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NON-LINEAR DYNAMIC BEHAVIOR OF A BALL FLOAT VALVE

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ABSTRACT

The dynamic behavior of the spherical body floating in a liquid, part of a ball float valve, is analyzed from the point of view of the dynamic systems theory. The study of the motion illustrates the differences between the linear and the non-linear approach. The time analysis of a significant variable and the phase plane representation were used. Based on the obtained solution, the paper deals with the possibility of onset of deterministic chaotic motions for special initial conditions and model assumptions is discussed.

KEYWORDS: chaotic motion, floating body, non-liner analysis, vibrations

1. DESCRIPTION OF THE DYNAMIC SYSTEM ANALYZED

The paper analyzes the dynamics of a ball float valve (Fig. 1). The valve is controlled by a spherical body floating in a liquid, which can close or open the hole of a tube, according to the free surface level. However, the considerations apply also to other devices with floating bodies, such as: sea planes, water level indicators (which produce warning whistles when reaching the hydrostatic level in boreholes or wells), the main parts of buoys, floating body attached to fishing nets, various devices used to feed pets, etc.

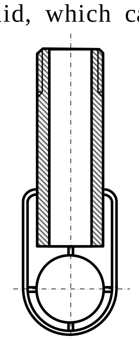


Fig. 1. Ball float valve

2. MODEL

The studied system is represented as a homogeneous spherical body (of radius R and density ρ), that is floating in a liquid (of density ρ_L), such that the equilibrium position is reached when the center of the sphere is at the liquid free level (Fig. 2). If a perturbation is applied in vertical direction, the immersed portion of the sphere is measured by variable $y(t)$. The perturbation acting on the body can be harmonic.

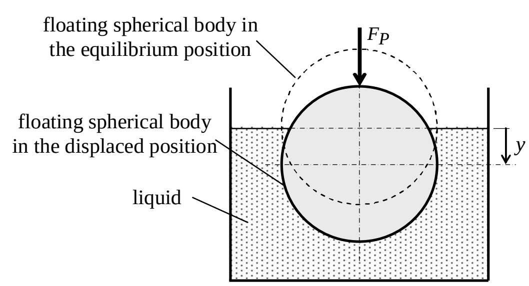


Fig. 2. Mechanical model

By applying the theorem of variation of the linear momentum [2], the equation of motion in vertical direction is obtained,

$$m\ddot{y} + \Delta m_L(y) g = F_P, \tag{1}$$

where the following notations have been used:

$m = \frac{4}{3}\pi\rho R^3$ - mass of the sphere;

$m_L(0) = \frac{2}{3}\pi\rho_L R^3$ - liquid mass dislocated in the equilibrium position;

$m_L(y) = \frac{1}{3}\pi\rho_L\left[\left(3R^2 - y^2\right)y + 2R^2\right]$ - total liquid mass dislocated in the displaced position;

$\Delta m_L(y) = \frac{1}{3}\pi\rho_L\left(3R^2 - y^2\right)y$ - supplementary liquid mass dislocated in the displaced position;

$F_P = F_0 \cos \Omega t$ is the perturbation force.

The equilibrium condition

$$mg = m_L(0)g \tag{2}$$

leads to:

$$\rho_L = 2\rho . \tag{3}$$

Therefore, the differential equation of motion can be rewritten as:

$$\ddot{y} + \frac{g}{2R}\left[3 - \left(\frac{y}{R}\right)^2\right]y = F_0 \cos \Omega t . \tag{4}$$

This equation will be further analyzed, both in linearized and non-linearized form.

2.1. Linear analysis of the motion

Retaining only the linear terms in (4), the equation of a forced undamped vibration is obtained [2], [4],

$$\ddot{y} + \alpha y = \delta \cos \Omega t , \tag{5}$$

where $\alpha = \frac{3g}{2R}$, $\delta = \frac{F_0}{m}$.

Diagrams in Figure 3 illustrate the characteristics of such a motion.

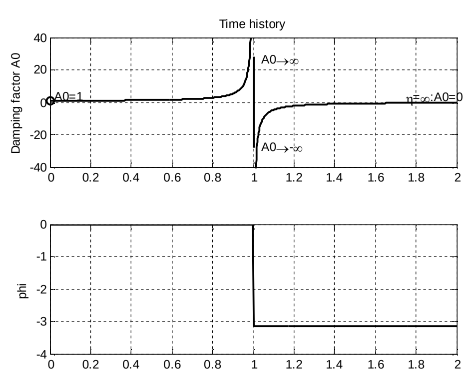
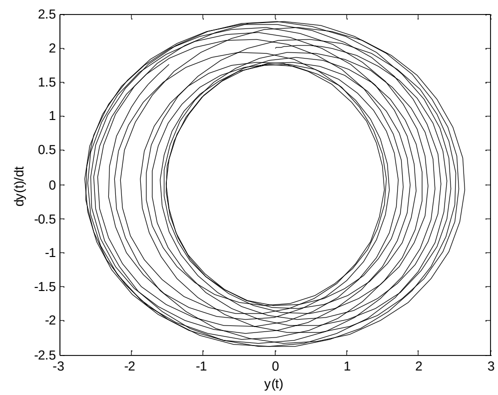
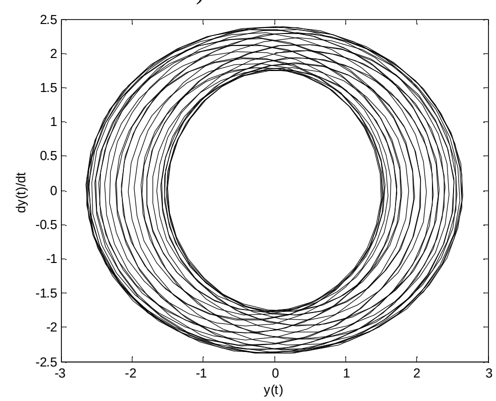


Fig. 3. Characteristics of the linear motion

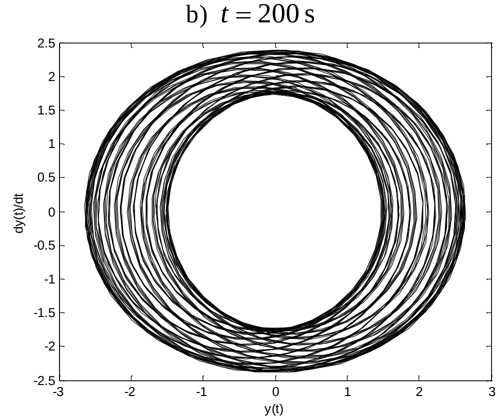
In the phase plane (Fig. 4), each moment of the motion is represented by a point of coordinates $y(t)$ and $\dot{y}(t)$, which provides more information regarding the system state than the time-history analysis of the variable $y(t)$, since the velocity is also indicated. Using the same system parameters ($\alpha = 1\,\mathrm{s}^{-2}$, $\delta = 0.4\,\mathrm{m/s}^2$, $\Omega = 0.55\,\mathrm{s}^{-1}$), but a different integration time ($t = 100\,\mathrm{s}$, $t = 200\,\mathrm{s}$ and $t = 500\,\mathrm{s}$, respectively), the closed curves presented in the figure, specific to periodic motions, have been obtained [1].



a) $t = 100\,\mathrm{s}$



b) $t = 200\,\mathrm{s}$



c) $t = 500\,\mathrm{s}$

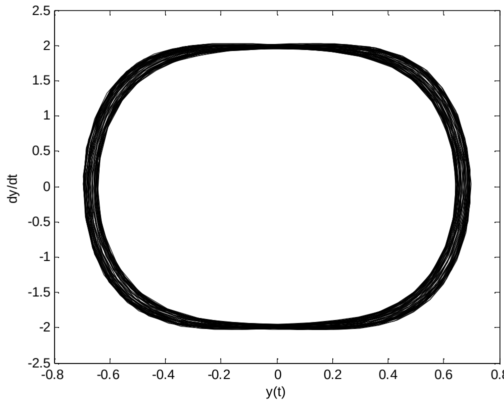
Fig. 4. Linear analysis

2.2. Non-linear analysis of the motion

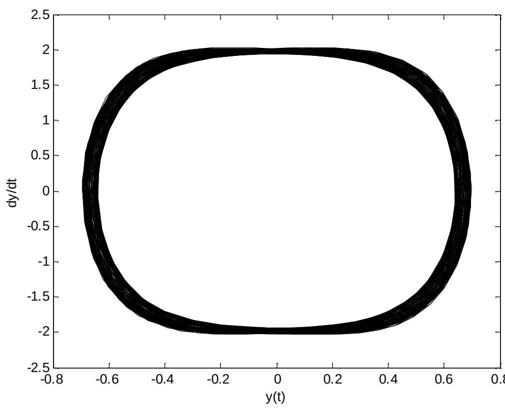
The non-linear differential equation (4), rewritten as,

$$\ddot{y} + \frac{3g}{2R}y - \frac{g}{2R^3}y^3 = F_0 \cos \Omega t \,, \qquad (6)$$

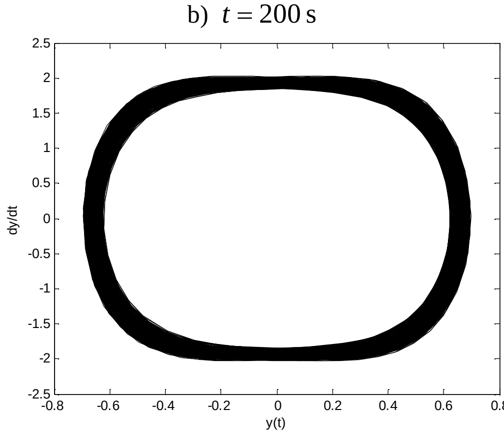
describes a non-linear undamped forced vibration.



a) $t = 100\,\text{s}$



b) $t = 200\,\text{s}$



c) $t = 500\,\text{s}$

Fig. 5. Undamped system - non-linear analysis

The second-order differential equation (6) can be transformed into an equivalent system of first-order differential equations,

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = -\alpha\, y_1 - \gamma\, y_1^3 + \delta \cos \theta \\ \dot{\theta} = \Omega, \end{cases} \qquad (7)$$

where $\alpha = \dfrac{3g}{2R}$, $\gamma = -\dfrac{g}{2R^3}$, $\delta = \dfrac{F_0}{m}$.

The differential equation system (7) has been integrated numerically. The phase plane analysis is considered more suitable to illustrate the dynamic behavior of the modeled system. The same values were chosen for the coefficients of the linear terms ($\alpha = 1\text{s}^{-2}$, $\delta = 0.4\text{m/s}^2$, $\Omega = 0.55\text{s}^{-1}$) and for the integration times, while for the non-liner term, value $\gamma = 34.53\text{m}^{-2}\text{s}^{-2}$ has been used. Diagrams in Figure 5 have been obtained.

For the chosen numerical values, the presence of a regular motion can be observed, even if the non-linear characteristic have been taken into account. The shape of the trajectory in the phase plane is close to an ellipse and it approaches a limit cycle.

2.3. Non-linear analysis of the damped motion

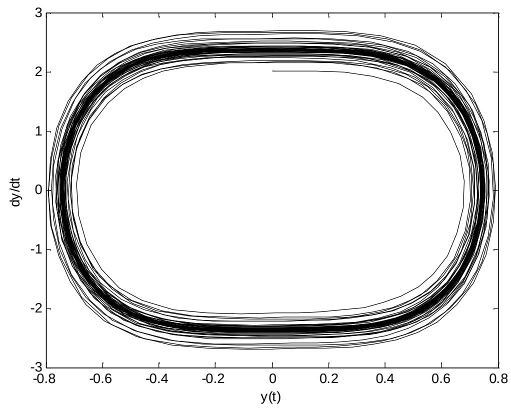
If viscous damping effects are taken into consideration, the motion of the body immersed in liquid is described by the differential equation system [3]

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = -\xi\, y_2 - \alpha\, y_1 - \gamma\, y_1^3 + \delta \cos \theta \\ \dot{\theta} = \Omega, \end{cases} \qquad (8)$$

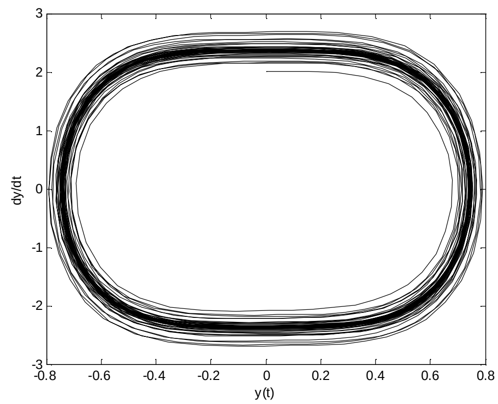
where ξ is the viscous damping coefficient, while all other coefficients have the same meaning as in the previously studied cases.

For consistency, the phase plane analysis is used again and the system of differential equations has been integrated with the same coefficients and using the same computer program, by choosing the same integration times as before. The value $\xi = 0.1\text{s}^{-2}$ has been chosen for the damping coefficient. Diagrams in Figure 6 have been obtained.

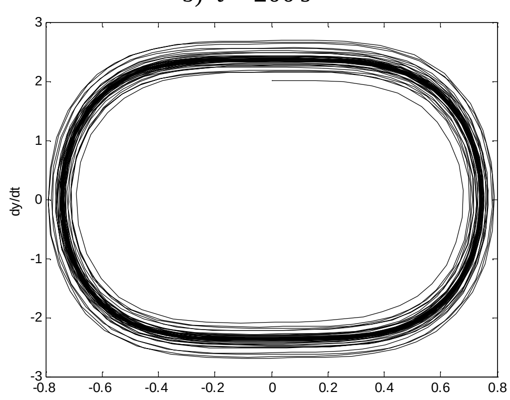
The obtained numerical results indicate that the motion is close to a harmonic oscillation, since the image in the phase plane is a closed curve, similar in shape to an ellipse. Increased integration time has led to a solution of the same type, in the sense that the result is still a closed curve, even if relatively more complicated, and the width of the curve is no longer infinitely small but it occupies a finite region in the phase plane. Such observations have led to the conclusion that the dynamic system is in the first two phases of evolution towards chaos (the beginning phase and the strong beginning phase) [3], that has prompted to increase the integration time just as in the previous two cases studied.



a) $t = 100\,\mathrm{s}$



b) $t = 200\,\mathrm{s}$



c) $t = 500\,\mathrm{s}$

Fig. 6. Damped system - non-linear analysis

Diagrams such as the one in Figure 7 have confirmed the observations regarding the stability of the motion, as a strong diffusion of trajectories in the phase plane with a relatively complex pattern can be seen. In the same plot, the attractor pools and the character of strange attractor (typical to a set of attractors with fractal character located in a finite domain) can be noticed. The dynamic system has reached the mature chaos phase of its evolution [3].

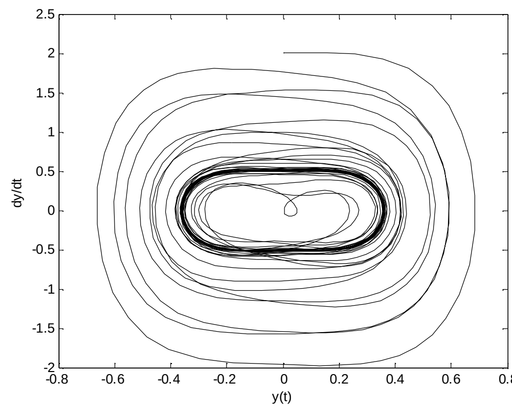


Fig. 7. Damped system - non-linear analysis for $\Omega = 1.74\mathrm{s}^{-1}$, $t = 2000\,\mathrm{s}$

Fig

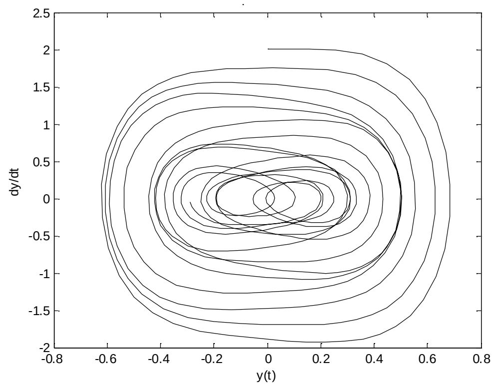
By changing the eigenfrequency Ω of the perturbation force in equation system (8) and by changing the integration time, the results in Figure 8 have been obtained.

The system dynamics study is extended by varying the eigenfrequency of the perturbation in system (8), in the range $\Omega = 0.15\,\mathrm{s}^{-1}$, $\Omega = 0.2\,\mathrm{s}^{-1}$, $\Omega = 0.3\mathrm{s}^{-1}$, $\Omega = 0.4\mathrm{s}^{-1}$, with a constant integration time ($t = 75\,\mathrm{s}$). The corresponding phase plane diagrams are shown in Figure 9.

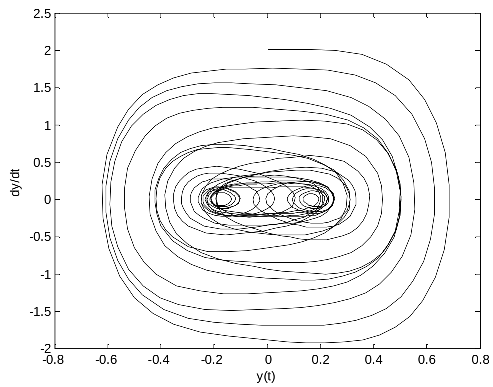
3. CONCLUSIONS

The paper studies the behavior of a dynamic system whose equation of motion is described by a nonlinear differential equation. The dynamics of the system is analyzed for the case of the linearized equation of motion and for the non-linear case, with and without viscous damping effects [1].

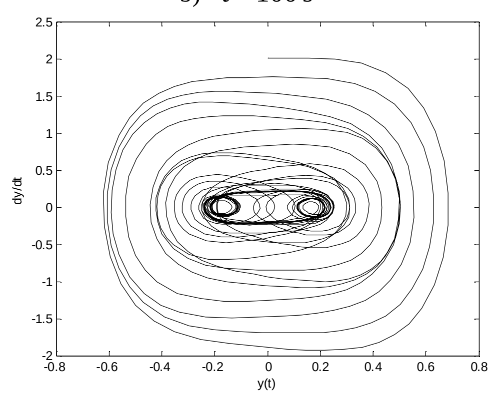
The study is developed for identical initial conditions and the effects of changing the integration time and the eigenfrequency of the perturbation force are analyzed, respectively.



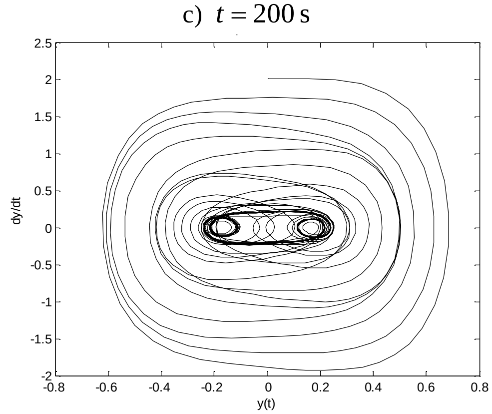
a) $t = 50\,\mathrm{s}$



b) $t = 100\,\mathrm{s}$

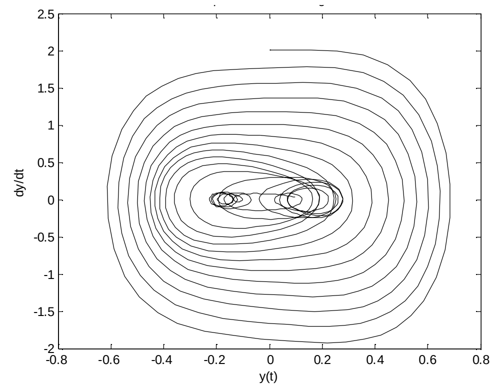


c) $t = 200\,\mathrm{s}$

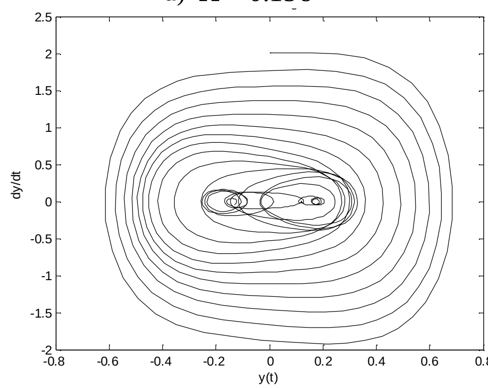


d) $t = 2000\,\mathrm{s}$

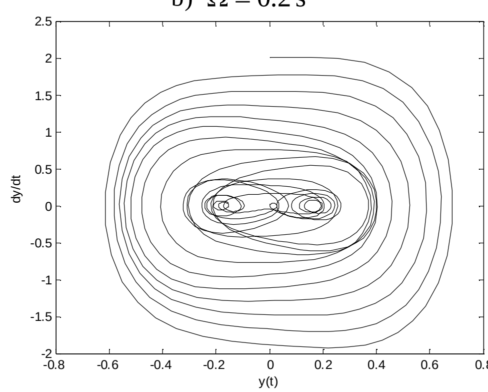
Fig. 8. Non-linear analysis - influence of the integration time



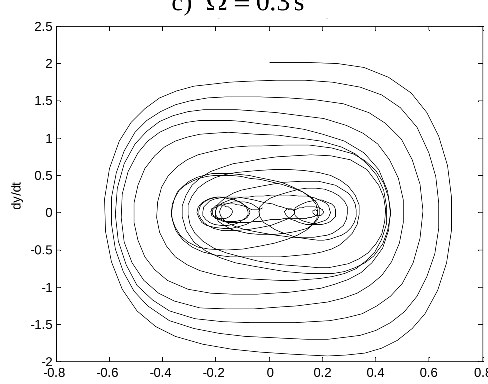
a) $\Omega = 0.15\,\mathrm{s}^{-1}$



b) $\Omega = 0.2\,\mathrm{s}^{-1}$



c) $\Omega = 0.3\,\mathrm{s}^{-1}$



d) $\Omega = 0.4\,\mathrm{s}^{-1}$

Fig. 9. Non-linear analysis - influence of Ω

Results of numerical integration in each case are illustrating the evolution of the system. Under certain conditions and for certain values of the coefficients in the differential equation of motion, various phases of deterministic chaos can be identified, in accordance with published definitions. Of the same importance is the observation that, from a qualitative point of view, deterministic chaotic motions occur when certain essential characteristic conditions are met: non-repeating behavior, great sensitivity to initial conditions, non-linear system of differential equation describing the motion and containing at least three independent variables, attractor of the motion with characteristics of a fractal.

It can be concluded that the linearized analysis of a dynamic system is relevant only as a first order approximation, while a more refined investigation requires a non-linear approach, since, even such simple systems may exhibit unpredictable evolutions for certain values of the definition parameters and for certain initial conditions.

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