}$$
$$y(t) = A_Y \sin(\omega t - \alpha) \tag{6}$$

where

- A_f - displacement amplitude of mass m
- A_Y - displacement amplitude of point **P**
- φ_0 - phase shift $z_f \leftrightarrow F$
- α - phase shift $y \leftrightarrow F$

With the displacements time functions (5) and (6), the differential equation (4) becomes:

$$\begin{aligned} &-mA\omega^2 \sin(\omega t - \varphi_0) + k_2 A_Y \sin(\omega t - \alpha) + \\ &+ k_1 A \sin(\omega t - \varphi_0) = F_0 \sin \omega t \end{aligned} \tag{7}$$

The dynamic balance differential equation of the point **P** can be written:

$$\begin{aligned} &b_2 \omega A_f \cos(\omega t - \varphi_0) - \\ &- b_2 \omega A_Y \cos(\omega t - \alpha) = k_2 A_Y \sin(\omega t - \alpha) \end{aligned} \tag{8}$$

Equations (7) and (8) are equivalent to a system of four trigonometric equations whose solution is $(A_f, \varphi_0, A_Y, \alpha)$.

The amplitude of the steady-state forced vibration of the mass m can be written [4]

$$A_f = \frac{F_0}{k_1} \sqrt{\frac{N^2 + \delta^2}{N^2 (1 - \Omega^2)^2 + \delta^2 (N + 1 - \Omega^2)^2}} \tag{9}$$

or

$$A_f = \frac{F_0}{k_1} \cdot A(\Omega, \delta, N) = A_{st} \cdot A(\Omega, \delta, N), \tag{10}$$

where:

$A_{st} = \dfrac{F_0}{k_1}$ is the static displacement

$A(\Omega, \delta, N)$ - amplitude factor

$$A(\Omega, \delta, N) = \sqrt{\frac{N^2 + \delta^2}{N^2 (1 - \Omega^2)^2 + \delta^2 (N + 1 - \Omega^2)^2}} \tag{11}$$

The parametric relation (10) is used to determined the variation graphs of the amplitude factor A function of the relative angular frequency Ω, for different values for structural damping ratio δ and elasticity coefficients ratio N.

5. AMPLITUDE FACTOR DIAGRAMS OF 1DOF SYSTEM FORCED VIBRATION

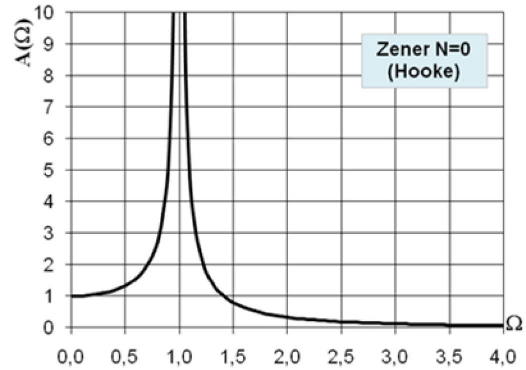


Fig. 6. Amplitude factor diagram Zener model - N=0 (Hooke model)

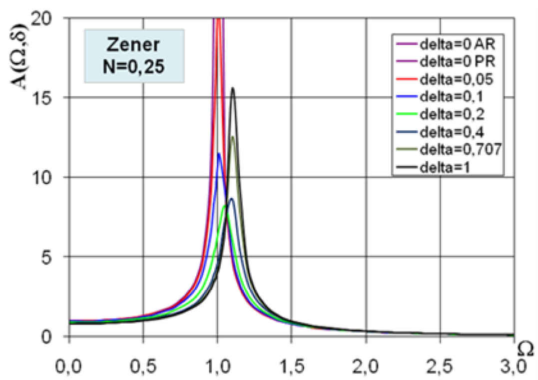


Fig. 7. Amplitude factor diagram Zener model - N=0.25

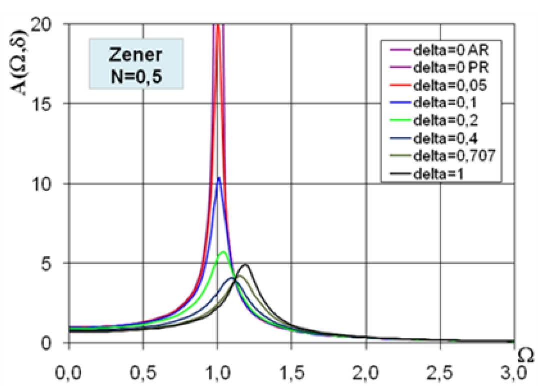


Fig. 8. Amplitude factor diagram Zener model - N=0.5

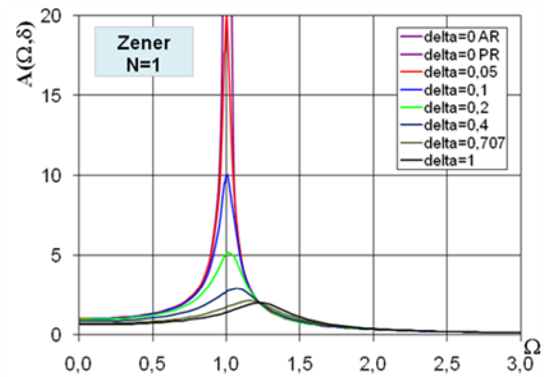


Fig. 9. Amplitude factor diagram
Zener model - N=1

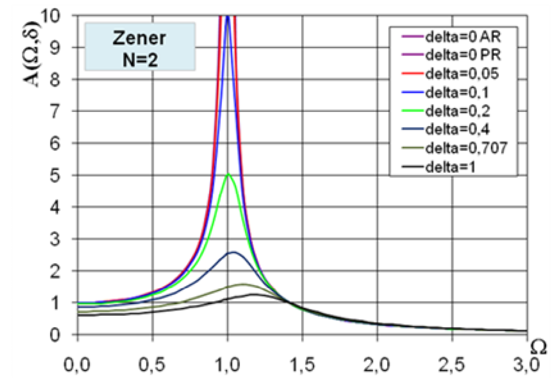


Fig. 10. Amplitude factor diagram
Zener model - N=2

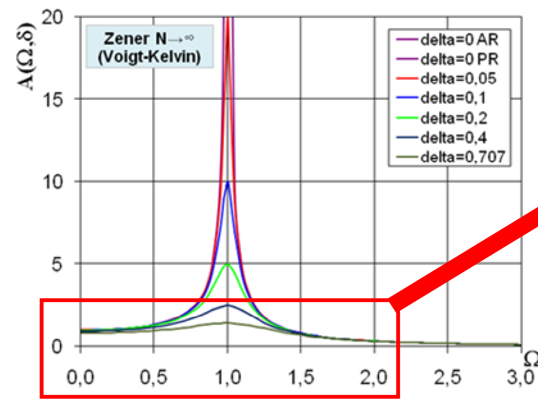


Fig. 11. Amplitude factor diagram
Zener model - N→∞ (Voigt-Kelvin model)

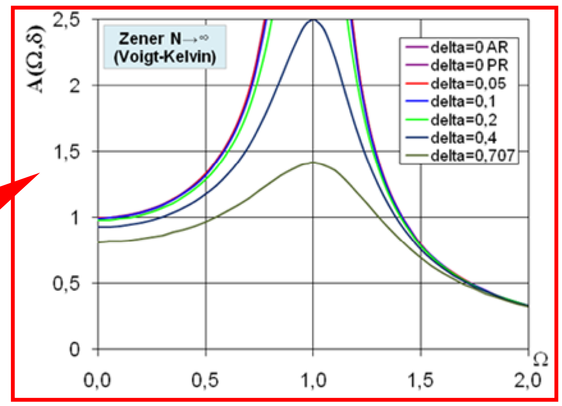


Fig. 12. Amplitude factor diagram detail
Zener model - N→∞ (Voigt-Kelvin model)

6. CONCLUSIONS

Analyzing the amplitude factor parametric relation (11), we can observe that:

a)for $N=0$ or $\delta=0$ (meaning Maxwell model cancellation), Zener model becomes Hooke model; the diagram is shown in figure 6 and the amplitude factor is as follows:

$$A_{N=0}(\Omega)=A_{\delta=0}(\Omega)=\frac{1}{\left|1-\Omega^2\right|} \tag{12}$$

b)for $N\to\infty$ (in Maxwell model, spring is replaced by a rigid connection, obtaining a viscous Newton model), Zener model becomes Voigt-Kelvin model; the diagram is shown in figure 11 and figure 12 and the amplitude factor is as follows:

$$A_{N\to\infty}(\Omega,\delta)=\frac{1}{\sqrt{\left(1-\Omega^2\right)^2+\delta^2}} \tag{13}$$

c)for simple rheological Hooke model, the amplitude resonance point is $\Omega=1$, see figure 6;

d)for simple rheological Voigt-Kelvin model, the amplitude resonance point is $\Omega=1$, see figure 11 and figure 12; if we consider a structural damping $\delta\neq 0$, the value for the amplitude factor at resonance is:

$$A_{V-K}^{resonance}(\delta)=\frac{1}{\delta} \tag{14}$$

e)for complex rheological Zener model, the amplitude resonance points are obtaining for $\Omega>1$ and it's value depends on the elasticity coefficients ratio N, see figures 7 to 10.

REFERENCES

[1] **Flugge, W.**, *Viscoelasticity*, Springer, New York, 1975.
[2] **Christensen, R.M.**, *Theory of Viscoelasticity*, Dover Publications Inc., New York, 2010.
[3] **Bratu, P.**, *Elastic Systems Vibrations*, Technical Publishing House, Bucharest, 2000.
[4] **Bratu, P.**, *Bearing Elastic Systems for Machines and Equipment*, Technical Publishing House, Bucharest, 1990.