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ON THE CONTROL OF THE ELEMENT FINITE MESHES WITH DIMENSIONS IN ARITHMETIC PROGRESSION

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ABSTRACT

The purpose of this article is to present the formulae for calculating the parameters defining the finite element meshes with variable dimensions in arithmetical progression along some user defined paths within the domain of analysis. Also, it is presented an analysis of the conditions of existence of the formulae previously deducted.
In the final part of the article, the deducted formulae are used in a case study for their validation and some conclusions are formulated.

KEYWORDS: finite element meshes, user defined paths

1. PRELIMINARY

It is known that one of the essential conditions for obtaining accurate results in finite element analysis is to generate an optimal mesh for the analysis domain. Therefore, the current finite element analysis environments provide many tools to control mesh generation. Generally, the internal mesh generators of the current finite element environments for mesh generation provide an automatic generation of mesh with finite elements having dimensions adapted to the geometry of the analysis domain.
However, automatic mesh generation can not ensure the existence of the nodes and / or of the finite elements along a certain path (straight or curved) in the analyzed domain. It is important to emphasize that the imposition of such conditions is dictated by the need to study the variation of the results values along certain paths in the analyzed domain. Also, in many cases, it is necessary a variable density of the finite elements (default of the nodes) along the imposed paths.

In this context, the algorithms used in automatic generation of meshes, no matter how proficient they are, can not replace the choosing and the discernment on the goals pursued by the analysis, of an eminently human process. Therefore, is reached the semi-automatic generation of the mesh, when the user must impose the size of the finite elements and how these dimensions are variable, along certain paths imposed in the analysis domain.

If we refer to the laws of variation of the finite element dimensions along specified paths in the analysis domain, most finite element analysis environments use two laws: variation in arithmetic progression and variation in geometric progression. When using any of the two laws mentioned above, user is prompted to enter the following information:
- information required for locating the path or the sub-domain subjected to the semiautomatic meshing,
- dimension of the "middle” finite element or the number of finite elements to be generated with nodes on the imposed path,
- the ratio between the maximum and minimum size of the finite elements which have nodes on the imposed path,
- information required for locating the agglomeration of the finite elements. In this context for straight or curved paths there are four possibilities for location of the agglomeration: at the beginning, at the end, at the beginning and the end, or in the middle of path.

Based on the foregoing aspects, it is noted that although the information inputted is necessary and sufficient for finite element environment to generate mesh, the user still does not have a complete view of this process. In this context, for a total control of the mesh, it is necessary to know the dimensions of all finite elements to be generated, the obtained level of agglomeration, and the necessary and sufficient conditions for the existence of the formulas that govern this process.

This paper provides the necessary results for completing the image on the semi-automatic generating process of the mesh. Also, are deducted the necessary formulae in the control of the parameters of the mesh, using the following hypotheses: the dimensions of finite elements with nodes on the imposed path are in arithmetic progression, the finite element agglomeration is located at the beginning of the curve representing the imposed path. For other cases (variation of the dimensions in geometrical progression and other locations for the finite elements agglomeration), the calculus formulae can be deducted based on a similar algorithm.

2. CALCULUS FORMULAE. ANALYSIS OF THE CONDITIONS OF EXISTENCE

After the indication of necessary inputs for the semiautomatic meshing, finite element analysis environment generates a mesh so that on the indicated path, distances between successive nodes, form an arithmetic progression.

The following notations will be used:

n - number of the finite elements on the specified path, to be generated in the meshing process, $n \in N, n \geq 2$

b - bias factor (ratio of maximum size and minimum size of the finite elements with nodes along the specified path), $b \in R, b > 1$

a - length of the curve representing the path along which will be generated nodes at distances in arithmetic progression

r - constant difference of the consecutive terms in arithmetic progression

a_k - length of any segment, located on the specified path, $k \in N, 1 \leq k \leq n$

Furthermore, are considered known a, b, n and the location of the finite element agglomeration (assumed at the beginning of the curve representing the path specified). With these inputs, we will determine the formula of the constant difference, r, and formula of the length a_k. Since the a_k lengths are terms of an arithmetic progression, there are the following relations:

$$a_k = a_1 + (k-1)r \tag{1}$$

and

$$a = \frac{n(a_1 + a_n)}{2} \tag{2}$$

Given the bias factor definition, we have:

$$a_n = b a_1 \tag{3}$$

If it is eliminated a_n between relations (2) and (3), we obtain:

$$a_1 = \frac{2a}{n(b+1)} \tag{4}$$

To determine the constant difference r, a_1 and a_n are eliminated between relations (1) for case k = n, (3) and (4). After calculations, we obtain:

$$r = \frac{2a}{n(n-1)} \cdot \frac{b-1}{b+1} \tag{5}$$

Taking into account the expressions of a_1 and r, given by relations (4) and (5), based on relation (1), we can determine the calculus formula for the length of any segment a_k, depending on known quantities:

$$a_k = \frac{2a}{n(b+1)} \cdot \left[1 + (k-1) \cdot \frac{b-1}{n-1}\right] \tag{6}$$

Remarks

- The condition imposed to the bias factor ($b \in R, b > 1$) is a necessary and sufficient condition for ensuring the variable length of the a_k segments. Indeed, if the bias factor tends to become equal to unity, by reaching the limit in (6), we obtain:

$$a_k = \lim_{b \to 1} \left\{ \frac{2a}{n(b+1)} \cdot \left[1 + (k-1) \cdot \frac{b-1}{n-1}\right] \right\} = \frac{a}{n} \tag{7}$$

The result obtained in (7) shows that all segments have the same length, in other words, in the meshing process are generated finite elements with nodes placed equidistantly on the imposed path. The same conclusion can be formulated when in the formula of the constant difference r - formula (5) - is reaching the limit:

$$r_{lim} = \lim_{b \to 1} \left[\frac{2a}{n(n-1)} \cdot \frac{b-1}{b+1} \right] = 0 \tag{8}$$

which means that the arithmetic progression degenerates into a constant series of real numbers.

- The level of the finite elements agglomeration at the beginning of the imposed path is given by the value of the bias factor and the number of finite elements with nodes located on the path. To characterize this agglomeration can be calculated the parameter $c < a$, representing the fraction of the total length of the path which is "covered" by the first m segments ($a_1, ..., a_m$). For this purpose, we use equation (2), properly transformed, where a_1 is calculated using the formula (4), and a_m is given by (6), where k index is replaced with m index. After performing the calculation, the following formula is obtained:

$$c = \frac{m}{n} \cdot \frac{a}{b+1} \cdot \left[2 + (b-1) \cdot \frac{m-1}{n-1} \right] \quad (9)$$

or:

$$\frac{c}{a} = \frac{m}{n} \cdot \frac{1}{b+1} \cdot \left[2 + (b-1) \cdot \frac{m-1}{n-1} \right] \quad (9a)$$

Based on the above, the high agglomerations are characterized by the small fractions c and by the large numbers m.

Assuming that it is desired a certain level of agglomeration of finite elements, it is necessary to impose fraction $c < a$ and the number of segments, $m < n$.

We propose, in what follows, to determine the bias factor and the existence conditions for a desired agglomeration of the finite elements on the imposed path. In this context, we emphasize the importance of the above mentioned conditions, because the simple requirement concerning the c length ($c < a$) and of the number m of segments ($m < n$) may be insufficient to achieve the desired level of the agglomeration.

The calculus formula of the required bias factor is based on the relation (9a) where, for a simpler form of the result, we use the following notations:

$$A = \frac{c}{a}, \; B = \frac{m}{n} \quad (10)$$

After performing the calculation, the following formula is obtained:

$$b = \frac{2nB(1-B)}{n(A-B^2) + B - A} - 1 \quad (11)$$

The first condition which is necessary to ensure the imposed level of agglomeration is:

$$c/a < m/n \Leftrightarrow A < B \quad (12)$$

Indeed, assuming that $c/a < m/n$ and taking into account (9a), it follows:

$$\frac{1}{b+1} \cdot \left[2 + (b-1) \cdot \frac{m-1}{n-1} \right] \geq 1, \quad (13)$$

from where, after some elementary calculations, is reached the absurd inequality $m \geq n$.

The second condition of existence is obtained from the imposition of a plausible bias factor ($b > 1$). Based on the relation (11), after some simple calculations, the following inequality is deducted:

$$\frac{B-A}{n(A-B^2) + B - A} > 0 \quad (14)$$

from which, taking into account (12), we obtain the existence condition required:

$$n(A-B^2) + B - A > 0 \quad (15)$$

or:

$$\frac{c}{a} > \frac{m(m-1)}{n(n-1)} \quad (15a)$$

To summarize, for an imposed level for the agglomeration of the finite elements, it is necessary and sufficient that the parameters a, c, m, n meet the following inequalities:

$$\frac{m(m-1)}{n(n-1)} < \frac{c}{a} < \frac{m}{n} \quad (16)$$

or:

$$a \cdot \frac{m(m-1)}{n(n-1)} < c < a \cdot \frac{m}{n} \quad (16a)$$

Practically, in order to achieve a desired level of agglomeration of the finite elements on the specified path, first it is imposed the m number, then, based on the double inequality (16) are determined the limits of the c fraction, lower values being associated with high agglomerations.

3. CASE STUDY

In order to illustrate the use of the results presented above, it is considered the case where is required the mesh generation for the plate from Fig. 1a. For given plate it is imposed to determine the equivalent stresses (based on the adoption of a strength theory) around the hole, on the following directions: the directions of the coordinate axes and the directions which are inclined at 45^0, 135^0, 225^0 and 315^0 towards OX axis.

For reasons of symmetry, for the study of the plate behaviour, it is sufficient to model with finite elements only a half of the plate, determined by the OX axis. Obviously, for nodes located on the OX axis the necessary conditions must be imposed for them to maintain on the longitudinal axis of symmetry.

The mesh which is associated with this model is obtained by reflecting relatively to OY axis of the mesh associated with plate subdomain which is located in the first quadrant (Fig. 1b).

For generating the mesh which is associated to the sub-domain in Fig. 1b, there must be considered the imposed directions for the study of the equivalent stresses variation.

To achieve the above listed objectives, for the sub-domain in Fig. 1b were generated two boundary surfaces, as follows:

- the boundary surface which is bounded by the curves 1, 3, 6, 4;
- the boundary surface which is bounded by the curves 2, 4, 7, 8, 5.

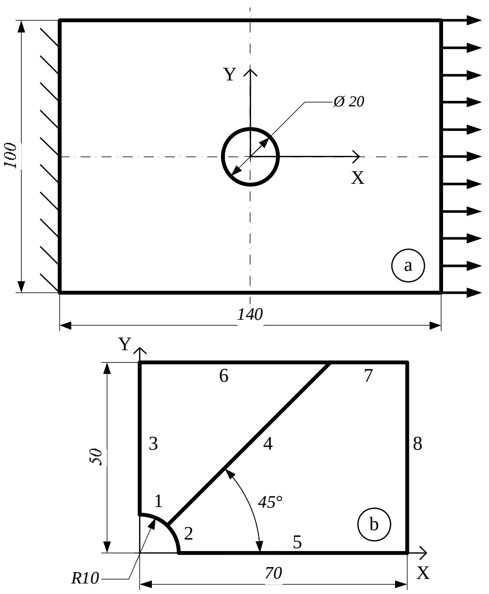

Fig. 1. Plate subjected to finite element analysis
a. Entire plate b. Sub-domain of the plate considered in modelling

Note that because it is imposed the variation study of the equivalent stresses along the oblique directions, is necessary to generate two boundary surfaces, which are adjacent along the curve 4.

The mesh which is associated to the sub-domain in Fig. 1b was obtained with imposing the following conditions:

- on arcs 1 and 2 were placed equidistant nodes. The number of nodes on each arc was chosen so that the arc be approximated by a polygonal line with six line segments. It follows that the distance between two consecutive nodes, located on these arcs, is:

$$a_k \cong 1.308\,mm, (\forall) k \in \{1,2,...,6\}$$

- on curves 6, 7 and 8 were placed equidistant nodes, the distance between two consecutive nodes being set to the value:

$$a_k = 2.5\,mm, (\forall) k \in \{1,2,...,n_s\}; s \in \{6,7,8\}$$

where n_s is the number of segments on s curve.

- on curves 3, 4, 5 nodes were placed so that the distance between consecutive nodes meets an arithmetic progression. To ensure that appearance and distortion of the finite elements is between the acceptable limits were considered, for each of these curves, the extreme segments with equal length with length of the segments which are on the curves located in proximate neighbourhood. It follows, for each curve the corresponding bias factor, where after, based on the formula (4), is obtained the number of the finite elements. Under these conditions, for curve 3 we have:

$a_1 = 1.308\,mm$, $a_n = 2.5\,mm$, $b \cong 1.91$, $n = 21$,

and for curves 4 and 5:

$a_1 = 1.308\,mm$, $b = 2.5 / 1.308 \cong 1.91$, $n = 32$.

After meshing, it was obtained a mesh, which is presented in Fig. 2.

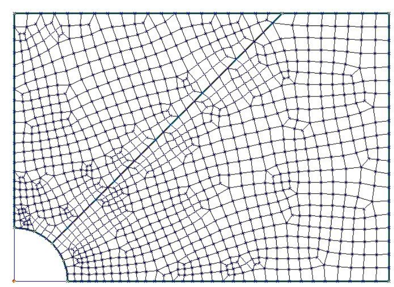

Fig. 2. The mesh associated to the sub-domain of the plate considered in modelling

4. CONCLUSIONS

Applying the above calculus formulae for controlling finite element meshes allows for the consideration of all aspects related to optimal meshes: perturbations, symmetries, creating paths for "collection" of results, etc.

Moreover, this procedure allows for the generation of meshes with controllable density, both that the number of finite elements, as well the finite element agglomeration, related to their location. This creates the premises for obtaining numerical models of computation with reasonable size while ensuring accurate results.

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