DYNAMIC CALCULATION OF THE MECHANICAL TRANSMISSIONS WITH GEARS AND ELASTIC SHAFTS

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ABSTRACT

This study presents the elaboration of the physical and mathematical dynamic models of the mechanical systems with elastic shafts and discrete mass elements subjected to dynamical loads, taking into consideration the equivalent mass and inertia, equivalent stiffness and equivalent torques. The result and final considerations have a real utility in fast and operational calculus of the natural frequencies of this kind of mechanical models with more degree of freedom. It had taken into account both the ideal mechanical systems (with no energy losses, η=1) and the real systems according to their mechanical efficiency ($0<\eta<1$).

KEYWORDS: dynamic calculus, equivalent stiffness, equivalent mass inertia, equivalent torque, mechanical efficiency

1. MASS INERTIA REDUCTION

In order to create the physical and the mathematical models of the transmission with step gears and elastic shafts, we consider a single step gear as in fig. 1. For this gear, we consider as known the transmission ratio $i \equiv i_{12} = \frac{\omega_1}{\omega_2}$, the mass inertia of the gearwheels J_1 and J_2 and the mechanical efficiency of the gear η.

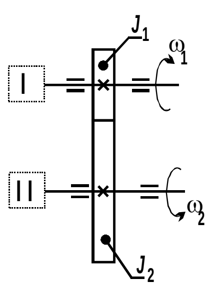


Figure 1 Physical model of a single step gear

According to [1] [2] [3], we can calculate the equivalent mass inertia of the two gearwheels on the other shaft, function of mechanical efficiency η of the step gear as follows:

1.a.Ideal gear transmission ($\eta = 1$)

$$J_{1\to II} \equiv J_{1eq} = J_1\left(\frac{\omega_1}{\omega_2}\right)^2 = J_1 i_{12}^2 = J_1 i^2 \quad (1)$$

$$J_{2\to I} \equiv J_{2eq} = J_2\left(\frac{\omega_2}{\omega_1}\right)^2 = J_2 i_{21}^2 = \frac{J_1}{i^2}, \quad (2)$$

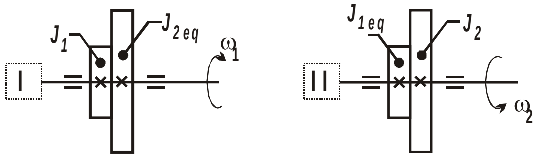
where J_{1eq} is the reduced/equivalent mass inertia of the gearwheel **1** to the wheel shaft **II** and J_{2eq} is the reduced/equivalent mass inertia of the gearwheel **2** to the wheel shaft **I**.

Taking into consideration that, after the reducing operation, both gearwheels are rotating with the same angular velocity, the total equivalent mass inertia of the step gear can be written as follows

$$J_{eqI} = J_1 + J_{2eq} = J_1 + \frac{J_2}{i^2}\ , \quad (3)$$

$$J_{eqII} = J_{1eq} + J_2 = J_1 i^2 + J_2 \,, \tag{4}$$

where J_{eqI} and J_{eqII} are the equivalent mass inertia on the wheel shaft **I** respectively on the wheel shaft **II** (see fig. 2).



a) **b)**

Figure 2 Step gear with equivalent inertia
a) on the wheelshaft **I**; **b)** on the wheelshaft **II**

1.b.Real gear transmission ($\eta < 1$**)**

In this case, function of the direction of power transmission (**DPT**, see fig. 3), the equivalent mass inertia of the gearwheels can be calculated with the next relations:
b.1)power transmission **1→2** (fig. 3.a)

$$J_{1\to II} \equiv J_{1eq} = J_1 \left(\frac{\omega_1}{\omega_2}\right)^2 \eta = J_1 i^2 \eta \tag{5}$$

$$J_{2\to I} \equiv J_{2eq} = \left(\frac{\omega_2}{\omega_1}\right)^2 \frac{J_2}{\eta} = \frac{J_2}{i^2 \eta} \tag{6}$$

b.2)power transmission **2→1** (fig. 3.b)

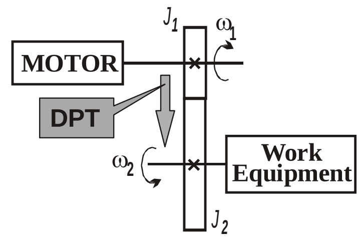
$$J_{1\to II} \equiv J_{1eq} = \left(\frac{\omega_1}{\omega_2}\right)^2 \frac{J_1}{\eta} = J_1 \frac{i^2}{\eta} \tag{7}$$

$$J_{2\to I} \equiv J_{2eq} = J_2 \left(\frac{\omega_2}{\omega_1}\right)^2 \eta = \frac{J_2}{i^2} \eta \tag{8}$$

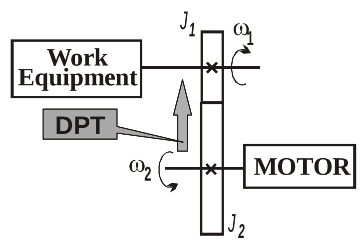
Considering a **DPT** as in fig. 3.a (if **DPT** is as in fig. 3.b, the equivalence operation is similar to fig. 3.a), the total equivalent mass inertia are as follows:

$$J_{eqI} = J_1 + J_{2eq} = J_1 + \frac{J_2}{i^2 \eta} \tag{9}$$

$$J_{eqII} = J_{1eq} + J_2 = J_1 \frac{i^2}{\eta} + J_2 \tag{10}$$



a)



b)

Figure 3 **D**irection of **P**ower **T**ransmission
a) from gearwheel **1** to gearwheel **2** (case b.1)
b) from gearwheel **2** to gearwheel **1** (case b.2)

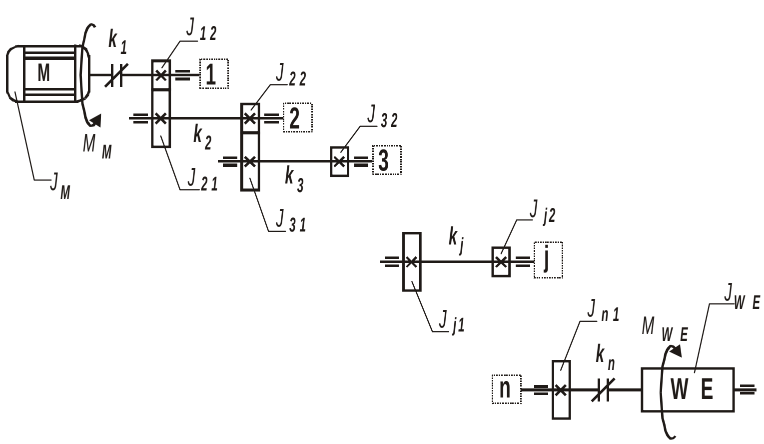


Figure 4 Mechanical transmission with $n-1$ steps spur gears

2. MODELING THE MECHANICAL SPUR GEAR TRANSMISSION WITH ELASTIC SHAFTS

Figure 4 presents a spur gear transmission of the mechanical power with the next components:

-driving motor with J_M mass inertia and M_M driving torque;

-working equipment with J_{WE} mass inertia M_{WE} resistant torque;

-n elastic shafts (stiffness k_j, $j=\overline{1,n}$) with steady state rotary motions (angular velocities ω_j, $j=\overline{1,n}$);

-$n-1$ spur gears with transmission ratio $i_{j,j+1}=\dfrac{\omega_j}{\omega_{j+1}}$, $j=\overline{1,n-1}$ and mechanical efficiency $\eta_{j,j+1}$, $j=\overline{1,n-1}$; the mass inertia of the geared wheels $J_{j,2} \leftrightarrow J_{j+1,1}$, $j=\overline{1,n-1}$.

We consider i_{1j} the transmission ratio from the wheel shaft **1** to wheel shaft **j** and η_{1j} the global mechanical efficiency of the transmission from shaft **1** to shaft **j**. With the above notations, we can write the formula of the transmission ratio and of the mechanical efficiency as follows:

$$i_j \equiv i_{1j} = \frac{\omega_1}{\omega_j} = \frac{\omega_1}{\omega_2}\cdot\frac{\omega_2}{\omega_3}\cdots\frac{\omega_{j-2}}{\omega_{j-1}}\cdot\frac{\omega_{j-1}}{\omega_j} = \prod_{k=1}^{j-1}\frac{\omega_k}{\omega_{k+1}} = \prod_{k=1}^{j-1} i_{k,k+1} \qquad (11)$$

$$\eta_j \equiv \eta_{1j} = \prod_{k=1}^{j-1}\eta_{k,k+1} \qquad (12)$$

3. CALCULUS OF THE EQUIVALENT TORQUES

Considering a single drive motor for the mechanical system and only one work equipment as in fig. 4, the equivalent model, with all characteristics reduced at the motor shaft is shown in fig. 5.

According to [4], the equivalent motor torque and the equivalent work equipment torque can be calculated as follows:

$$M_{WEeq} = \frac{M_{WE}}{\prod_{j=1}^{n-1} i_{j,j+1}\cdot\prod_{j=1}^{n-1}\eta_{j,j+1}} = \frac{M_{WE}}{i_{1n}\cdot\eta_{1n}} = \frac{M_{WE}}{i_n\eta_n}, \qquad (13)$$

where η_n is the global mechanical efficiency of the gear transmission.

4. CALCULUS OF THE EQUIVALENT STIFFNESS

According to [5], we can calculate the equivalent stiffness of the wheel shafts function of the mechanical stiffness as follows:

-for the wheel shaft **2**

$$k_{2eq} = \frac{k_2}{i_{12}^2\eta_{12}^2} = \frac{k_2}{i_2^2\eta_2^2} \qquad (14)$$

-for the wheel shaft **3**

$$k_{3eq} = \frac{k_3}{i_{23}^2 i_{12}^2\eta_{23}^2\eta_{12}^2} = \frac{k_3}{i_{13}^2\eta_{23}^2} = \frac{k_3}{i_3^2\eta_3^2} \qquad (15)$$

-for the wheel shaft **j**, $j=\overline{2,n}$

$$k_{jeq} = \frac{k_j}{\prod_{r=1}^{j-1} i_{r,r+1}^2 \prod_{r=1}^{j-1}\eta_{r,r+1}^2} = \frac{k_j}{i_{1j}^2\eta_{1j}^2} \qquad (16)$$

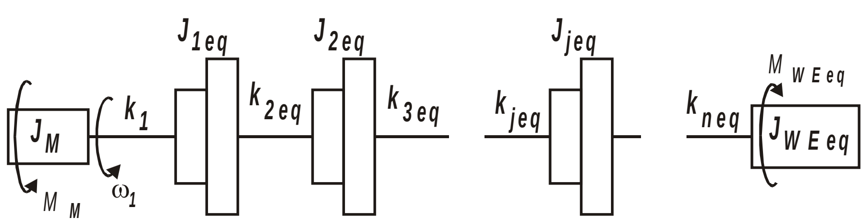


Figure 5 The equivalent spur gears mechanical transmission

-for the wheel shaft **n**

$$k_{neq} = \frac{k_n}{\prod_{j=1}^{n-1} i_{j,j+1}^2 \prod_{j=1}^{n-1} \eta_{j,j+1}^2} = \frac{k_n}{i_{1n}^2 \eta_{1n}^2} \quad (17)$$

5. CALCULUS OF THE EQUIVALENT MASS INERTIA OF THE GEARS, DRIVE MOTOR AND WORKS EQUIPMENT

According to [3], we can calculate the equivalent mass inertia of the wheel gears as follows:

-for the gears on the wheel shaft **2**

$$J_{2keq} = \frac{J_{2k}}{i_{12}^2 \eta_{12}} = \frac{J_{2k}}{i_2^2 \eta_2},\ k = \overline{1,2}\ , \quad (18)$$

-for the gears on the wheel shaft **3**

$$J_{3keq} = \frac{J_{3k}}{i_{23}^2 \eta_{23} i_{12}^2 \eta_{12}} = \frac{J_{3k}}{i_{13}^2 \eta_{13}} = \frac{J_{3k}}{i_3^2 \eta_3} \quad (19)$$

-for the gears on the wheel shaft **j**, $j = \overline{2,n}$

$$J_{jkeq} = \frac{J_{jk}}{\prod_{r=1}^{j-1} i_{r,r+1}^2 \prod_{r=1}^{j-1} \eta_{r,r+1}^2} = \frac{J_{jk}}{i_{1j}^2} = \frac{J_{jk}}{i_j^2} \quad (20)$$

-for the gear on the wheel shaft **n** (WE shaft)

$$J_{n1eq} = \frac{J_{n1}}{\prod_{j=1}^{n-1} i_{j,j+1}^2 \prod_{j=1}^{n-1} \eta_{j,j+1}^2} = \frac{J_{n1}}{i_{1n}^2 \eta_{1n}^2} \quad (21)$$

-for the works equipment

$$J_{WEeq} = \frac{J_{WE}}{\prod_{j=1}^{n-1} i_{j,j+1}^2 \prod_{j=1}^{n-1} \eta_{j,j+1}^2} = \frac{J_{WE}}{i_n^2 \eta_n^2} \quad (22)$$

6. CALCULUS OF THE TOTAL EQUIVALENT MASS INERTIA OF THE SPUR GEARS

The total mass inertia of the pairs of involved gears can be calculated taking into account that, after the reducing operations, both of them have the same angular velocity (in this case the angular velocity of the motor ω_1); the total mass inertia can be calculated by summing the equivalent mass inertia of geared tooth wheels, calculus using relations (18)-(21), as follows:

-for gear wheels **1↔2**

$$J_{1eq} = J_{12} + J_{21eq} = J_{12} + \frac{J_{21}}{i_2^2 \eta_2} \quad (23)$$

-for gear wheels **2↔3**

$$J_{2eq} = J_{22eq} + J_{31eq} = \frac{J_{22}}{i_2^2 \eta_2} + \frac{J_{31}}{i_3^2 \eta_3} \quad (24)$$

-for gear wheels **j↔j+1**, $j = \overline{2,n-1}$

$$J_{jeq} = J_{j2eq} + J_{j+1,1eq} = \frac{J_{j2}}{i_j^2 \eta_j} + \frac{J_{j+1,1}}{i_{j+1}^2 \eta_{j+1}} \quad (25)$$

-for gear wheels **n-1↔n**

$$J_{n-1eq} = J_{n-1,2eq} + J_{n1eq} = \frac{J_{n-1,2}}{i_{n-1}^2 \eta_{n-1}} + \frac{J_{n1}}{i_n^2 \eta_n} \quad (26)$$

7. CONCLUSIONS

The equivalence operation for mass inertia, stiffness and torques can be done, in the same way, to the **W**orks **E**quipment shaft (numbered with **n**), or to any shaft of the gear transmission.

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