THE ANNALS OF "DUNAREA DE JOS" UNIVERSITY OF GALATI
FASCICLE XIV MECHANICHAL ENGINEERING, ISSN 1224-5615
2011

MODAL DAMPING RATIO CALCULATION FOR FINITED OF MECHANICAL SYSTEMS WITH VISCOUS ELASTIC BEARINGS USING VIRTUAL INSTRUMENTATION

Assoc. Prof. Dr. Eng. Nicusor DRAGAN
MECMET - The Research Center of Machines, Mechanic and Technological Equipments
"Dunarea de Jos" University of Galati

ABSTRACT

This paper proposes an approach of a finite DOF dynamic model for the rigid-solid with multiple neoprene apparatus bearings modeled as viscous elastic dampers. After a short presentation of the physical model of the solid rigid with multiple bearings, the correspondent linear mathematical model in generalized coordinates is determined. The decoupled differential equations of motion are obtained by using linear modal transformation $\boldsymbol{q}=\underline{\boldsymbol{V}}^{-1}\boldsymbol{\eta}$ *considering that all the dissipations are linear (structural damping). The article presents the virtual instrument developed on the basis of LabView® ver. 8.5 from National Instruments, which was used for the modal analysis in site of a Romanian highway bridge and for the identification of modal damping ratio of the neoprene bearing system.*

KEYWORDS: modal coordinates, modal damping, virtual instrumentation, FFT analysis, eigenvalues, natural frequencies

1. INTRODUCTION

In order to create the mathematical model of the finite DOF mechanical system, we use the physical model of the rigid solid with six degrees of freedom (6DOF) with a finite number of viscous-elastic bearings [1], [2], [3], [4]. Figure 1 presents the model of the rigid solid with *n* triorthogonal elastic bearings and *m* triorthogonal viscous bearings.

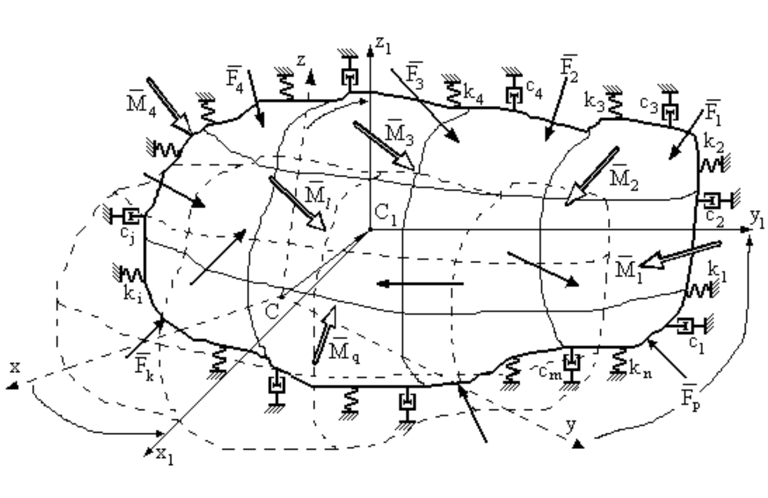


Figure 1 Physical model of the 6DOF solid rigid with multiple elastic and viscous bearings

According to [5], [6], [7], [8] and [9], the differential equations of the free movements of the rigid solid with viscous-elastic bearings are coupled by stiffness and damping coefficients. The system of the equations can be written as follows:

$$\underline{\underline{A}}\ddot{\underline{\underline{q}}} + \underline{\underline{B}}\dot{\underline{\underline{q}}} + \underline{\underline{C}}\underline{\underline{q}} = \underline{\underline{0}} \ , \tag{1}$$

where $\underline{A}$ is the inertia matrix;

$\underline{B}$ is the viscous damping matrix (damping coefficients);

$\underline{C}$ is the elasticity matrix (stiffness coefficients);

$\underline{\underline{q}} / \dot{\underline{\underline{q}}} / \ddot{\underline{\underline{q}}}$ are generalized displacements / velocities / accelerations vectors;

$\underline{\underline{0}}$ is the null vector.

2. MODAL DAMPING FOR THE *n*DOF MECHANICAL SYSTEMS

We consider the general case of a finite DOF mechanical system with elastic and viscous damping bearings. For reasons of generalization of the problem, we consider that the system has *n*DOF. In this case, the differential equations of the free movements vibration of the mechanical system are acc. to (1), where $\underline{A}$, $\underline{B}$ and $\underline{C}$ are $n \times n$ quadratic matrices and $\underline{\underline{q}}$, $\dot{\underline{\underline{q}}}$, $\ddot{\underline{\underline{q}}}$ and $\underline{\underline{0}}$ are column vectors with n components.

The modal analysis of the *n*DOF mechanical system leads to the natural pulsations $p_i \quad i = \overline{1,n}$ and eigenvectors $\underline{\underline{v}}_i \quad i = \overline{1,n}$, which verify the matrix equation

$$\underline{D}\underline{\underline{v}}_i = p_i^2 \underline{\underline{v}}_i \quad i = \overline{1,n} \ , \tag{2}$$

where $\underline{D} = \underline{A}^{-1}\underline{C}$ is the dynamic matrix of the system.

We consider the linear modal transformation of the eq. (1)

$$\underline{A}\underline{V}\ddot{\underline{\underline{\eta}}} + \underline{B}\underline{V}\dot{\underline{\underline{\eta}}} + \underline{C}\underline{V}\underline{\underline{\eta}} = \underline{\underline{0}} \ , \tag{3}$$

where: $\underline{\underline{\eta}} = \underline{V}^{-1}\underline{\underline{q}}$ is the modal vector (containing the modal coordinates $\eta_i \quad i = \overline{1,n}$)

$\underline{V} = \lfloor \underline{\underline{v}}_1 \ \ \underline{\underline{v}}_2 \ \ \cdots \ \ \underline{\underline{v}}_i \ \ \cdots \ \ \underline{\underline{v}}_n \rfloor$ is the modal matrix.

Considering the transposed modal matrix

$$\underline{V}^{\mathrm{T}} = \begin{bmatrix} \underline{\underline{v}}_1^{\mathrm{T}} \\ \underline{\underline{v}}_2^{\mathrm{T}} \\ \cdots \\ \underline{\underline{v}}_i^{\mathrm{T}} \\ \cdots \\ \underline{\underline{v}}_n^{\mathrm{T}} \end{bmatrix} \ , \tag{4}$$

we can left side matrix multiply all eq. (3) as follows:

$$\times \underline{V}^{\mathrm{T}} \Big| \ \underline{A}\underline{V}\ddot{\underline{\underline{\eta}}} + \underline{B}\underline{V}\dot{\underline{\underline{\eta}}} + \underline{C}\underline{V}\underline{\underline{\eta}} = \underline{\underline{0}} \ , \tag{5}$$

After matrix multiplication, we get the modal differential equation as follows

$$\underline{M}\ddot{\underline{\underline{\eta}}} + \underline{V}^{\mathrm{T}}\underline{B}\underline{V}\dot{\underline{\underline{\eta}}} + \underline{K}\underline{\underline{\eta}} = \underline{\underline{0}} \ , \tag{6}$$

where $\underline{M}$ is the modal matrix of inertia and $\underline{K}$ is the modal matrix of elasticity.

The above modal matrices are both diagonal

$$\underline{M} = \underline{V}^{\mathrm{T}}\underline{A}\underline{V} = DIAG[a_1, a_2, \cdots, a_i, \cdots, a_n] \tag{7}$$

$$\underline{K} = \underline{V}^{\mathrm{T}}\underline{C}\underline{V} = DIAG[c_1, c_2, \cdots, c_i, \cdots, c_n] \ , \tag{8}$$

where a_i are the modal coefficients of inertia and c_i are the modal coefficients of elasticity with the calculus relations as follows:

$$a_i = \underline{\underline{v}}_i^{\mathrm{T}}\underline{A}\underline{\underline{v}}_i \quad i = \overline{1,n} \tag{9}$$

$$c_i = \underline{\underline{v}}_i^{\mathrm{T}}\underline{C}\underline{\underline{v}}_i \quad i = \overline{1,n} \tag{10}$$

Considering structural damping for all bearings, the dissipation coefficients are proportional with the elasticity coefficients

$$b_{ij} = kc_{ij} \quad i, j = \overline{1,n} \ , \tag{11}$$

where k is the proportionality factor between damping and stiffness.

In this case, the second matrix product from eq. (6) becomes

$$\underline{V}^{\mathrm{T}}\underline{B}\underline{V} = k\underline{V}^{\mathrm{T}}\underline{C}\underline{V} = k\underline{K} = \underline{H} \ , \tag{12}$$

where $\underline{H}$ is the diagonal modal damping matrix

$$\underline{H} = DIAG[h_1, h_2, \cdots, h_i, \cdots, h_n], \quad (13)$$

and h_i are the modal damping coefficients:

$$h_i = \underline{\underline{v}}_i^T \underline{B} \underline{\underline{v}}_i \quad i = \overline{1,n} \quad (14)$$

With the diagonal modal damping matrix $\underline{H}$, the differential equations in modal coordinates become

$$\underline{\underline{M}}\ddot{\eta} + \underline{\underline{H}}\dot{\eta} + \underline{\underline{K}}\eta = \underline{\underline{0}} \quad (15)$$

and, as all matrix from (15) are diagonal, the system of differential equations are decoupled into n independent equations as follows:

$$a_i \ddot{\eta}_i + h_i \dot{\eta}_i + c_i \eta = 0 \quad i = \overline{1,n} \quad (16)$$

Dividing the eq. (16) by the modal coefficient of inertia $a_i \quad i = \overline{1,n}$, it can be write function of the modal damping ratio $\zeta_i \quad i = \overline{1,n}$ and the undamped natural pulsation $p_i \quad i = \overline{1,n}$

$$\ddot{\eta}_i + 2\zeta_i p_i \dot{\eta}_i + p_i^2 \eta = 0 \quad i = \overline{1,n} \quad (17)$$

where:

$$\zeta_i = \frac{h_i}{2\sqrt{a_i c_i}} \quad i = \overline{1,n} \quad (18)$$

and

$$p_i = \sqrt{\frac{c_i}{a_i}} \quad i = \overline{1,n} \quad (19)$$

The modal damping ratio ζ_i can be calculate from the plot of the time domain vibration signal for the displacement of an underdamped mechanical elastic system through the logarithmic decrement δ_i as follows

$$\zeta_i = \frac{\delta_i}{\sqrt{4\pi^2 + \delta_i^2}} \quad i = \overline{1,n} \quad (20)$$

$$\delta_i = \frac{1}{N} ln \frac{A_j}{A_{j+N}} \quad i = \overline{1,n} \quad (21)$$

where A_j is the displacement amplitude of a peak at a reference time t and A_{j+N} is the displacement amplitude of a peak N periods away.

Taking into consideration that the frequency of the under damped vibration of the 1DOF mechanical system is smaller than the frequency vibration of the same system but without damping, we can calculate the undamped natural frequency as follows

$$f_n = \frac{f_{ud}}{\sqrt{1-\zeta^2}}, \quad (21)$$

where f_{ud} is the frequency of the underdamped system vibration and ζ is the damping ratio of the system.

3. CASE STUDY. MODAL DAMPING RATIO CALCULATION FOR A VIADUCT USING VIRTUAL INSTRUMENTATION TECHNIQUE

In order to analyze the dynamic parameters of vibration, there were designed and built two virtual tools:

-one for acquiring and saving acceleration values taken during experiments;

-second to analyze the acceleration parameters.

The acquisition of vibration analog signals is made by acceleration transducers (accelerometers). The analog signals are transmitted to DAQ, which may be inside(internal) or outside (external) of the PC. The DAQ performs a signal conditioning and conversion in the digital format. Digital values are transmitted directly or through a cable into a USB port on the PC which is running the signals' acquisition program. The acquisition program has some features which provides the opportunity for memorizing information about the measurement conditions.

The virtual instrument developed on the basis of LabView® ver. 8.5 was used to perform the dynamic tests in site on the viaduct situated on km 29+602,75 - km 29+801,25 on the A3 Romanian highway. Dynamic actions were generated by a running four axle 41 ton truck over standardized height h = 4 cm obstacles on the road at different standardized speeds (10km/h to 50 km/h). The experimental data were obtained on three channels (accelerations on the axis x, y and z) by a fourth channel DAQ from National Instrument (NI 9233) through the USB port of a PC workstation. The used transducer was a triaxial accelerometer Bruel&Kjaer type 003 4506 B series 10145, fixed in the middle sectional plane of the viaduct; the axis of the transducer are oriented as follows: x axis - parallel with the longitudinal axis; y axis - horizontal transverse axis; z axis - parallel with the vertical axis.

Figure 2 shows a partial view of the frontal panel of the virtual instrument used for time domain and frequency domain analysis of the vibration signals on three direction. The displacement signals were obtained by digital integration of the acceleration signals obtained from the triaxial accelerometer B&K.

Figures 3, 4 and 5 show the time and frequency domain representations of the unfiltered displacement signals of the longitudinal, lateral and vertical vibration.

In order to calculate the damping ratio for natural modes of vibration, we have to get the time domain representation of the under damped vibration, separately for each eigenfrequency. Because the time domain representation of the displacements of the viaduct results as a linear summation of the eigenvectors, it’s necessary a signal filtering for each eigenfrequency domain. The signal filtering is made by the Virtual Instrument which has a dedicated internal module, with different types of filtration bandwidth and algorithm type.

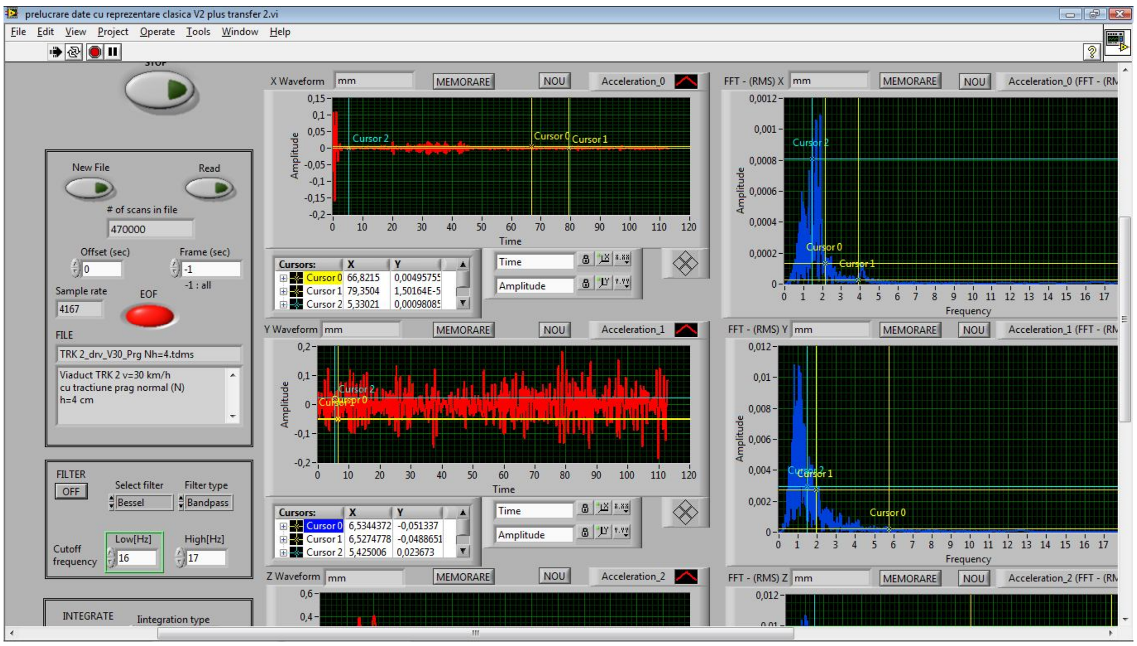
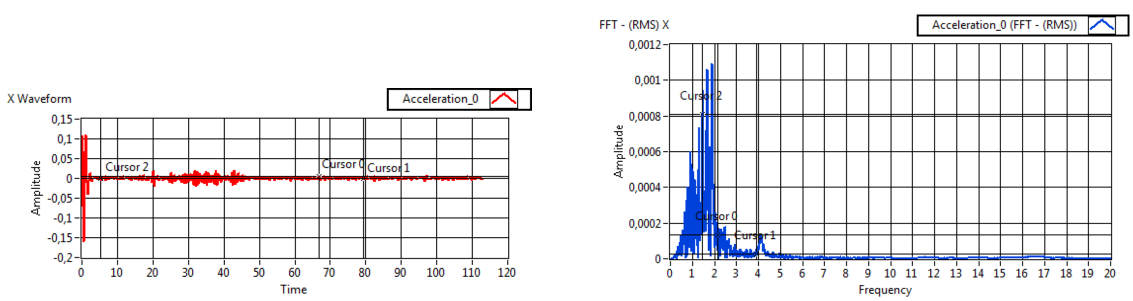
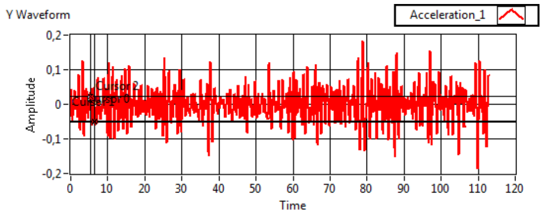


Figure 2 Frontal panel of the Virtual Instrument for *v=30km/h*, *h=4mm*, with traction (displacement signal on three channels – time and FFT representation)

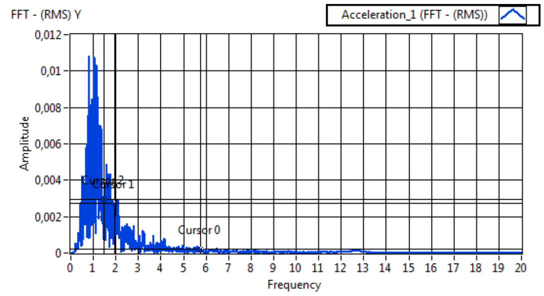


a) time domain [s] representation *b) frequency domain [Hz] representation*

Figure 3 Longitudinal displacement [mm] representation

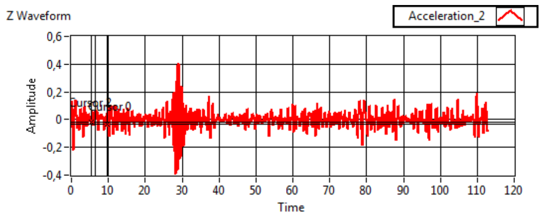


a) time domain [s] representation

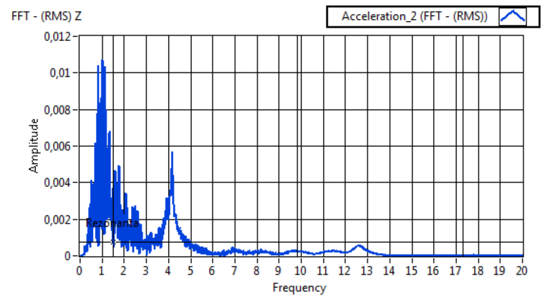


b) frequency domain [Hz] representation

Figure 4 Lateral displacement [mm] representation



a) time domain [s] representation



b) frequency domain [Hz] representation

Figure 5 Vertical displacement [mm] representation

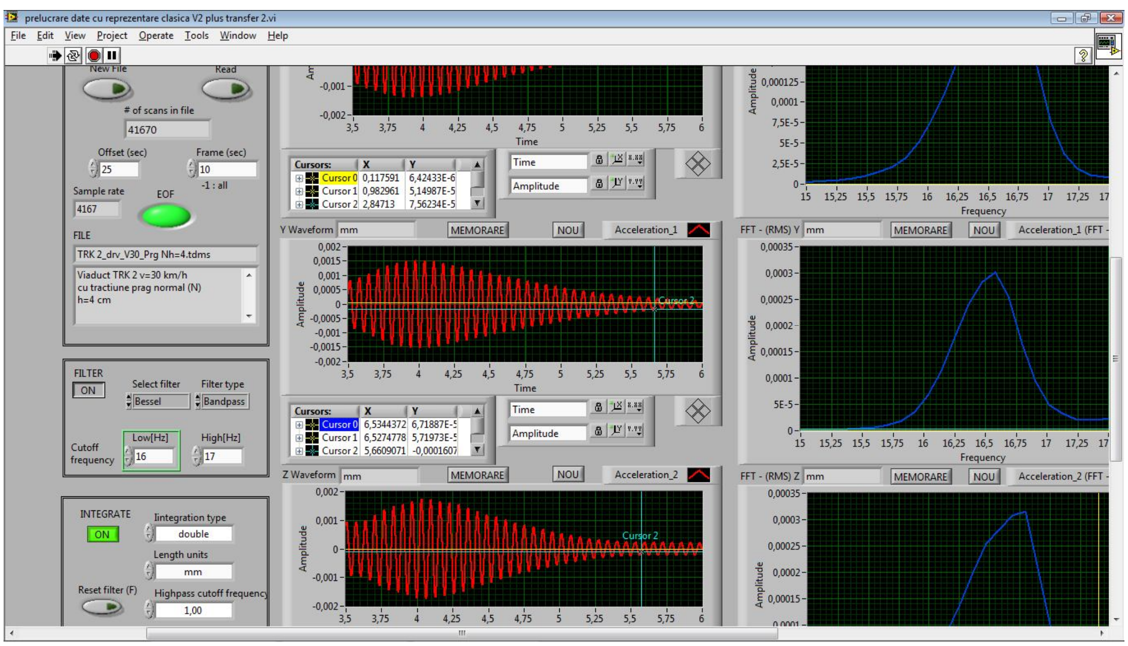
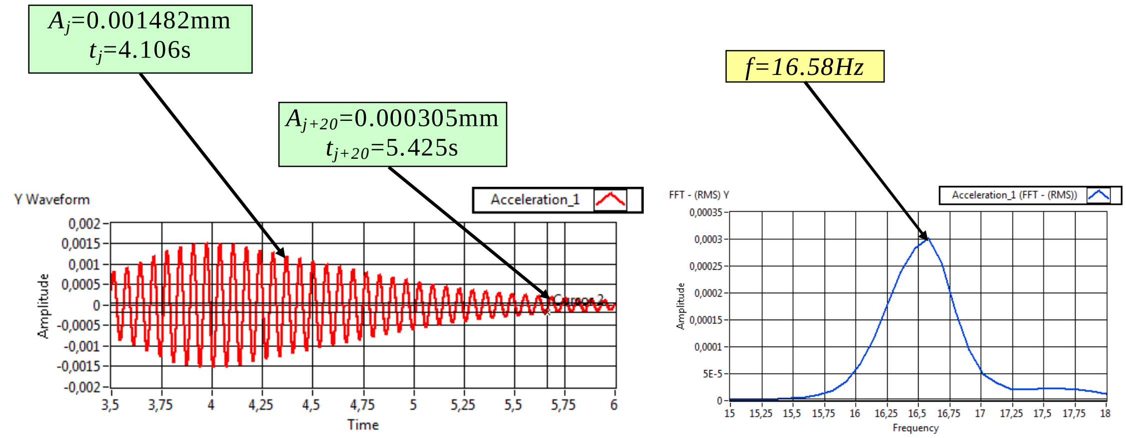


Figure 6 Frontal panel of the Virtual Instrument for *v=30km/h*, *h=4mm*, with traction (filtered signals, Bessel filter, 16-17 Hz bandpass filter type)



a) time domain [s] representation *b) frequency domain [Hz] representation*

Figure 7 Lateral displacement [mm] representation (12-13 Hz bandwidth pass Bessel filter type)

Figure 6 shows the frontal panel of the Virtual Instrument analyzer (partial view) for the free under damped vibration of the viaduct with the with following settings:

-dynamic excitation: $v=30km/h$, $h=4mm$;
-displacements obtained through double integration of the accelerations signals (with 1Hz high pass cutoff frequency);
-time duration of FFT analysis: 10 sec.;
-filtering method: 16-17 Hz bandwidth pass Bessel filter type.

Figure 7 shows the displacements representation (time and frequency domain representations) of the viaduct lateral vibration. With the highlighted values on the on the time domain representation, we can calculate the logarithmic decrement δ, the damping ratio ζ and the eigenfrequency f_n for the under damped free vibration with the frequency $f_{ud}=16.58Hz$ as follows:

$$\delta = \frac{1}{20} ln \frac{0.001482}{0.000305} = 0.079$$

$$\zeta = \frac{0.079}{\sqrt{4\pi^2 + 0.079^2}} = 0.0126$$

$$f_n = \frac{16.58}{\sqrt{1-0.0126^2}} = 16.581Hz$$

4. CONCLUSIONS

The steps to determine the modal damping by Virtual Instrumentation are:

-FFT analysis of the vibration signal (→natural frequencies);
-bandwidth pass filtering the signal (→modal vibration);
-logarithmic decrement δ, damping ratio ζ and eigenfrequency f_n calculation.

REFERENCES

[1] **Bratu, P.**, *Izolarea şi amortizarea vibraţiilor la utilajele de construcţii*, Redacţia publicaţiilor pentru construcţii, Bucureşti, 1982.
[2] **Bratu, P.**, *Sisteme elastice de rezemare pentru maşini şi utilaje*, Editura Tehnică, Bucureşti, 1990.
[3] **Buzdugan, Gh.**, *Izolarea antivibratorie*, Ed. Academiei Române, Bucureşti, 1993.
[4] **Rădoi, M., Deciu, E.**, *Mecanica*, Editura Didactică şi Pedagogică, Bucureşti, 1977.
[5] **Bratu, P.**, *Vibraţiile sistemelor elastice*, Editura Tehnică, Bucureşti, 2000.
[6] **Bratu, P., Drăgan, N.**, *L'analyse des mouvements désaccouplés appliquée au modèle de solide rigide aux liaisons élastiques*, Analele Universităţii "Dunărea de Jos" din Galaţi, Fascicula XIV, 1997.
[7] **Buzdugan, Gh., Fetcu, L., Radeş, M.**, *Vibraţii mecanice*, Ed. Didactică şi Pedagogică, Bucureşti, 1982.
[8] **Drăgan, N.**, *Contribuţii la analiza şi optimizarea procesului de transport prin vibraţii - teză de doctorat*, Universitatea "Dunărea de Jos", Galaţi, 2001.
[9] **Harris, C.M., Crede, C.E.**, *Şocuri şi vibraţii* vol. I-III, Ed. Tehnică, Bucureşti, 1967-1969.