

## ASPECTS REGARDING WORM GEARING DEFORMATION

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### ABSTRACT

*The paper is a review of studies and research done in the worm gearing teeth deformation. It is obvious that teeth deformation is present during meshing, with negative consequences on the running accuracy of the worm gearing.*

KEYWORDS: worm gearing, deformation, contact line, contact pattern, stress

### 1. INTRODUCTION

If the worm gearing teeth are, theoretically, rigid and assembling and processing errors are null, then the contact between worm and wheel is a lineworm and worm wheel (Figure 1).

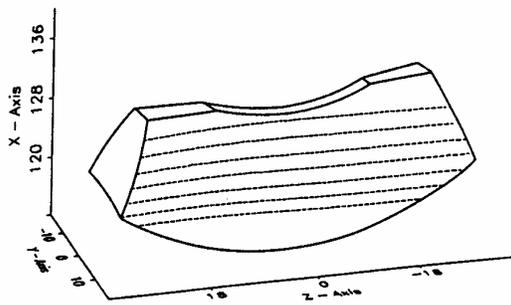


Fig. 1 [1] Theoretical worm gearing contact

The lines of contact can be determined based on the enveloping conditions and enveloped surface equation:

$$\begin{aligned} \bar{n} \cdot \bar{v}_{12} &= 0; \\ f &= 0, \end{aligned} \quad (1)$$

where:  $\bar{n}$  is the normal vector at the contact point of the two surfaces;

$\bar{v}_{12}$  - is the relative velocity vector of the two surfaces in contact.

### 2. CONTACT OF THE DEFORMABLE AND ERRORS FREE WORM GEARING

The computerized research has made it possible to obtain significant results in the study of worm gearing. Computer programs have been

developed to analyze the contact and contact pressure distribution of a worm gearing., Figure 2 shows three coordinate systems necessary mathematical model, to determine the contact lines and surfaces toothed System S is fixed, the system S1 is the attached worm and S2 is the attached wheel. The distance between axes is "a" [2]. The teeth surface of the worm in the coordinate system S1 is given by the position vector  $\bar{r}_1$ ,

$$\bar{r}_1 = \bar{r}_1(\xi, \eta). \quad (2)$$

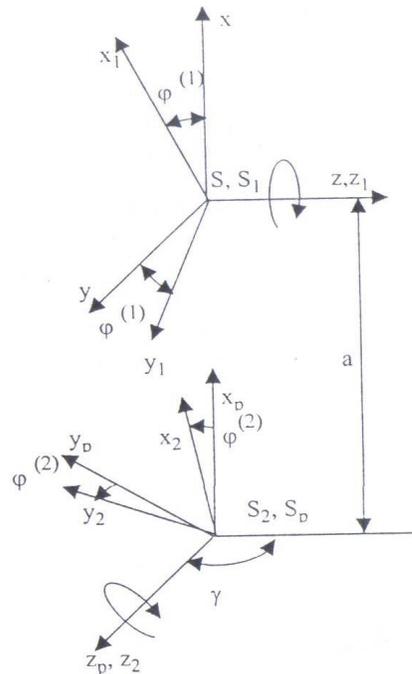


Fig. 2 [2] Coordinate systems of the worm gearing

The same surface in  $S$  and  $S_2$  systems is given using the transfer matrices  $M_{01}$  and  $M_{21}$ :

$$\bar{r} = M_{01} \cdot \bar{r}_1; \tag{3}$$

$$\bar{r}_2 = M_{21} \cdot \bar{r}_1. \tag{4}$$

In the case of the studied worm gearing, the transfer ratio [2] is:

$$i_{21} = \frac{d\varphi^{(2)}}{d\varphi^{(1)}} = \frac{\omega^{(2)}}{\omega^{(1)}}. \tag{5}$$

The contact lines are described by the enveloping conditions [2]:

$$\bar{n} \cdot \bar{v}^{(12)} = 0, \tag{6}$$

where:  $\bar{n} = \frac{\partial \bar{r}}{\partial \xi} \times \frac{\partial \bar{r}}{\partial \eta}; \tag{7}$

$$\bar{v}^{(12)} = [(\bar{\omega}^{(1)} - \bar{\omega}^{(2)}) \times \bar{r}] - (\bar{a} \times \bar{\omega}^{(2)}). \tag{8}$$

The surface of the contact pattern (fig. 3, [3]), consisting of the instantaneous contact lines, is represented by the equation (3) and the following equation:

$$f(\xi, \eta, \varphi^{(1)}) = 0. \tag{9}$$

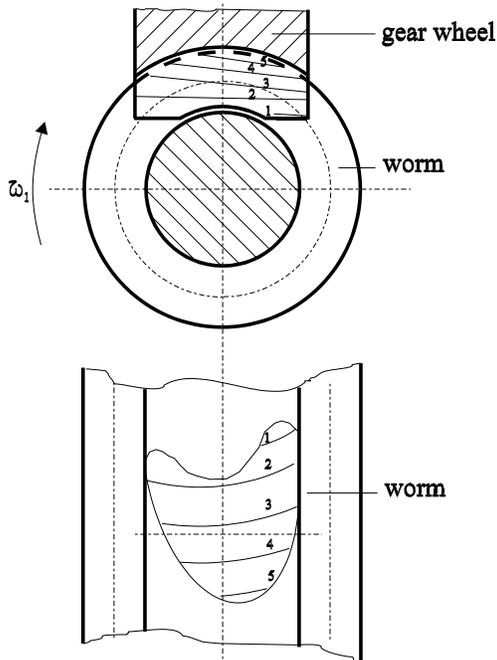


Fig. 3 [3] Contact pattern of the worm gearing

The teeth surface of the worm wheel, as the enveloping surface of the instantaneous contact lines, is represented by equations (4) and (9).

These equations and the geometric relations of the analyzed worm gearing were used to develop the necessary geometric model used in the finite element analysis to investigate the worm and worm wheel deformation, as well as the contact pressure distribution on the contact area.

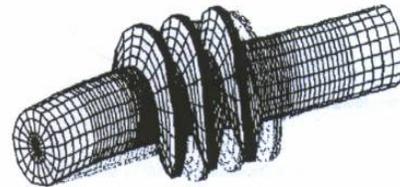


Fig. 4 [2] Worm deformation



Fig. 5 [2] Von Mises stress distribution for the worm

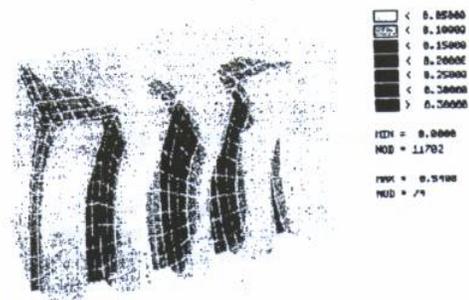


Fig. 6 [2] Von Mises stress distribution for the worm wheel

In figures 4, 5 and 6 [2] are presented the motions and tensions calculated in the meshing position  $\varphi_1 = 60^\circ$ .

From [2] the conclusion, based on obtained results, is that the maximum stress is caused by the teeth hertz contact along the contact line and these stresses change depending on the meshing position.

Although the worm shaft bending gives the maximum deflection, the local deformation nearby the contact pattern is dominant for the developed model because this model describes only the contact of one pair of teeth in meshing [4].

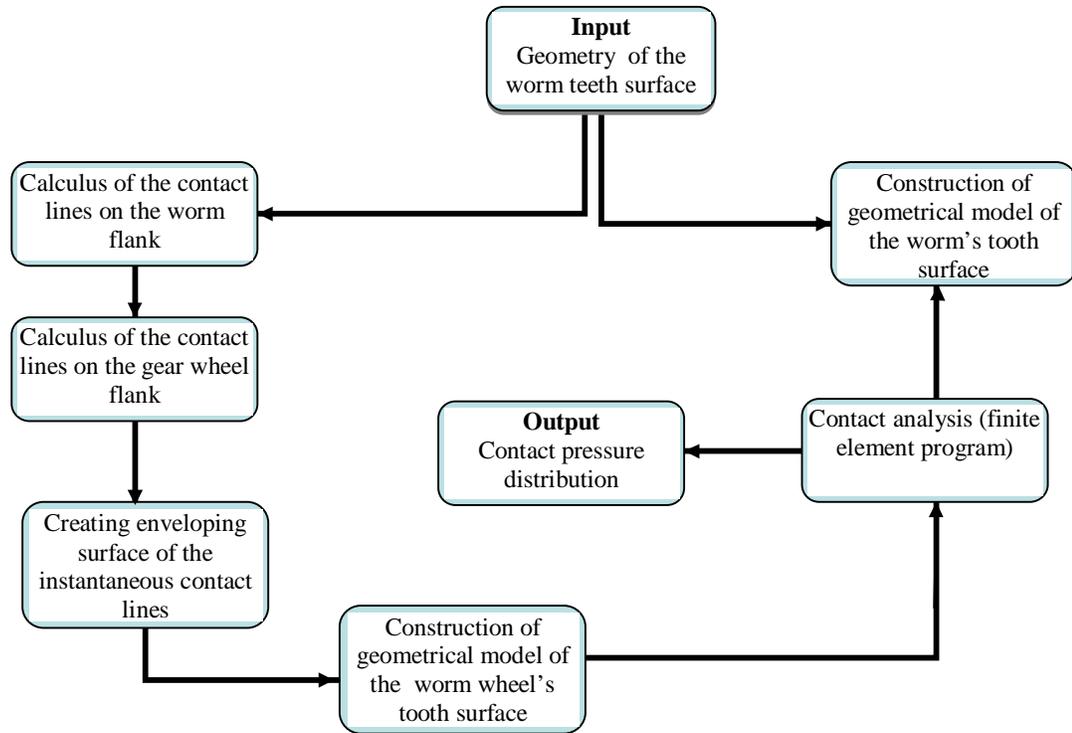


Fig. 7 [2] Block diagram of the computer program for worm gearing

Figure 7 [2] is a block diagram of a computer program to analyze the contact and contact pressure distribution of a worm gearing with the circular profile in the axial section. It was felt that the gear is deformable and errors free [2].

### 3. CONTACT OF THE DEFORMABLE WORM GEARING WITH ERRORS

The investigation of the contact conditions, the errors' effect on them and the gears' optimizing have difficulties because the contact lines move continuously and change shape during meshing.

At the same time, the kinematic relations influenced by many design parameters and processing are also different during meshing.

Another problem of the meshing research is real gearing, which is different from the theoretical model. For example, a real worm gearing can not be treated like a perfect conjugated pair of wheels without errors and with rigid bodies.

Solving these problems can only be done by developing a mathematical and physical model and applying modern numerical methods and computer techniques. The mathematical model must include error components, consider deformations that occur under the load.

This path was followed by the authors of [5]. Starting from the tool surface  $r_{S1,2}$  equation, considering errors tool parameter and machine tools, considering the generation mechanism in accordance with the processing technology applied, these equations were obtained for teeth surfaces [5]:

$$r_1 = r_{10} + r_{1\Delta} = M_S \cdot r_{S1}(u, \vartheta) = (M_{S0} + M_{S\Delta})[r_{S10}(u, \vartheta) + r_{S1\Delta}(u, \vartheta, \Delta)] \quad (10)$$

$$r_2 = r_{20} + r_{2\Delta} = M_{2S} \cdot r_{S2}(v, \psi) = (M_{2S0} + M_{2S\Delta})[r_{2S0}(v, \psi) + r_{2S\Delta}(v, \psi, \Delta)] \quad (11)$$

and meshing equations:

$$f_1(u, \vartheta, \Delta(\rho_1)) = n_1 \cdot v_1^{(S1)} = n_S \cdot v_S^{(S1)} = (n_{10} + n_{1\Delta})(v_{10}^{(S1)} + v_{1\Delta}^{(S1)}) = 0 \quad (12)$$

$$f_2(v, \psi, \Delta(\rho_2)) = n_2 \cdot v_2^{(S2)} = n_S \cdot v_S^{(S2)} = (n_{20} + n_{2\Delta})(v_{20}^{(S2)} + v_{2\Delta}^{(S2)}) = 0 \quad (13)$$

If in the equations (10) and (11)

$$r_{1,2\Delta} = 0,$$

then the effect of the tool errors and the adjustment errors is cancelled, resulting theoretical teeth surfaces,  $r_{1,20}$ .

Obviously, that we can make an analysis of the effect of the tool errors and machine tools adjustment on the teeth precision.

Also due to these errors, authors of [5] found it necessary to exclude the possibility of intersection of the contact surfaces. In the case of determining the contact points, the surfaces do not intersect if the sign of the reduced  $\chi^{(p)}$  normal curvature remains the same throughout all normal intersections.

Therefore, adds to relations (10) ... (13) the following equation [5]:

$$f_3(u, \vartheta, v, \psi, \Delta_1, \Delta_2, \Delta_{1,2}, \rho_1, \rho_2 = i_{21}\rho_1) = 0 \quad (14)$$

$$\text{sign}|\chi^{(p)}| = \text{sign}|\chi^{(1)} - \chi^{(2)}| = \text{const.} \quad (15)$$

To determine the contact between worm and worm wheel, a finite element computer system and a special algorithm were used.

This algorithm allowed the assessment of contact pressure distribution and the effect of the error on the contact pattern.

It is a complex approach that is a cause-effect analysis in the case of each error or a combination of errors, for both worm and worm wheel.

#### 4. CONCLUSIONS

1. If the worm gearing teeth are theoretically rigid theoretical and the assembling and processing errors are null, then the contact between worm and wheel is a line.

2. Computer programs have been developed to analyze the contact and contact pressure distribution of a worm gearing.

3. A problem of the meshing research is real gearing, which is different from the theoretical model, i.e. real worm gearing can not be treated like a perfect conjugated pair of wheels without errors and with rigid bodies.

4. The mathematical model must include error components, consider deformations that occur under the load.

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