DYNAMIC MODEL FOR A MOVING PARTICLE ON TECHNOLOGICAL ROTATING WORKING BODY OF AN AGRICULTURAL MACHINE

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ABSTRACT

This paper deals with the movement dynamics of a soil particle or a seed on the working body of an agricultural machine having uniform rotational motion. It presents the dynamic equations that describe the instant positions of the particle. From these equations, the kinematical characteristics can be obtained. The goal of this research is to dignify the influences of the complementary forces about the particle motion, which are usually neglected for these cases of analysis.

KEYWORDS: soil particle, agricultural machine, particle motion, dynamic model

1. INTRODUCTION

In order to achieve the specific technological processes, agricultural machines have used equipments with radial palettes or cutters, rotated with angular speed generally acting on perpendicular direction onto port-cutting blades disc. The materials for these technological equipments can be soil (terrain), granulated materials or crystal-shape materials.

This category of agricultural machines contain equipments for fertilizer dissipation, equipments for dip burrow, rotating hoes, rotating discs harrows, mixers, seeders, etc. It is obvious the fact that for each type of these equipments previously presented the particle movement on the active working body (palette, cutter, etc.) has a specific character that imposes a particular approach. The common aspect of these equipments is that they contain palettes or cutters with rotational movement on orthogonal direction relative to the disposing plane. The specific aspect consists of the access way of the particles into the equipment and of the spatial disposing way of the equipment (horizontal or vertical).

In such conditions, it can be compiled a common dynamic model and the particularities can be imposed through the initial conditions of the particles access into the equipment, the spatial disposing way of the equipment and the tilt angle of the palette.

These are special aspects presented in this paper, which try to dignify a generalized model intended for particles movement while taking into account the complementary forces (transport, Coriolis) that appear at relative motion of a particle related to the active working body.

The final purpose of this study is to obtain the proper conditions for which the material particle leaves the active element of the equipment (palette, cutter, etc.) when the interest is focused on the kinematical characteristics (speed modulus and direction, trajectory, etc.) or on the dynamics (kinetic energy, impulse, etc.). These characteristics are necessary for the study of the general movement of the particle in the air.

2. PROBLEM BASICS AND HYPOTHESIS

The schematic diagram of the proposed model id depicted in Fig. 1. The material particle M, with mass m, being in contact with active working body with (a, h) dimensions has constant angular speed ω. Tilt angle of the active working body with respect to the rotation axis is denoted by angle α. The instant position of the particle M with respect to the palette is given by the instant coordinates x=x(t) and z=z(t).

The y axis coordinate is null (y = 0) taking into account that particle M continuously keeps the contact with the active working body until this material particle leaves the active surface (a.h).

The friction between the particle and the active working body surface is denoted by the friction coefficient μ, which is specific for each pair of materials in interaction.

The position of the active working body with respect to the rotation axis is given by the inner radius r_o and outer radius $r_e = \overline{r_o} + a$.

The reference systems related to the motion analysis are $x_1oy_1z_1$ – the fixed reference system and xoyz (xAy'z') - the mobile reference system linked with the active working body.

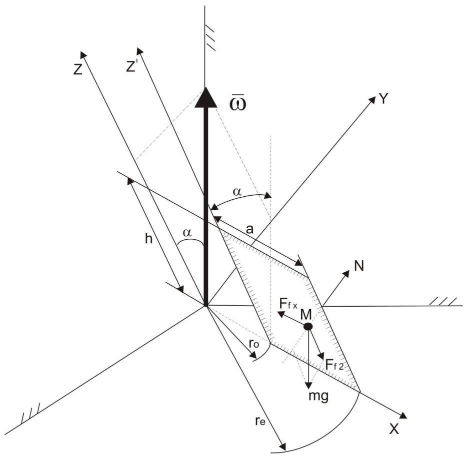

Fig.1. Schematic diagram of the active working body

The notations and symbols used in Fig. 1 have significations as follows: G=mg – particle M weight; F_{fx} – friction force at particle sliding along x direction; F_{fz} – friction force at particle sliding along z direction; N – normal reaction force on active working body surface; G – active force of the particle; F_{fx}, F_{fz}, N – linkage forces between the particle and the working body.

3. MOTION EQUATIONS FOR A SINGLE PARTICLE CASE

The vectorial equation that describes the motion laws of the particle M on the surface of the active working body (depicted by ABCD in Fig.1) is

$$m\overline{a} = \sum \overline{F}^a + \sum \overline{F}^l + \overline{F}_t + \overline{F}_c ; \qquad (1)$$

where

$\sum \overline{F}^a$ - denote the active forces applied directly to the particle M; in this case the only active force is the weight G of the particle.

$\sum \overline{F}^l$ - denotes the linkage forces of the particle related to the active surface of the working body.

$\overline{F}_t$ - denotes the complementary force related to particle movement.

$\overline{F}_c$ - denotes the Coriolis complementary force.

The transport complementary force equation is

$$\overline{F}_t = -m\overline{a}_t = -m\left[\overline{\omega} \times (\overline{\omega} \times \overline{r})\right]; \qquad (2)$$

where ω = constant, which denotes the permanent regime movement of the active body.

The expression of the Coriolis complementary force is

$$\overline{F}_C = -m\overline{a}_C = -m\left[2\overline{\omega} \times \overline{v}_r\right]; \qquad (3)$$

The notations and symbols in Eqns. (2) and (3) have significations as follows

m – particle mass;

$\overline{\omega}$ - angular speed of the active body, which is defined as a vectorial parameter into the mobile reference system xoyz. This parameter can be defined using an expression as follows

$$\overline{\omega} = \omega \cdot \sin\alpha\,\overline{j} + \omega \cdot \cos\alpha\,\overline{k}\,; \qquad (4)$$

$\overline{r}$ - the vector of the instant position of the particle m, with the expression

$$r = x\overline{i} + z\overline{k}\,; \qquad (5)$$

$\overline{v}_r$ - the relative velocity of the particle with respect to the active working body

$$\overline{v}_r = \dot{x}\overline{i} + \dot{z}\overline{k}\,; \qquad (6)$$

In the previous equations the $\overline{i}, \overline{j}, \overline{k}$ denote the versors of the mobile reference system xoyz.

Using the Eqns. (2) and (3) for the parameters defined by expressions (4), (5) and (6), and taking into account the vectorial product significations it results

$$\begin{aligned}\overline{F}_t &= mx\omega^2\overline{i} - mz\omega^2 \sin\alpha\cos\alpha\,\overline{j} + \\ &+ mz\omega^2 \sin^2\alpha\,\overline{k}\end{aligned}\;; \qquad (7)$$

$$\begin{aligned}\overline{F}_c &= -2m\omega\dot{z}\sin\alpha\,\overline{i} - 2m\omega\dot{x}\cos\alpha\,\overline{j} + \\ &+ 2m\omega\dot{x}\sin\alpha\,\overline{k}\end{aligned}\;; \qquad (8)$$

which represents the vectorial expressions of the moving and Coriolis complementary forces.

Supposing the vectorial expressions of active and linkage forces defined by the following equations

$$\begin{aligned}&\overline{N} = N\overline{J}; \quad \overline{G} = -mg\sin\alpha\,\overline{j} - mg\cos\alpha\,\overline{k}; \\ &\overline{F}_{fx} = -\mu N\overline{i}; \overline{F}_{fz} = -\mu N\overline{k}\end{aligned}\;; \qquad (9)$$

and by the Eqns. (7) and (8), and using the Eqn. (1), after reordering the terms with respect to ox, oy, oz axis, it results

$$\begin{cases} m\ddot{x} = mx\omega^2 - 2m\omega\dot{z}\sin\alpha - F_{fx} \\ m\ddot{y} = -mz\omega^2\sin\alpha\cos\alpha - \\ -2m\omega\dot{x}\cos\alpha - mg\sin\alpha + N \\ m\ddot{z} = mz\omega^2\sin^2\alpha + 2m\omega\dot{x}\sin\alpha - \\ -mg\cos\alpha - F_{fz} \\ F_{fx} = F_{fz} = \mu N \end{cases} ; \quad (10)$$

While the particle kept the permanent contact with the surface of the active body, respectively y=0; $\dot{y}$=0; $\ddot{y}$=0, from the second equation of the differential system (10) results the expression of the normal force N as follows

$$N = mz\omega^2\sin\alpha\cos\alpha + 2m\omega\dot{x}\cos\alpha) + mg\sin\alpha ; \quad (11)$$

Rejoining the Eqn. (10) and taking into account the normal force (11) and the friction forces configuration, results the generalized mathematical model for the particle movement with respect to the active working body as follows

$$\begin{cases} \ddot{x} + 2\mu\omega\cos\alpha\,\dot{x} - \omega^2 x + 2\omega\sin\alpha\,\dot{z} + \\ +\mu\omega^2\sin\alpha\cos\alpha z + \mu g\sin\alpha = 0 \\ \ddot{z} + \omega^2(\mu\sin\alpha\cos\alpha - \sin^2\alpha)z - \\ -2\omega(\sin\alpha - \mu\cos\alpha)\dot{x} + \\ +g(\cos\alpha + \mu\sin\alpha) = 0 \end{cases} ; \quad (12)$$

Solving the differential system (12) with initial conditions particularized for each application enables the evaluation of the kinematical characteristics for particle movement related to the active body of the technological equipment.

4. SPECIFIC CASES

The model differential equations (12) can be assessed for particular cases with respect to tilt angle of the active working body. There result two limit cases related to the active bodies of seeders and granular fertilizer dissipaters - for $\alpha = 0$, or the active bodies of rotating hoes, rotating discs harrows and rotating discs of dust fertilizer dissipaters - $\alpha = \pi/2$. Hereby, the specific value of angle α leads to the following situations

- α=0, which denotes the case of the palette or cutter with vertical configuration, imposes sin α=0 and cos α=1, and the model becomes

$$\begin{cases} \ddot{x} + 2\mu\omega\dot{x} - \omega^2 x = 0 \\ \ddot{z} + 2\mu\omega\dot{x} + g = 0 \end{cases} ; \quad (13)$$

with the normal reaction force $N = 2m\omega\dot{x}$.

- $\alpha = \pi/2$, which denotes the case of the palette or cutter with horizontal configuration, imposes sin α=1; cos α=0, and the model becomes

$$\begin{cases} \ddot{x} - \omega^2 x + 2\omega\dot{z} + \mu g = 0 \\ \ddot{z} - \omega^2 z - 2\omega\dot{x} + \mu g = 0 \end{cases} ; \quad (14)$$

with the normal reaction force N=mg.

The mathematical models with Eqn. (13), respectively (14) take the particular initial conditions according the type of the technological process as follows

at $$t = 0;\ x = r_i;\ z = h;\ \dot{x} = 0;\ \dot{z} = v_0;\ for\ \alpha = 0; \quad (15.1)$$

and

at $$t = 0;\ x = r_e;\ z = 0;\ \dot{x} = v_0;\ \dot{z} = 0;\ for\ \alpha = \pi/2; \quad (15.2)$$

5. ANALYTICAL SOLUTIONS

Solving Eqn. (12) for the general case supposes following mathematical configuration

$$\begin{cases} \ddot{x} + a_1\dot{x} + b_1 x + c_1\dot{z} + d_1 z + e_1 = 0 \\ \ddot{z} + b_2 z + c_2\dot{x} + e_2 = 0 \end{cases} ; \quad (16)$$

where $a_1; b_1; c_1; d_1; e_1; b_2; c_2; e_2$ refer to coefficients with the particular expressions from Eqn. (12).

From the second equation of the system (16) it results

$$\dot{x} = \frac{1}{c_2}\ddot{z} + \frac{b_2}{c_2}z + \frac{e_2}{c_2}$$

or, with suiting notations

$$\dot{x} = \alpha_1\ddot{z} + \beta_1 z + \chi_1 ; \quad (17)$$

Applying the differential operator to the Eqn. (17) it results

$$\ddot{x} = \alpha_1 \dddot{z} + \beta_1 \dot{z}; \quad \dddot{x} = \alpha_1 \ddddot{z} + \beta_1 \ddot{z}; \quad (18)$$

From the first equation of the system (16) and using the derivative operator, it results

$$\dddot{x} + a_1 \ddot{x} + b_1 \dot{x} + c_1 \ddot{z} + d_1 \dot{z} = 0; \quad (19)$$

Replacing Eqns. (17) and (18) by Eqn. (19), it results an inhomogeneous 4th-order differential equation as follows

$$\ddddot{z} + a\dddot{z} + b\ddot{z} + c\dot{z} + dz = e; \quad (20)$$

where a, b, c, d, e denote the coefficients with the expressions

$$a = a_1; b = (\beta_1 + b_1\alpha_1 + c_1)/\alpha_1; c =$$
$$= (a_1 b_1 + d_1)/\alpha_1; d =$$
$$= b_1\beta/\alpha_1; e = b_1\chi/\alpha_1$$

With respect to the roots type of the characteristic equation coupled with Eqn. (20) follow general solutions result:

- if the solutions $r_1; r_2; r_3; r_4$ of the characteristic equation are real results

$$z(t) = c_1 e^{r_1 t} + c_2 e^{r_2 t} + c_3 e^{r_3 t} + c_4 e^{r_4 t} + et; \quad (21)$$

- if the solutions $r_1; r_2; r_3; r_4$ are complex conjugated roots with the expressions

$r_{1,2} = \xi_1 \pm i\eta_1; r_{3,4} = \xi_2 \pm i\eta_2$, results

$$z(t) = (c_1 \sin\eta_1 t + c_2 \cos\eta_1 t)e^{\xi_1 t} + \\ + (c_3 \sin\eta_2 t + c_4 \cos\eta_2 t)e^{\xi_2 t} + et \quad ; \quad (22)$$

- if the solutions $r_1; r_2$ are real and $r_3; r_4$ are complex conjugated results

$$z(t) = c_1 e^{r_1 t} + c_2 e^{r_2 t} + \\ + (c_3 \sin\eta t + c_4 \cos\eta t)e^{\xi t} + et \quad ; \quad (23)$$

Taking into account the solutions (21), (22) or (23) and rejoining the substitution (17) it results the solution set of x(t) with the expression

$$x(t) = B_1 e^{r_1 t} + B_2 e^{r_2 t} + B_3 e^{r_3 t} + \\ + B_4 e^{r_4 t} + et^2/2 + B_6 t + B_7 \quad ; \quad (21^*)$$

$$x(t) = (B_1 \cos\eta_1 t + B_2 \sin\eta_2 t)e^{\xi_1 t} + \\ + (B_3 \cos\eta_2 t + B_4 \sin\eta_2 t)e^{\xi_2 t} + \quad ; (22^*) \\ + \frac{et^2}{2} + B_6 t + B_7$$

$$x(t) = B_1 e^{r_1 t} + B_2 e^{r_2 t} + \\ + (B_3 \cos\eta t + B_4 \sin\eta t)e^{\xi t} + ; \quad (23^*) \\ + \frac{et^2}{2} + B_6 t + B_7$$

The coefficients $C_1 \div C_7; \quad B_1 \div B_7$ result from the initial condition imposition for the movement process and have particular values for each supposed case.

6. CONCLUSIONS

The movement of the material particle that can simulate the ground (soil) grains, seeds, granular or crystalline fertilizer over the rotating working body of an agricultural equipment is a complex motion characterized by simultaneous displacements of the particle along the x and z direction of the active body - see system (12).

Solving the system (12) related to general conditions leads to a solution with expressions (21,22*) or (23, 23*) with respect to the type of the solutions of the characteristic equation coupled with Eqn. (20).

The analysis of the solutions type reveal that the material particle movement is very sensible to the values of the coefficients in Eqn. (12), respectively (16) and to the roots type of the characteristic equation used for solving the system (20).

As a concluding remark, with respect to the equipment type in this study and to the category of material particles used by the equipment it is necessary a movement analysis, separately for each case, thus the coefficients can be exactly settled.

This general case, which was analyzed and presented in this paper, consists of a global frame about the complex problematic of the moving particle onto the active body surface with palette or cutter shape. This general model through a proper approach will derive the particular cases with the essential coefficients replaced by realistic values.

7. REFERENCES

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