RHEOLOGICAL SLS MODEL. DYNAMIC PARAMETERS OF THE MECHANICAL SYSTEMS WITH VISCOUS DAMPING. PART 2: TRANSMISSIBILITY FACTOR AND ISOLATION DEGREE

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ABSTRACT

The paper presents the dynamic model of a 1DOF elastic mechanical system with viscous damping rheologically modeled as SLS (Standard Linear Solid) model. The SLS rheological model is a Maxell model in parallel with a Hooke model. The perturbation force $F_0 sin\omega t$ is transmitted to the base through both of the elements (Hooke and Maxwell). The analyzed dynamic characteristics are the transmissibility ratio $T(\Omega,\zeta)$ and the isolation degree $I(\Omega,\zeta)$, function of the viscous damping ratio ζ and the relative frequency Ω.

KEYWORDS: 1DOF, viscous damping, SLS model, transmissibility ratio, isolation degree

INTRODUCTION. THE DYNAMIC MODEL OF 1DOF MECHANICAL SYSTEM WITH SLS VISCOUS DAMPING

Considering the calculus scheme of the 1DOF mechanical system with SLS viscous damping support from figure 1, perturbed by the variable harmonic force $F(t) = F_0 \, sin\omega t$, the structural linear characteristics are:

►the supported mass m;
►the visco-elastic characteristics of SLS rheological model:

-the stiffness coefficient k^1 of the Hooke model

-the stiffness coefficient $k^2 = Nk^1$ of the Maxwell model

-the viscous damping coefficient b^2 of the Maxwell model

The kinematic parameters of the mechanical system are:

►displacement z_f, speed $\dot{z}_f$ and acceleration $\ddot{z}_f$ of mass m

►displacement y and speed $\dot{y}$ of the connection point of the Maxwell model (between the Hooke model and the Newton model).
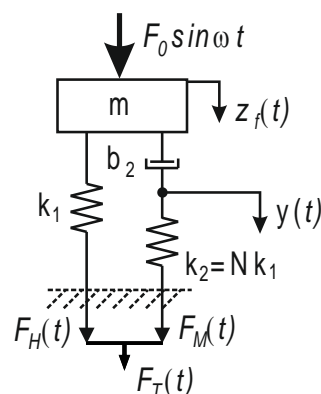
Fig. 1. Model of 1DOF mechanical system with viscous damping (SLS rheological model) perturbed by a harmonic force [1] [2] [3]

TRANSMISSIBILITY RATIO

The perturbation harmonic force is transmitted from the mass m to the base through the Hooke model and the Maxwell model. The transmitted force to the base is the sum of the Hooke force and the Maxwell force as follows [4] [5]:

$$F_T(t) = F_H(t) + F_M(t) \qquad (1)$$

Since the components of SLS rheological model are linear, the kinematic and dynamic parameters of the system have harmonic time variation, with the same angular frequency ω as the perturbation force, with different values for the phase shift [4] [5]:

$$z_f(t) = A_f \, s \, (\omega t - \varphi_0) \qquad (2)$$

$$y(t) = A_Y \, s \, (\omega t - \alpha) \qquad (3)$$

$$F_T(t) = F_{0T} \, s \, (\omega t - \beta) \qquad (4)$$

The moving equations and the transmitted force can be written as follows [1] [2] [3]:

$$\begin{cases} m\ddot{z} + b_2(\dot{z}_f - \dot{y}) + k_1 z_f = F_0 \, s \, \omega t \\ b_2(\dot{z}_f - \dot{y}) = k_2 y \\ F_T(t) = k_1 z_f + k_2 y \end{cases} \qquad (5)$$

Taking into consideration (2) (3) (4), the transmitted force to the base is:

$$F_{0T} \, s \, (\omega t - \beta) = \\ = k_1 A_f \, s \, (\omega t - \varphi_0) + k_2 A_Y \, s \, (\omega t - \alpha) \qquad (6)$$

The amplitude F_{0T} of the transmitted force can be written

$$F_{0T} = F_0 \sqrt{\dfrac{1 + \left(\dfrac{2\zeta\Omega}{N}\right)^2 (N+1)^2}{\left(1 - \Omega^2\right)^2 + \left(\dfrac{2\zeta\Omega}{N}\right)^2 \left(N + 1 - \Omega^2\right)^2}} \qquad (7)$$

where: N is the stiffness coefficients ratio

ζ - viscous damping ratio (Maxwell model)

$$\Omega = \frac{\omega}{p} \text{ - relative (angular) frequency}$$

p - natural frequency (Hooke model)

The amplitude of the transmitted force F_{0T} can be written function of the amplitude of the perturbation force F_{0T} and the transmissibility ratio $T(\Omega, \zeta, N)$ as follows:

$$T(\Omega, \zeta, N) = \sqrt{\dfrac{1 + \left(\dfrac{2\zeta\Omega}{N}\right)^2 (N+1)^2}{\left(1 - \Omega^2\right)^2 + \left(\dfrac{2\zeta\Omega}{N}\right)^2 \left(N + 1 - \Omega^2\right)^2}} \qquad (8)$$

$$F_{0T} = F_0 \cdot T(\Omega, \delta, N) \qquad (9)$$

ISOLATION DEGREE

Function of transmissibility ratio, the isolation degree is:

$$I = (1 - T) \times 1 \; 0 \; [\%] \qquad (10)$$

TRANSMISSIBILITY RATIO DIAGRAMS OF THE MECHANICAL SYSTEM WITH SLS MODEL

Figures 2 to 8 show the transmissibility ratio diagrams function of the relative frequency Ω with parametric variables ζ and N.
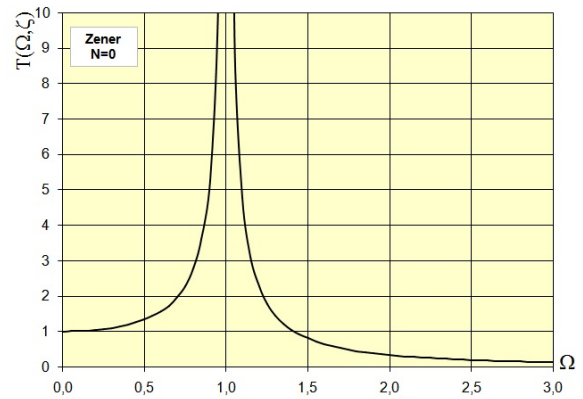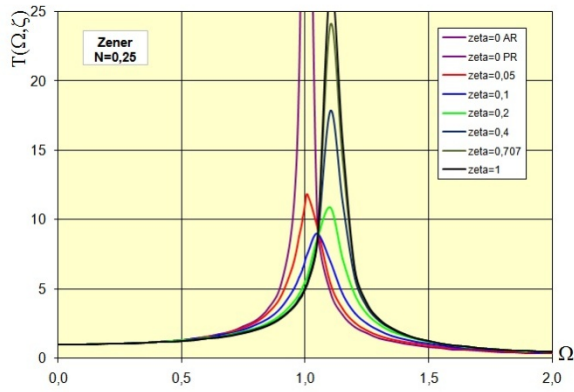

Fig. 2. Transmissibility SLS model - N=0 (Hooke model)
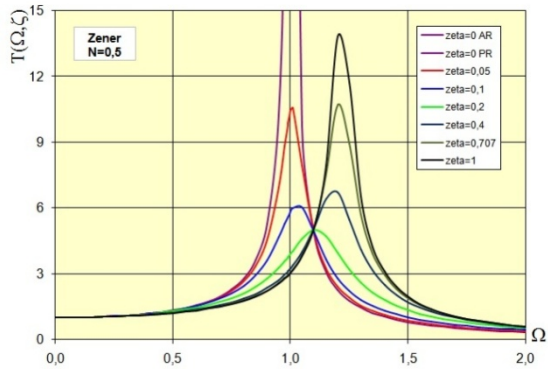
Fig. 3. Transmissibility SLS model - N=0.25

Fig. 4. Transmissibility SLS model - N=0.5
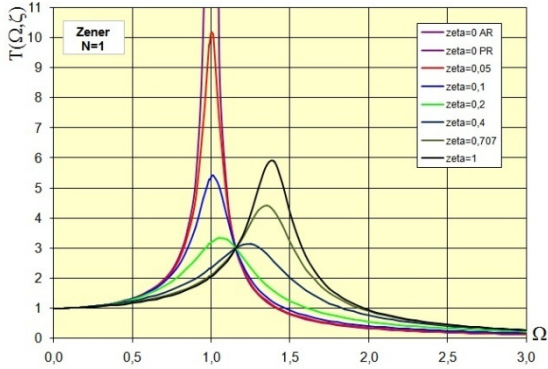

Fig. 5. Transmissibility SLS model - N=1

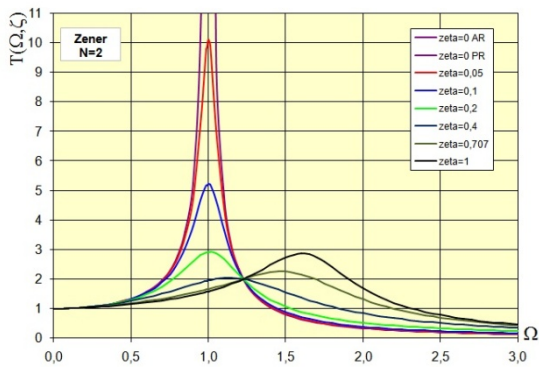Fig. 6. Transmissibility SLS model - N=2
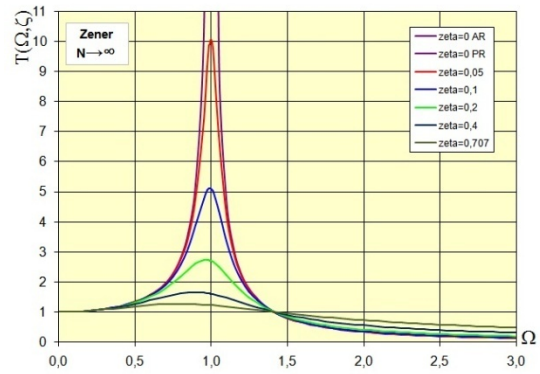

Fig. 7. Transmissibility SLS model - N→∞
(Voigt-Kelvin model)

ISOLATION DEGREE DIAGRAMS OF THE MECHANICAL SYSTEM WITH SLS MODEL

Figures 8 to 13 show the isolation degree diagrams function of the relative frequency Ω for different values of the parametric variables ζ and N.
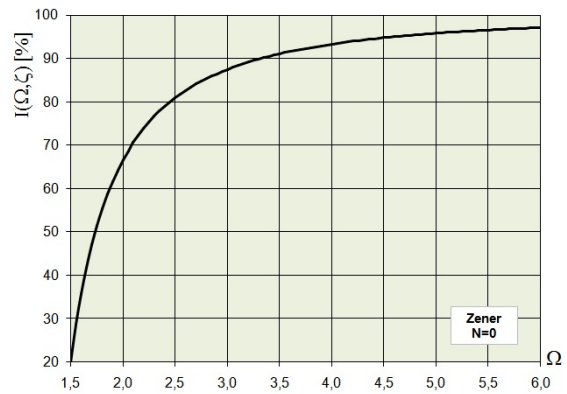

Fig. 8. Isolation degree SLS model - N=0
(Hooke model)
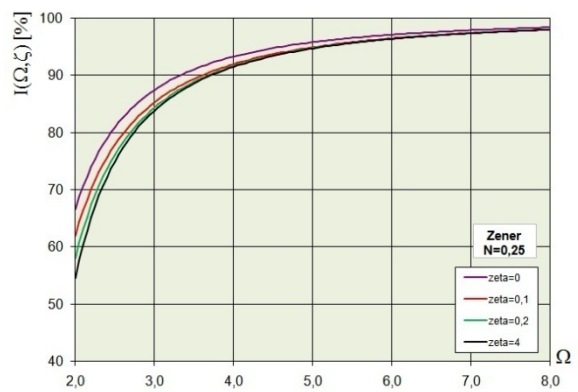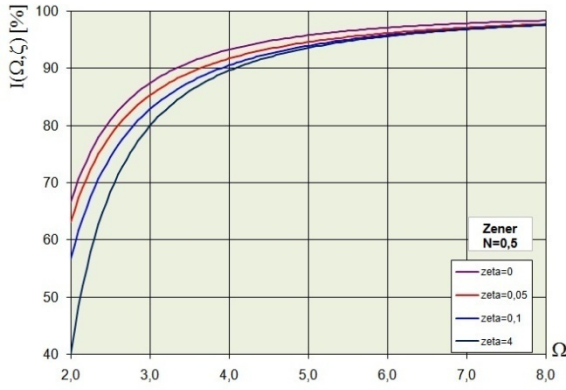

Fig. 9. Isolation degree SLS model - N=0.25

Fig. 10. Isolation degree SLS model - N=0.5
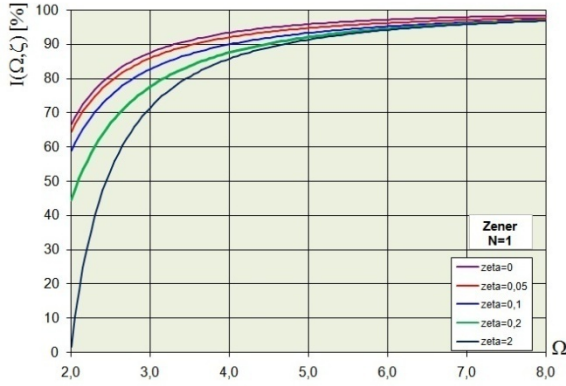

Fig. 11. Isolation degree SLS model - N=1

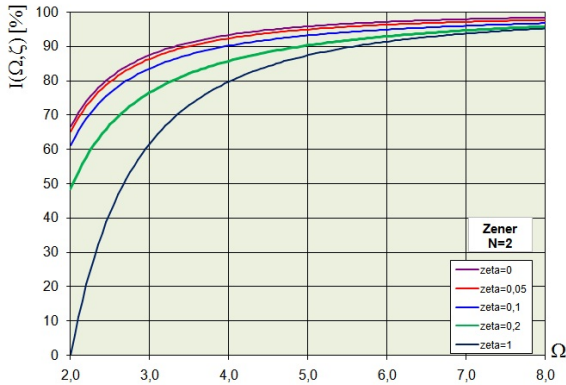Fig. 12. Isolation degree SLS model - N=2

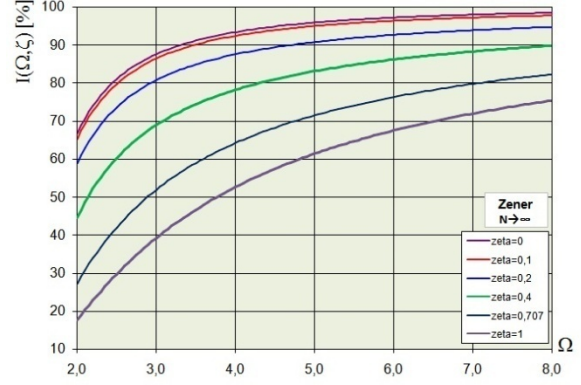Fig. 13. Isolation degree SLS model - N→∞
(Voigt-Kelvin model)

CONCLUSIONS

►if $N=0/k_2=0$ or $\zeta=0$, SLS model becomes the Hooke model; the transmissibility ratio diagram is shown in figure 2; the transmissibility formula becomes:

$$T_{N=0}(\Omega) \equiv T_{\zeta=0}(\Omega) = \frac{1}{\left|1-\Omega^2\right|} \qquad (11)$$

►if $N \rightarrow \infty / k_2 >> k_1$ (the spring from the Maxwell model becomes a rigid connection), SLS rheological model becomes the Voigt-Kelvin model; the transmissibility ratio diagram is shown in figure 7; the transmissibility ratio formula becomes:

$$T_{N\rightarrow\infty}(\Omega,\zeta) = \sqrt{\frac{1+4\zeta^2\Omega^2}{\left(1-\Omega^2\right)^2 + 4\zeta^2\Omega^2}} \qquad (12)$$

►for the Hooke rheological model, the maximum value for transmissibility ratio is obtained for $\Omega=1$, see figure 2;
►for the complex SLS rheological model, the maximum values for transmissibility ratio depends on the stiffness coefficients ratio N (figures 3 to 6); the maximum values are obtained for $\Omega>1$;
►the isolation degree depends on the parametric variables ζ and N ;
►we can obtain good isolation degrees ($I>95\%$) only for bigger relative frequency ($\Omega>3,5...4,0$) and only if viscous damping is smaller than 0.2.

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