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# ABOUT THE HYSTERESIS-LOOP OF THE MECHANICAL OSCILLATORS

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## ABSTRACT

*The work presents firstly the representation of the typical Hysteresis-loop as function between the indoor forces of the oscillator as friction and elastic forces dependant on displacement and, otherwise, as function between the outdoor forces as excitation dependant on displacement, at linear or non-liner oscillators. Secondly is presented another type of Hysteresis-loop, as functions dependant on speed instead of the displacement under the same conditions. The axis of the loop can describe mechanical properties of the elements of the oscillator as elasticity or friction.*

KEYWORD: oscillator , loop, hysteresis

## 1. INTRODUCTION

The movement of the non-linear mechanical oscillator excited through harmonics forces is described from the non-linear differential equation

$$m\cdot\ddot{x}+F_f(x,\dot{x})+F_e(x,\dot{x})=F\cdot\sin\omega\cdot t, \quad (1)$$

where m=mass in movement , x=displacemen, $F_f$ = the dumping force, $F_e$ = the restoring force.

From the hysteresis-loop with the coordinates force-displacement or force-speed, we may find the equations of the restoring characteristics and dumping characteristics of an oscillator.

Drawing the hysteresis-loop will be made by measuring the acceleration of the mechanical system and knowing the excitation force[1].

## 2. NUMERICAL EXAMPLE

There are presented some samples of hysteresis-loop.

It is studied a mechanical oscillator with a non-linear restoring characteristic, with the mass m=1kg excited harmonic force with F=1000*sin(100t), with two values for the dumping force, $F_f$=0 respectively $F_f$=300Ns/m.

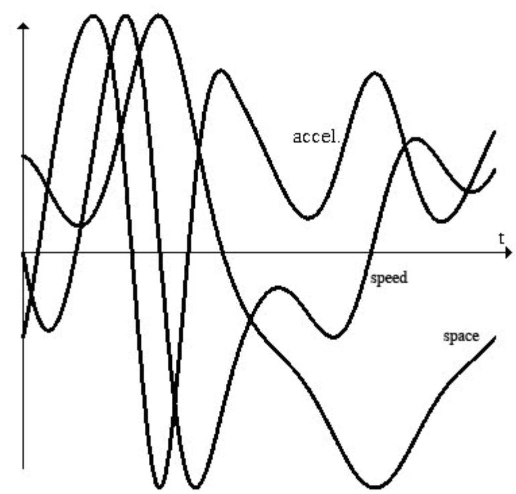


Fig. 1. The movement graphs, without dumping force

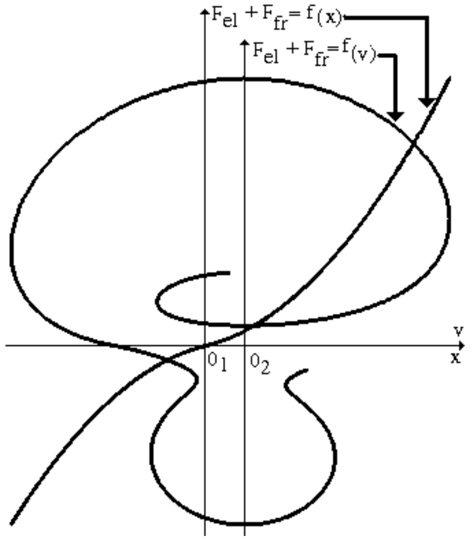


Fig. 2. The hysteresis-loops, without dumping force

In the case without dumping force (fig.1 and fig.2) it is shown the hysteresis-loop with force-movement coordinates; it has the surface equal to null but with force-speed coordinates the hysteresis-loop can't close because the oscillator regime is not stable on the excitation pulse.

With the dumping force, the graphs from fig. 3 and fig. 4 were drawn after 200 periods of excitation, time in which the movement is already stable[2].

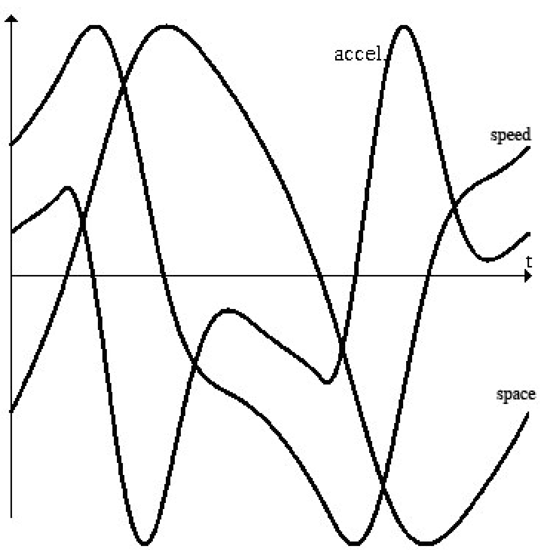


Fig. 3. The movement graphs, with dumping force

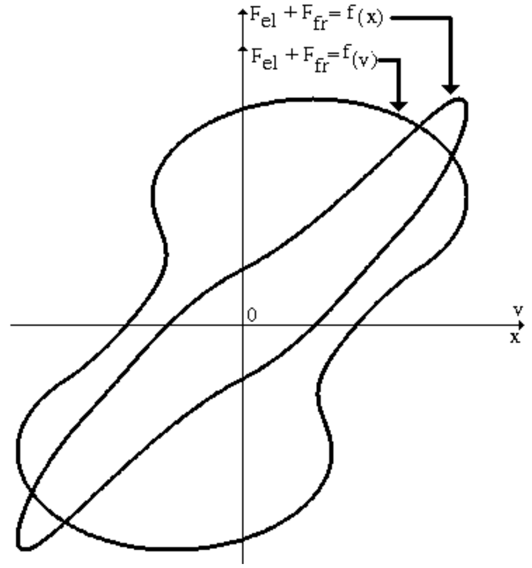


Fig. 4. The hysteresis-loops, with dumping force

The movement of the linear oscillator, excited through harmonic forces, is described from the almost known equation

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = \frac{F_0}{m}sin(\omega t), \qquad (2)$$

where m = mass in movement, c = friction quotient, k = elastic quotient, x = displacement.

The law of the stabilised movement is

$$\begin{cases} x = A\,sin(\omega t - \varphi) \\ \dot{x} = \omega A\,cos(\omega t - \varphi) \end{cases}, \qquad (3)$$

where

$$\begin{cases} A = \dfrac{h}{\sqrt{\left(p^2 - \omega^2\right)^2 + 4n^2\omega^2}} \\ tan(\varphi) = \dfrac{2n\omega}{p^2 - \omega^2} \end{cases} \qquad (4)$$

and

$$c/m = 2n\,,\ k/m = p^2\,,\ F_0/m = h\,. \qquad (5)$$

## 3. HYSTERESIS-LOOP

If the time is eliminated between the displacement x from (2) and the elastic and friction forces, [2], the equation of the hysteresis-loop is obtained. At our example, that is

$$\begin{cases} x = A\sin(\omega t - \varphi) \\ F_{sp} = \dfrac{F}{m} = 2n\dot{x} + p^2 x = \\ = A\left[2n\omega\cos(\omega t - \varphi) + p^2\sin(\omega t - \varphi)\right] \end{cases} \quad (6)$$

where $F$ =indoor forces, $F_{sp}$ =indoor specific forces or

$$F_{sp}(x) = p^2 x \pm 2n\omega\sqrt{A^2 - x^2}\,. \quad (7)$$

The time can be eliminated not only between expressions from (6), but between another two expressions, namely the displacement x and excitation

$$\begin{cases} x = A\sin(\omega t - \varphi) \\ F_{e\,sp} = h\sin(\omega t) \end{cases}, \quad (8)$$

where $F_{e\,sp}$ =outdoor (external) forces, as excitation. In these conditions, we will obtain another hysteresis-loop, as

$$F_{e\,sp}(x) = \left(p^2 - \omega^2\right)x \pm 2n\omega\sqrt{A^2 - x^2}\,. \quad (9)$$

In (6), (7), (8) and (9) the size of $F_{sp}$ is acceleration, but this can be a specific force, *N/Kg*.

If the quotient *p*, *n* and *h* may be written explicitly, the equations can express really a relation between forces and displacement.

## 4. PROPERTIES

The loops described from (7) and (9) have some properties:

-The area of the loops, calculated with

$$Area = \oint F(x)\cdot dx = \int_0^T F(x)\cdot\dot{x}\cdot dt \quad (10)$$

is equal in the two cases, respectively

$$Area = 2\pi n\omega A^2 \quad (11)$$

-The shape of the loops is different and dependant on the pulsation $\omega$. At very little value of $\omega$,

$$\omega < p \Rightarrow \omega^2 << p^2 \quad (12)$$

the shape of the loops is like infig. 5.a.

-The axis of the loop from (7) gives the slope the mechanical elasticity, respectively the elastic quotient k. The equation of the symmetrical axis of the loop becomes from (7) as

$$\begin{aligned} F_{sy} &= \frac{1}{2}\left(p^2 x + 2n\omega\sqrt{A^2 - x^2}\right) + \\ &+ \frac{1}{2}\left(p^2 x - 2n\omega\sqrt{A^2 - x^2}\right) = p^2 x = \frac{k}{m}x \end{aligned} \quad (13)$$

or

$$F = kx\,. \quad (14)$$

This quotient can be found practically easier from (9), at very little pulsation , $\omega << p$, the parameters of the excitation (amplitude and pulsation) can be mostly known easier as the friction forces, the displacement can be easily experimentally determined.

A non-linear oscillator can be similar to more linear oscillators with a different elasticity. Its elastic behaviour can be a sum from the linear elastic quotients. In this case, the symmetry axis of the loop describes the elastic characteristic of the oscillator.

If instead of the displacement $x$ we chose the speed $\dot{x}$ as independent variable, we obtain another hysteresis-loop. Based on the same considerations, it is easy to demonstrate that its symmetrical axis describes the friction properties, linear or non-linear and, at very little pulsation, these properties are to be found from loop of the external forces and speed. The relations are

$$F_{sp}(\dot{x}) = 2n\dot{x} \pm \frac{p^2}{\omega}\sqrt{A^2\omega^2 - \dot{x}^2} \quad (15)$$

respectively

$$F_{sp}(\dot{x}) = 2n\dot{x} \pm \frac{p^2 - \omega^2}{\omega}\sqrt{A^2\omega^2 - \dot{x}^2}\,. \quad (16)$$

## 5. EXAMPLES

A linear oscillator has *m*=100Kg, *c*=1000 Ns/m, *k*=1000000 N/m, $F_0$=1000N. In this case , *p*=86.6 rad/s.

Fig. 5 shows the loops for $\omega$=0.1*p* (fig. 5.a) , $\omega$=*p* (fig. 5.b), $\omega$=5*p* (fig. 5.c).

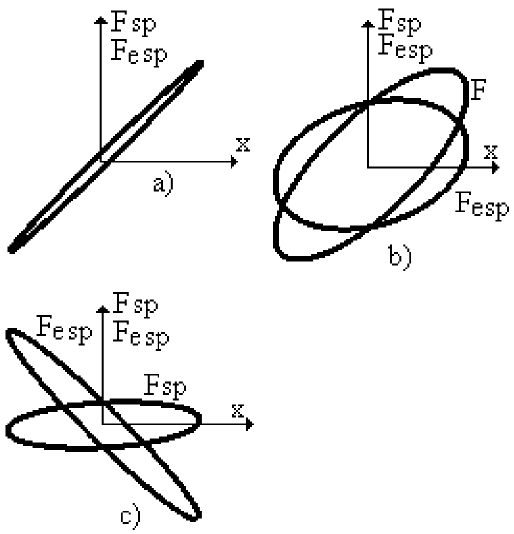


Fig. 5. Hysteresis-loops for linear oscillator

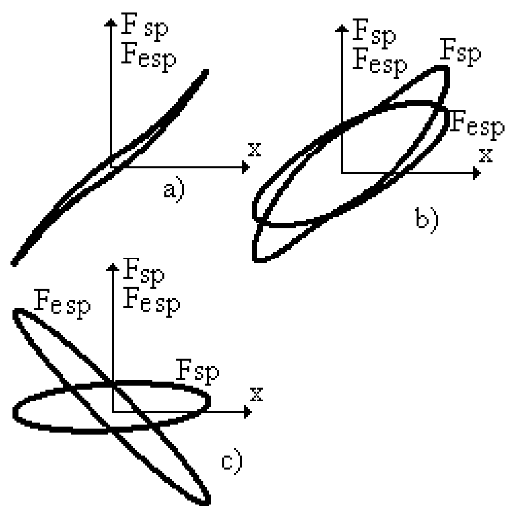


Fig. 6. Hysteresis-loops for non-linear oscillator

If the elastic forces are non-linear, as

$$F_{elastic} = \left(10^6 |x| + 10^9 \left|x^2\right|\right) SGN(x) \quad (17)$$

and the differential equation of the oscillator becomes

$$\ddot{x} + \frac{c}{m}\dot{x} + F_{elastic} = \frac{F_0}{m}\sin(\omega t), \quad (18)$$

where *SGN*=signum function, the results show that the hysteresis-loops are very near at little pulsation, fig. 6, in the same conditions for $\omega$; $\omega$=0.1*p* (fig. 6.a), $\omega$=*p* (fig. 6.b), $\omega$=5*p* (fig. 6.c).

## 6. CONCLUSION

The linear or non-linear friction characteristic induces little influence on the stabilised movement of the mechanical oscillator.

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