THE ANNALS OF "DUNAREA DE JOS" UNIVERSITY OF GALATI
FASCICLE XIV MECHANICHAL ENGINEERING, ISSN 1224-5615
2017
https://doi.org/10.35219/im.2017.2.03

A COMMON MODE FOR CALCULATING THE KINEMATICS OF MECHANISMS ACTIVATED WITH HYDRAULIC CYLINDERS

Assoc. Prof. Dr. Eng. Niculai HAUK
„Dunărea de Jos" University of Galaţi

ABSTRACT

The actuation of the mechanisms using the hydraulic cylinders is performed because they can achieve large forces at small dimensions and have small relative masses. The increase of the computing power has made possible a mathematical modelling closer to the reality of the behaviour during the work of this type of mechanical-hydraulic systems.
There are still difficulties in approaching the construction of the models because of the variety of mechanical systems, the different types of hydraulic cylinders used and the types of simpler or more complex hydraulic schemes used.
This work aims at achieving a uniformization of the approach of the models of mechanisms used in the case of the use of different technical solutions of construction and of location of hydraulic cylinders.

KEYWORDS: hydraulic cylinder, actuated mechanism

INTRODUCTION

The most common types of hydraulic power cylinders are shown in Figures 1 and 2.

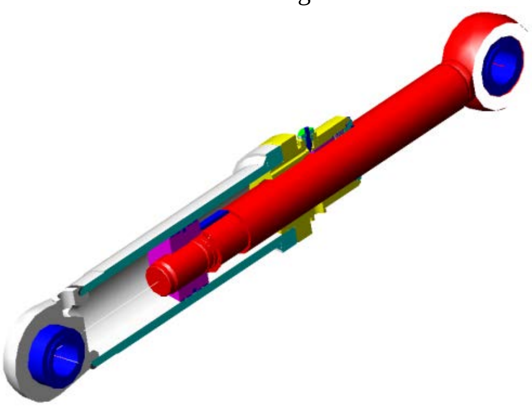

Fig. 1 Hydraulic cylinder articulated at both ends

At this type of hydraulic cylinder the joints at the ends can be:

- cylindrical joints (easy to achieve but at risk of being blocked in operation);
- spherical joints (most often used because they easily take on constructive errors and deformations of the mechanism structure).

In the hydraulic cylinder in figure 2 the joint on the cylinder body can be conveniently mounted.

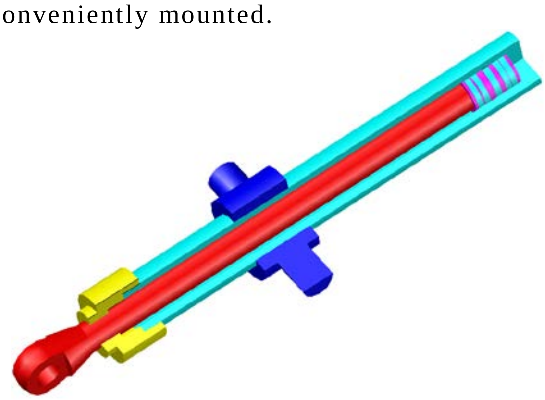

Fig. 2 Hydraulic cylinder with one joint made on the body and the other at the end of the rod

These two schemes have the major

advantage of not introducing large forces on the piston and cap guide elements.

This paper aims at presenting a unitary way of calculating the kinematics of the mechanisms operated with such hydraulic cylinders, irrespective of the direction of the movement of the rod and the direction of the rotation of the driven element.

Figures 3 and 4 show the main geometric elements of the hydraulic cylinders in Figures 1 and 2, which relate to the kinematics of the actuated mechanisms.

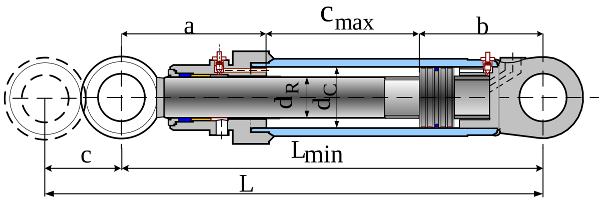

Fig. 3 The main kinematic dimensions of a hydraulic cylinder hinged at both ends

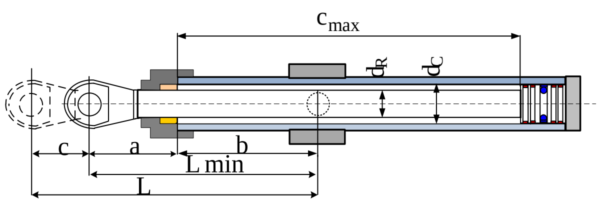

Fig. 4. The main kinematic dimensions of a hydraulic cylinder hinged on the cylinder tube

The maximum achievable stroke depends on the geometry of the hydraulic cylinder components.

An example of the use of hydraulic cylinders is shown in Figure 5. The triangle of the vectors which describe the components of the mechanism, drawn in the plane of symmetry of the excavator's working equipment, is highlighted.

Figure 6 shows the excavation equipment of a hydraulic excavator that includes three subsystems operated with hydraulic cylinders.

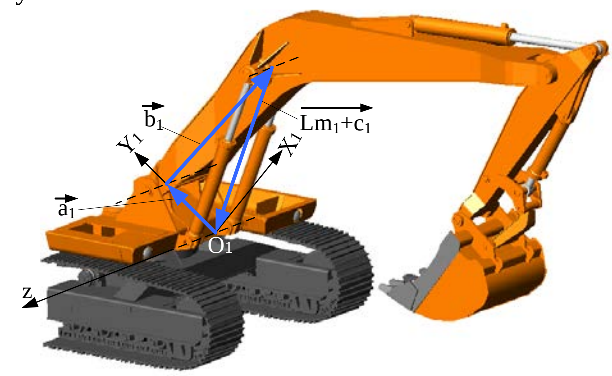

Fig. 5 The system of hydraulic cylinders, articulated at both ends, which actuate the boom of an excavator represented vectorially

Three mobile reference systems were considered corresponding to the degree of mobility of the excavation equipment mechanism.

2. KINEMATICS OF MECHANICAL SYSTEMS OPERATED WITH HYDRAULIC CYLINDERS

For the kinematic analysis of the subsystems operated with hydraulic cylinders are considered the situations presented in figures 7 and 8. The dimensioning system was chosen so that, in both cases, the same way of calculating the kinematics could be used. In both cases:

- $L = Lmi + c$;

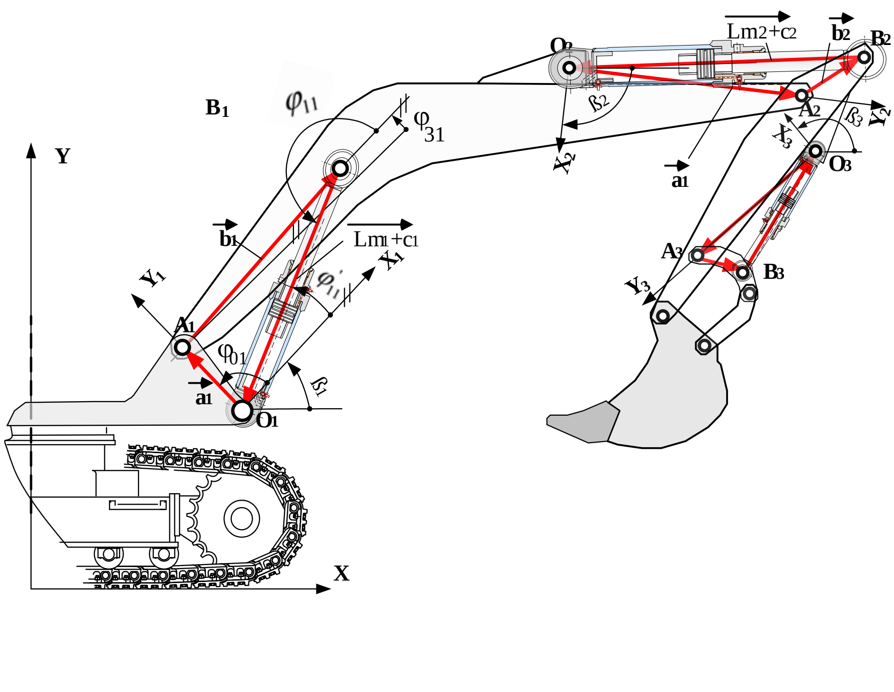

Fig. 6 Subsystems of a work equipment of an excavator with reverse bucket actuated by hydraulic cylinders

- the axis Oi of the local coordinate system was fixed in the articulation of the cylinder body;
- the axis Oi - Yi passes through Ai;
- the rotations of the vectors associated with the movable elements are considered in relation to the axis Oi - Xi.

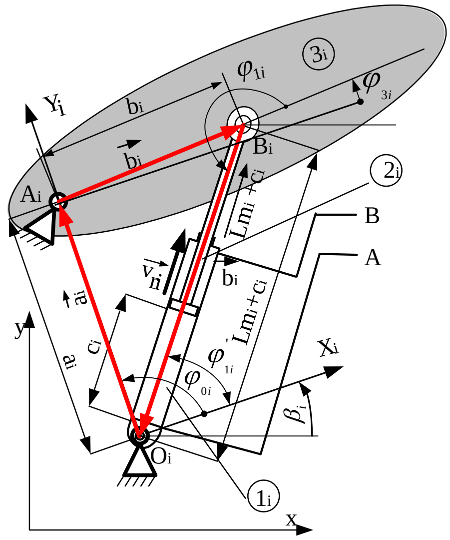

Fig. 7 Kinematic scheme in which the rigid element 3, with fixed ax, is rotated by the hydraulic cylinder, consisting of elements 1i, body, and 2i piston-rod

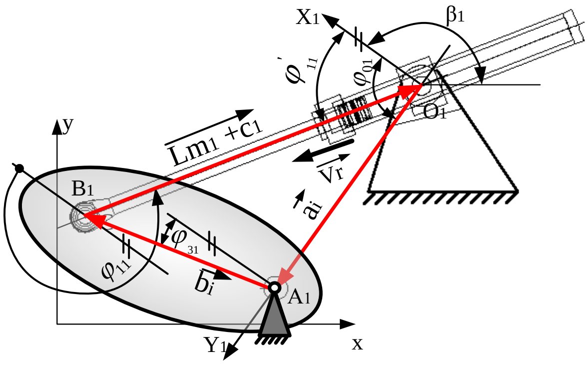

Fig.8 Subsystem driven with hydraulic cylinder with a joint disposed on the cylinder body

Position calculation

In the system of mobile coordinate axes X_i, O_i, Y_i is considered the closed polygonal contour formed by the vectors: $\overline{a}_i$, $\overline{b}_i$, $\overline{L_{mi}+c_i}$

The equation of the closed polygonal contour is:

$$\overline{a}_i+\overline{b}_i+\overline{L_{mi}+c_i}=\overline{0} \qquad (1)$$

This equation is projected on the axes of the mobile coordinate system and it is obtained:

$$\begin{cases} a_i\cos\varphi_{0i}+b_i\cos\varphi_{3i}+(L_{mi}+c_i)\cos\varphi_{1i}=0 \\ a_i\sin\varphi_{0i}+b_i\sin\varphi_{3i}+(L_{mi}+c_i)\sin\varphi_{1i}=0 \end{cases} \qquad (2)$$

With $\varphi_{0i}=\pi/2$ and $\varphi_{1i}=\varphi_{2i}=\pi+\varphi_i'$ the system (2) becomes

$$\begin{cases} b_i \cos\varphi_{3i} = +\left(L_{mi}+c_i\right)\cos\varphi'_{1i} \\ a_i + b_i \sin\varphi_{3i} = \left(L_{mi}+c_i\right)\sin\varphi'_{1i} \end{cases} \qquad (3)$$

and

$$\sin\varphi_{3i} = \frac{\left(L_{mi}+c_i\right)^2 - b_i^2 - a_i^2}{2a_i b_i}$$

$$\sin\varphi'_{1i} = \frac{a_i^2 + \left(L_{mi}+c_i\right)^2 - b_i^2}{2a_i\left(L_{mi}+c_i\right)} \qquad (4)$$

The position of the B_i joint in relation to the mobile coordinate axis system is:

$$\begin{cases} x_{B_i} = \left(L_{mi}+c_i\right)\cos\varphi'_{1i} \\ y_{B_i} = \left(L_{mi}+c_i\right)\sin\varphi'_{1i} \end{cases} \qquad (5)$$

In relation to the fixed system of axes, the coordinates are:

$$\begin{cases} x = x_{0i} + \left(L_{mi}+c_k\right)\cos\left(\varphi'_{1i}+\beta_i\right) \\ y = y_{0i} + \left(L_{mi}+c_k\right)\sin\left(\varphi'_{1i}+\beta_i\right) \end{cases} \qquad (6)$$

Calculation of speeds

For the calculation of the angular velocities we derive system (2). Taking into account the fact that angles are functions of time we obtain:

$$\begin{cases} -b_i\omega_{3i}\sin\varphi_{bi} = \dot{c}_i\cos\varphi'_{1i} - \left(L_{mi}+c_i\right)\omega_{1i}\sin\varphi'_{1i} \\ b_i\omega_{3i}\cos\varphi_{bi} = \dot{c}_i\sin\varphi'_{1i} + \left(L_{mi}+c_i\right)\omega_{1i}\cos\varphi'_{1i} \end{cases} \qquad (7)$$

$\dot{c}_k$ is the relative speed between elements 1i and 2i.

$$\dot{c}_{ri} = v_{1i,2i} = \frac{Qiq}{S_{iq}} \qquad (8)$$

with ***q*** = 1 if the rod gets out of the body and ***q***=2 when the rod enters the body.

The index q individualizes the surface as follows:

- for ***q*** = 1, with $S_{iq1} = \frac{\pi}{4}\cdot D_{Ci}^2$;
- for ***q*** = 2, with $S_{iq2} = \frac{\pi}{4}\cdot\left(D_{Ci}^2 - D_{Ri}^2\right)$.

From system (7) result the angular velocities:

$$\omega_{1i} = \frac{\dot{c}_i}{\left(L_{mi}+c_i\right)\cdot tg\left(\varphi'_{1i}-\varphi_{3i}\right)}$$

$$\omega_{3i} = \frac{\dot{c}_i\sin\varphi'_{1i} + \omega_{1i}\left(L_{mi}+c_i\right)\cos\varphi'_{1i}}{b_i\cos\varphi_{31}} \qquad (9)$$

Calculation of accelerations

To determine the accelerations, the system (7) is derived and the following equations are obtained:

$$\begin{cases} -b_i\varepsilon_{3i}\sin\varphi_{3i} - b_i\omega_{3i}^2\cos\varphi_{3i} = \\ \ddot{c}_i\cos\varphi'_{1i} - \dot{c}_i\omega_{1i}\sin\varphi'_n - \dot{c}_i\omega_{1i}\sin\varphi'_{1i} \\ -(L_{mi}+c_i)\cdot(\varepsilon_n\sin\varphi'_{1i} + \omega_{1i}^2\cos\varphi'_{1i}) \\ b_i\varepsilon_{31}\cos\varphi_{3i} - b_i\omega_{3i}^2\sin\varphi_{3i} = \\ \ddot{c}_i\sin\varphi'_{1i} + \dot{c}_i\omega_{1i}\cos\varphi'_{1i} + \dot{c}_i\omega_{1i}\cos\varphi'_{1i} + \\ (L_{mi}+c_i)\cdot(\varepsilon_{1i}\cos\varphi'_{1i} - \omega_{1i}^2\sin\varphi'_{1i}) \end{cases} \quad (11)$$

With notations:

$$\begin{aligned} t_1 &= b_i\omega_{3i}^2\cos\varphi_{3i} + \ddot{c}_i\cos\varphi'_{1i} - \\ &2\dot{c}_i\omega_{1i}\sin\varphi'_{1i} - (L_{m1}+c_i)\,\omega_{1i}^2\cos\varphi'_{1i} \\ t_2 &= b_i\omega_{3i}^2\sin\varphi_{3i} + \ddot{c}_i\sin\varphi'_{1i} + \\ &2\dot{c}_i\omega_{1i}\cos\varphi'_{1i} - (L_{mi}+c_i)\,\omega_{1i}^2\sin\varphi'_{1i} \end{aligned} \quad (12)$$

It results the system:

$$\begin{cases} -\varepsilon_{3i}b_i\sin\varphi_{3i} + \varepsilon_{1i}(L_{mi}+c_i)\sin\varphi'_{1i} = t_1 \\ \varepsilon_{3i}b_i\cos\varphi_{3i} - \varepsilon_{1i}(L_{mi}+c_i)\cos\varphi'_{1i} = t_2 \end{cases} \quad (13)$$

Solving the system (13), it results:

$$\begin{aligned} \varepsilon_{1i} &= \frac{t_1 + t_2 tg\varphi_{3i}}{(L_{mi}+c_i)(\sin\varphi'_{1i} - \cos\varphi'_{1i} tg\varphi_{3i})} \\ \varepsilon_{3i} &= \frac{\varepsilon_{1i}(L_{mi}+c_i)\cos\varphi'_{1i} + t_2}{b_i\cos\varphi_{3i}} \end{aligned} \quad (14)$$

where $\ddot{c}_k$ is the modulus of the relative linear acceleration between the elements 1i and 2i, with

$$\ddot{c}_i = a_{1i,2i} = \frac{\ddot{Q}_i}{S_{ci}}$$

Kinematic calculations can be developed in two hypotheses:

a) - the movement of the leading element is assumed to be known. In this case, the positions, speeds and accelerations of the points / elements that are of interest can be determined by classical vector calculations. Calculations can also be developed if the mobility of the mechanism is greater than one;

b) - it is desired to establish the laws of motion of the elements of the mechanism starting from the balance of forces and the continuity of the fluidic media. These conditions are written in the form of differential-integral equations. For their integration, the numerical procedures are used. One possible method is Runge-Kutta of order 4.

With notations

$$z_1 = c(t),\ z_2 = \dot{c}(t) \text{ and } z_3 = p(t) \quad (15)$$

the parameters from one step $n+1$ can be calculated based on those from step n with the relation

$$\begin{vmatrix} z_1 \\ z_2 \\ z_3 \end{vmatrix}_{n+1} = \begin{vmatrix} z_1 \\ z_2 \\ z_3 \end{vmatrix}_n + \frac{1}{6}\left|K_0 + 2K_1 + 2K_2 + K_3\right|_n \quad (16)$$

where the K_i coefficients are method specific.

In this case, it is necessary to know the behaviour of the hydraulic drive system.

3. CONCLUSIONS

The use of this type of analysis of the mechanisms operated with hydraulic cylinders allows for;

-the use of calculation relationships regardless of the type of hydraulic cylinders used;

-the analysis of this type of planar mechanisms regardless of the calculation hypotheses from which it starts;

-the automatization of calculations through the creation of specialized programs.

REFERENCES

[1] **A.S. Hall, P.R. McAree,** *Robust bucket position tracking for a large hydraulic excavator*, Mechanism and Machine Theory 40 (2005) 1–16 Elsevier

[2] **Hauk N.,** *Contributii teoretice si experimentale privind comportarea in regim dinamic a cilindrilor hidraulici montati pe excavatoare*, Doctoral dissertation – University “Dunarea de Jos” Galati, 1998.

[3] **Soo-Jin Lee, Pyung-Hun Chang**, *Modeling of a hydraulic excavator based on bond graph method and its parameter estimation*, Journal of Mechanical Science and Technology 26 (1) (2012) 195~204

[4] **Yang Liu, Mohammad Shahidul Hasan1, Hong-Nian Yu1** *Modelling and Remote Control of an Excavator* International Journal of Automation and Computing, 7(3), August 2010, 349-358, DOI: 10.1007/s11633-010-0514-8.