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COMPARATIVE MODAL ANALYSIS OF A RIGID STRUCTURE WITH CONSERVATIVE INSULATION

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ABSTRACT

This paper deals with dynamics of compact rigid structures with elastic insulation and subjected to various vibratory, shock or seismic actions. It was supposed a simple rigid model with six degrees of freedom, affected by one or both vertical symmetry planes. Modal analysis was performed for six different rigidity cases. Concluding remarks dignify the correlation between isolation characteristics and natural pulsation in terms of eigenvalues and eigenvectors.

KEYWORDS: modal analysis, dynamic behaviour, rigid structure, eigenvalue, eigenvector

1. INTRODUCTION

The structural systems are made of material systems as solid bodies (rigid or deformable) with elastic or viscoelastic linkages subjected to external dynamic actions, having as consequences the generation of inertial effects.

Dynamic behavior of structural systems is described by mathematical equations that take different forms from a specific case to another. The representation in calculations of the solid bodies (rigid or deformable), of the linkages and of the whole system is based on the concept of dynamic system and dynamic model.

The dynamic system is an abstraction of the physical and mechanical characteristics of the structural system whose mechanical condition changes during time.

Any dynamical system is characterized by some specific qualitative properties (inertial, dissipative, elastic) represented by the values of measurable parameters (mass, moments of inertia, the damping coefficient, the rigidity/ flexibility coefficient).

The dynamic model is essentially an idealized form, simplified or schematized of a dynamic system in order to reduce the numerical analysis operations without that the real processes (qualitative and quantitative) being significantly modified.

The dynamic response is the instantaneous state of a dynamic system over which have been applied external dynamic actions, real and variable during time. The dynamic response can be expressed through fundamental kinematic parameters (displacement, velocity, accelera-tion) or through derived parameters (energy, sectional strains, stresses, deformations, generalized forces).

2. THEORETICAL APPROACHES

It is proposed the complex dynamic model of a rigide structure with six degrees of freedom which consists of three translational linear coordinates x, y, z and three rotational angular coordinates $\varphi_x, \varphi_y, \varphi_z$. For the pro-posed model, we will study the behavior of the structure under the vibration action and in the presence of elastic elements.

Thus, the motion equations of the rigid with elastic linkages are written:

$$\underline{\underline{A}}\ddot{\underline{q}} + \underline{\underline{C}}\underline{q} = \underline{0} \tag{1}$$

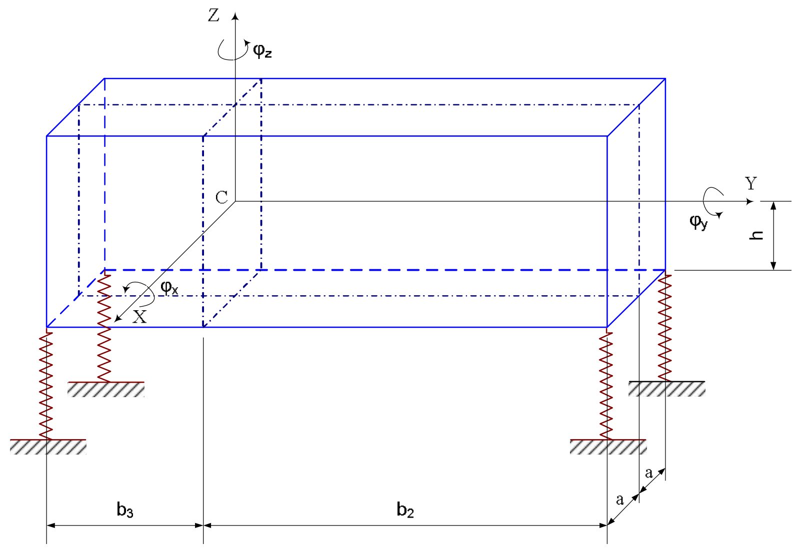

Fig.1. Model of the rigid system with six degrees of freedom, elastic supported in four points on inferior base, with a longitudinal vertical plane of symmetry

In an analytical form, the system is:

$$\begin{cases} m\ddot{x} + x\sum k_{ix} + \varphi_y \sum k_{ix} z_i - \varphi_z \sum k_{ix} y_i = 0 \\ m\ddot{y} + y\sum k_{iy} - \varphi_x \sum k_{iy} z_i + \varphi_z \sum k_{iy} x_i = 0 \\ m\ddot{z} + z\sum k_{iz} + \varphi_x \sum k_{iz} y_i - \varphi_y \sum k_{iz} x_i = 0 \\ J_x\ddot{\varphi}_x - y\sum k_{iy} z_i + z\sum k_{iz} y_i + \varphi_x \sum \left(k_{iy} z_i^2 + k_{iz} y_i^2\right) - \varphi_y \sum k_{iz} x_i y_i - \varphi_z \sum k_{iy} z_i x_i = 0 \\ J_y\ddot{\varphi}_y + x\sum k_{ix} z_i - z\sum k_{iz} x_i - \varphi_x \sum k_{iz} x_i y_i + \varphi_y \sum \left(k_{iz} x_i^2 + k_{ix} z_i^2\right) - \varphi_z \sum k_{ix} y_i z_i = 0 \\ J_z\ddot{\varphi}_z - x\sum k_{ix} y_i + y\sum k_{iy} x_i - \varphi_x \sum k_{iy} z_i x_i - \varphi_y \sum k_{ix} y_i z_i + \varphi_z \sum \left(k_{ix} y_i^2 + k_{iy} x_i^2\right) = 0 \end{cases} \quad (2)$$

The system (2) is difficult to solve analytically or using the matriceal formalism because it requires a large amount of calculation, and the sixth degree polynomial equation of the natural pulsations involves difficulties in solving and analysis. The solution is the automatic numerical calculus of the differential motion equations system of second degree, resulting a system with 12 differential equations of first degree, which can be integrated without difficulty. On the other hand, at the use of numerical analysis appears as a disadvantage the highlighting of the influence of the dynamic system physical characteristics. Thus, the analysis is done by repeated tests, using different sets of values for the input data. To this end, both for the elimination of coupling movements and to analytically solve the dynamic system model, certain sized and structure requirements may be imposed to the system, leading to a decoupling of the equation system into simple subsystems, easier to integrate.

As discussed above, we consider the case of the rigid structure, elastically supported in four points on inferior base, with a longitudinal vertical plane of symmetry yCz as in Figure 1. In this situation a few simplifying assumptions are valid:

- the dimensions of the analyzed rigid structure are symmetrical in relation to the considered plane
- the elastic linkages are identical, have symmetrical positions and are located in the same horizontal plane

Due to the mentioned symmetries, a part of coupling terms from stiffness matrix are canceled, and we have:

$$\begin{aligned} &\sum k_{iy} x_i = 0 \\ &\sum k_{iz} x_i = 0 \\ &\sum k_{iz} x_i y_i = 0 \\ &\sum k_{iy} z_i x_i = 0 \end{aligned} \quad (3)$$

Through the disappearance of the coupling terms, the system decouples into two subsystems described by coordinates (y, z, φ_x) and (x, φ_y, φ_z).

For the two decoupled subsystems can be written the equations of the free vibrations. Thus, we have for the subsystem (y, z, φ_x):

$$\begin{cases} m\ddot{y} + 4k_y y + 4hk_y \varphi_x = 0 \\ m\ddot{z} + 4k_z z + 2k_z (b_3 - b_2)\varphi_x = 0 \\ J_x \ddot{\varphi}_x + 4hk_y y + 2k_z (b_3 - b_2) z + 2\left[k_z \left(b_2^2 + b_3^2\right) + 2h^2 k_y\right]\varphi_x = 0 \end{cases} \quad (4)$$

and for the subsystem (x, φ_y, φ_z):

$$\begin{cases} m\ddot{x} + 4k_x x - 4hk_x \varphi_y - 2k_x (b_3 - b_2)\varphi_z = 0 \\ J_y \ddot{\varphi}_y - 4hk_x x + 4\left(h^2 k_x + a^2 k_z\right)\varphi_y + 2hk_x (b_3 - b_2)\varphi_z = 0 \\ J_z \ddot{\varphi}_z - 2k_x (b_3 - b_2) x + 2hk_x (b_3 - b_2)\varphi_y + 2\left[2a^2 k_y + k_x \left(b_2^2 + b_3^2\right)\right]\varphi_z = 0 \end{cases} \quad (5)$$

Further it is proposed an analyze of the vibrations of the two subsystems characterized each by three dynamic coordinates (degrees of freedom) coupled.

For each of the two subsystems, with elastic linkages and three degrees of freedom, the vector of the generalized coordinates is:

$$\underline{\underline{q}} = \left[q_1, q_2, q_3\right]^T \quad (6)$$

Using the classical mathematical apparatus, were written the quadratic forms of the system energies and then the case II Lagrange equations were used for obtaining motion equations, written matriceal in the form (1). The solution for the system (1) has been sought as:

$$\underline{\underline{q}} = \underline{\underline{a}} \sin pt = \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix} \sin pt \quad (7)$$

where $\underline{\underline{a}} = \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix}$ is the vector of the motion amplitudes.

Taking into account the proposed form of the system solution, equation (1) becomes:

$$\left(\underline{C} - p^2 \underline{A}\right)\underline{\underline{a}} = \underline{\underline{0}} \quad (8)$$

Equation (8) has nonzero solutions only if the determinant of the matrix is zero. This equation is third degree with p^2 as variable and represents the equation of the natural pulsations of the dynamic system with three degrees of freedom. Through analytically or numerically resolving the equation (8), we obtain the three natural pulsations of the system p_1, p_2, p_3.

3. CASE STUDY

The case study was made for two types of symmetry of the proposed structure, namely:

- considering the structure having a longitudinal vertical plane of symmetry, case for which as numerical values were proposed:

  a=7.5 m
  b_3=12 m
  b_2=8 m
  h=7 m
  $J_x=42 \times 10^6$ kgm^2
  $J_y=25 \times 10^6$ kgm^2
  $J_z=17.5 \times 10^6$ kgm^2

- considering the structure with two vertical planes of symmetry, one longitudinal and one transversal, case for which as numerical values were proposed:

  a=7.5 m
  b_3=10 m
  b_2=10 m
  h=7 m
  $J_x=35 \times 10^6$ kgm^2
  $J_y=25 \times 10^6$ kgm^2
  $J_z=17.5 \times 10^6$ kgm^2

For both sets of values, the mass of the analyzed structure was considered with the value m = 3×10^6 kg. Also, in both cases were proposed for study six sets of values of stiffness coefficients as follows in Table 1.

After completing the mathematical calculus, we obtain values for the parameters initially proposed - the eigenvalues and eigenvectors of the analyzed system. These values are summarized in Tables 2, 3, 4, 5.

Table 1 The proposed sets of values for the stiffness coefficients

	Var 1	Var 2	Var 3	Var 4	Var 5	Var 6
k_x [N/m]	2x10^6	4x10^6	8x10^6	16x10^6	32x10^6	64x10^6
k_y [N/m]	2x10^6	4x10^6	8x10^6	16x10^6	32x10^6	64x10^6
k_z [N/m]	8x10^6	16x10^6	32x10^6	64x10^6	128x10^6	256x10^6

Table 2 The system parameters assessment for the subsystem I in the case with one vertical plane of symmetry

Case with a longitudinal vertical plane of symmetry - subsystem I									
	eigen val p^2	nat puls p	freq f	eigen vect 1 μ_1	eigen vect 2 μ_2	eigen vect 3 μ_3	eigen vect 1 norm μ_{1n}	eigen vect 2 norm μ_{2n}	eigen vect 3 norm μ_{3n}
V1	89.3	9.4484	1.5038	0.2037	0.9990	0.0458	1.0000	1.0000	1.0000
	2.4	1.5376	0.2447	0.2564	0.0416	-0.9988	1.2592	0.0416	-21.8008
	10.3	3.2044	0.5100	0.9449	-0.0162	0.0187	4.6396	-0.0162	0.4072
V2	178.5	13.3621	2.1266	0.2037	0.9990	0.0458	1.0000	1.0000	1.0000
	4.7	2.1745	0.3461	0.2564	0.0416	-0.9988	1.2592	0.0416	-21.8008
	20.5	4.5317	0.7212	0.9449	-0.0162	0.0187	4.6396	-0.0162	0.4072
V3	357.1	18.8968	3.0075	0.2037	0.9990	0.0458	1.0000	1.0000	1.0000
	9.5	3.0752	0.4894	0.2564	0.0416	-0.9988	1.2592	0.0416	-21.8008
	41.1	6.4088	1.0200	0.9449	-0.0162	0.0187	4.6396	-0.0162	0.4072
V4	714.2	26.7241	4.2533	0.2037	0.9990	0.0458	1.0000	1.0000	1.0000
	18.9	4.3490	0.6922	0.2564	0.0416	-0.9988	1.2592	0.0416	-21.8008
	82.1	9.0634	1.4425	0.9449	-0.0162	0.0187	4.6396	-0.0162	0.4072
V5	1428.4	37.7936	6.0150	0.2037	0.9990	0.0458	1.0000	1.0000	1.0000
	37.8	6.1504	0.9789	0.2564	0.0416	-0.9988	1.2592	0.0416	-21.8008
	164.3	12.8176	2.0400	0.9449	-0.0162	0.0187	4.6396	-0.0162	0.4072
V6	2856.7	53.4483	8.5066	0.2037	0.9990	0.0458	1.0000	1.0000	1.0000
	75.7	8.6980	1.3843	0.2564	0.0416	-0.9988	1.2592	0.0416	-21.8008
	328.6	18.1268	2.8850	0.9449	-0.0162	0.0187	4.6396	-0.0162	0.4072

Table 3 The system parameters assessment for the subsystem II in the case with one vertical plane of symmetry

Case with a longitudinal vertical plane of symmetry - subsystem II									
	eigen val p^2	nat puls p	freq f	eigen vect 1 μ_1	eigen vect 2 μ_2	eigen vect 3 μ_3	eigen vect 1 norm μ_{1n}	eigen vect 2 norm μ_{2n}	eigen vect 3 norm μ_{3n}
V1	2.1	1.4601	0.2324	-0.9996	-0.2160	-0.0023	1.0000	1.0000	1.0000
	90.0	9.4861	1.5098	-0.0256	0.9079	-0.2669	0.0256	-4.2028	116.9657
	71.5	8.4550	1.3457	-0.0105	0.3592	0.9637	0.0105	-1.6626	-422.2837
V2	4.3	2.0649	0.3286	-0.9996	-0.2160	-0.0023	1.0000	1.0000	1.0000
	180.0	13.4153	2.1351	-0.0256	0.9079	-0.2669	0.0256	-4.2028	116.9657
	143.0	11.9571	1.9030	-0.0105	0.3592	0.9637	0.0105	-1.6626	-422.2837
V3	8.5	2.9202	0.4648	-0.9996	-0.2160	-0.0023	1.0000	1.0000	1.0000
	359.9	18.9721	3.0195	-0.0256	0.9079	-0.2669	0.0256	-4.2028	116.9657
	285.9	16.9100	2.6913	-0.0105	0.3592	0.9637	0.0105	-1.6626	-422.2837
V4	17.1	4.1298	0.6573	-0.9996	-0.2160	-0.0023	1.0000	1.0000	1.0000
	719.9	26.8306	4.2702	-0.0256	0.9079	-0.2669	0.0256	-4.2028	116.9657
	571.9	23.9143	3.8061	-0.0105	0.3592	0.9637	0.0105	-1.6626	-422.2837
V5	34.1	5.8405	0.9295	-0.9996	-0.2160	-0.0023	1.0000	1.0000	1.0000
	1439.8	37.9442	6.0390	-0.0256	0.9079	-0.2669	0.0256	-4.2028	116.9657
	1143.8	33.8199	5.3826	-0.0105	0.3592	0.9637	0.0105	-1.6626	-422.2837
V6	68.2	8.2597	1.3146	-0.9996	-0.2160	-0.0023	1.0000	1.0000	1.0000
	2879.5	53.6612	8.5405	-0.0256	0.9079	-0.2669	0.0256	-4.2028	116.9657
	2287.6	47.8286	7.6122	-0.0105	0.3592	0.9637	0.0105	-1.6626	-422.2837

Table 4 The system parameters assessment for the subsystem I in the case with two vertical planes of symmetry

Case with two vertical planes of symmetry (longitudinal and vertical) - subsystem I						
	eigenval p^2	nat puls p	freq f	eigenvect 1 μ_1	eigenvect 2 μ_2	eigenvect 3 μ_3
V1	2.4	1.5391	0.2450	-0.9999	-0.1830	0
	102.9	10.1453	1.6147	0	0	1.0000
	10.7	3.2660	0.5198	0.0160	-0.9831	0
V2	4.7	2.1766	0.3464	-0.9999	-0.1830	0
	205.9	14.3476	2.2835	0	0	1.0000
	21.3	4.6188	0.7351	0.0160	-0.9831	0
V3	9.5	3.0782	0.4899	-0.9999	-0.1830	0
	411.7	20.2905	3.2293	0	0	1.0000
	42.7	6.5320	1.0396	0.0160	-0.9831	0
V4	19.0	4.3532	0.6928	-0.9999	-0.1830	0
	823.4	28.6952	4.5670	0	0	1.0000
	85.3	9.2376	1.4702	0.0160	-0.9831	0
V5	37.9	6.1563	0.9798	-0.9999	-0.1830	0
	1646.8	40.5811	6.4587	0	0	1.0000
	170.7	13.0639	2.0792	0.0160	-0.9831	0
V6	75.8	8.7064	1.3857	-0.9999	-0.1830	0
	3293.6	57.3903	9.1340	0	0	1.0000
	341.3	18.4752	2.9404	0.0160	-0.9831	0

Table 5 The system parameters assessment for the subsystem II in the case with two vertical planes of symmetry

Case with two vertical planes of symmetry (longitudinal and vertical) - subsystem II						
	eigenval p^2	nat puls p	freq f	eigenvect 1 μ_1	eigenvect 2 μ_2	eigenvect 3 μ_3
V1	2.2	1.4757	0.2349	-0.9997	0.2133	0
	88.2	9.3898	1.4944	-0.0262	-0.9770	0
	71.4	8.4515	1.3451	0	0	1.0000
V2	4.4	2.0869	0.3321	-0.9997	0.2133	0
	176.3	13.2792	2.1135	-0.0262	-0.9770	0
	142.9	11.9523	1.9023	0	0	1.0000
V3	8.7	2.9514	0.4697	-0.9997	0.2133	0
	352.7	18.7797	2.9889	-0.0262	-0.9770	0
	285.7	16.9031	2.6902	0	0	1.0000
V4	17.4	4.1739	0.6643	-0.9997	0.2133	0
	705.4	26.5585	4.2269	-0.0262	-0.9770	0
	571.4	23.9046	3.8045	0	0	1.0000
V5	34.8	5.9027	0.9394	-0.9997	0.2133	0
	1410.7	37.5593	5.9778	-0.0262	-0.9770	0
	1142.9	33.8062	5.3804	0	0	1.0000
V6	69.7	8.3477	1.3286	-0.9997	0.2133	0
	2821.4	53.1169	8.4538	-0.0262	-0.9770	0
	2285.7	47.8091	7.6091	0	0	1.0000

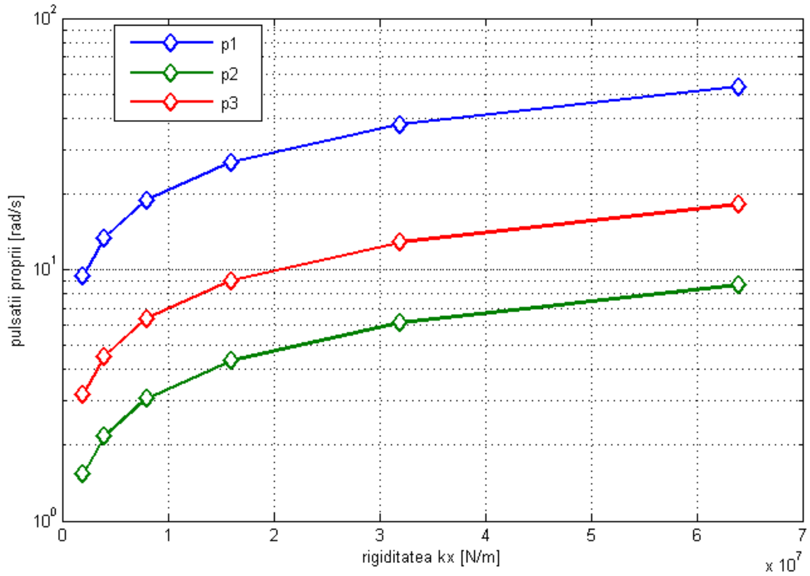

Fig. 2. The dependence between the natural pulsations of the system and the rigidity in the horizontal direction

Based on the values obtained for the three natural pulsations of the system in the six cases proposed, it was realized a graphic (Figure 2) of the dependence between the natural pulsations and the rigidity in the horizontal direction, denoted by k_x.

4. CONCLUSIONS

It should be mentioned that the stiffness in the x direction is equal to that in the y direction and stiffness in the z direction is a linear combi-nation of the two others. Therefore we obtained for the eigenvectors identical values, regardless of the values of the stiffness coefficient consi-dered in calculus.

As an independent variable for the representation of the pulsation evolution was chosen the stiffness in the x direction, denoted by k_x.

The evolution of the pulsations corresponding to the eigenvalues follows the natural tendency imposed by the pairs of values considered for rigidities. The correlative analysis of each set of eigenvalues induces the following conclusion, namely that linear combinations between the stiffness in the horizontal plane and the one in the vertical plane require similar evolutions.

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