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INFLUENCE OF THE BEHAVIOR OF NONLINEAR VISCOELASTIC SYSTEMS BEARING ON THE DYNAMICS OF BRIDGE STRUCTURES

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ABSTRACT

Modern structures of bridges and viaducts require dynamic isolation systems, designed to eliminate or mitigate the destructive effects of intense traffic and seismic activity on these constructions. This paper studies the problem of viscoelastic type systems bearing in terms of viscoelastic links after wear viscoelastic elements were envisaged. It will demonstrate, in theory, the connection between the behaviors of viscoelastic nonlinear systems and the modification of kinematic and energy parameters of structure dynamic response loading by force pulse.

KEYWORDS: dynamic, viscoelastic, vibration, nonlinear, rubber

1. Introduction

In this paper we propose a theoretical methodology for assessing the degree of wear of the isolation dynamic systems, corresponding to their nonlinear behavior, by identifying changes that occur in the dynamic response of the structure loaded by strong impulsive type. From the theoretical point of view, we will look into the same physical model but with the following assumptions:

- elastic and viscous linear forces
- elastic and viscous forces of nonlinear type.

The next step is the experimental validation of the methodology as follows:

- experimental measurements will be made when commissioning the viscoelastic systems
- are regularly performed the same types of experimental measurements under the same conditions
- there will be carried out a comparative analysis of parameters determined experimentally in order to identify deviations from normal operation of isolation dynamic systems.

2. Physical and mathematical modeling of a deck of bridge section

To develop a physical model as close to the real situation, we started from an existing viaduct located on highway A3 Transylvania in Romania, at km 29 602.75 801.25 ↔ 29, fig. 1 (at Savadisla between Tirgu Mures and Cluj) [2]. In a simplified way a deck of bridge section can be considered as a rigid solid with triortogonal viscoelastic bearing, fig. 2.

Fig. 1 Viaduct on the A3 Transylvania highway

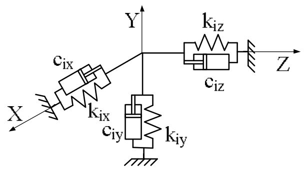

Fig. 2 Triortogonal viscoelastic bearing

Each section of the bridge is leaning through 16 dynamic isolation systems made of laminated rubber, fig. 3.

Fig. 3 Viscoelastic supports (rubber)

The matrix expression of the equation that characterizes the oscillatory movement of the system can be written as, [1]:

$$\underline{\underline{I}}\,\underline{\underline{\ddot{q}}}+\underline{\underline{C}}\,\underline{\underline{\dot{q}}}+\underline{\underline{K}}\,\underline{\underline{q}}=\underline{\underline{f}} \qquad (1)$$

where: $\underline{\underline{q}}$ - generalized coordinates vector; $\underline{\underline{\dot{q}}}$ - generalized speeds vector; $\underline{\underline{\ddot{q}}}$ - generalized accelerations vector;

f - generalized forces vector; $\underline{I}$ - inertia matrix; $\underline{C}$ - amortizations matrix; $\underline{K}$ - stiffness matrix.

The main elastic axes of elastic bearing are parallel to the reference axes. In this case, the movements represented by the variation of coordinates, corresponding to the six degrees of freedom, may be released as follows:

- coupled translational motion along the axis X and Y (X, φ_y) axis rotation;
- coupled translational movement along the Y axis and rotation around the X (Y, φ_x) axis;
- translational movement along the Z axis independent of the other ways;
- rotation around the z(φ_z) axis independent of other modes.

In this case, the system of differential equations can be structured as follows:

Coupled mode (X, φ_y)

$$\begin{cases} m\ddot{X}+\dot{X}\sum_{1}^{16}c_{ix}+\dot{\phi}_y\sum_{1}^{16}z_ic_{ix}+X\sum_{1}^{16}k_{ix}+ \\ +\phi_y\sum_{1}^{16}z_ik_{ix}=0 \\ J_y\ddot{\phi}_y+\dot{X}\sum_{1}^{16}z_ic_{ix}+\dot{\phi}_y\sum_{1}^{16}\left(c_{iz}x_i^2+c_{ix}z_i^2\right)+ \\ +X\sum_{1}^{16}z_ik_{ix}+\phi_y\sum_{1}^{16}\left(k_zx_i^2+k_xz_i^2\right)=e_xF_z \end{cases} \qquad (2)$$

Coupled mode (Y, φ_x)

$$\begin{cases} m\ddot{Y}+\dot{Y}\sum_{1}^{16}c_{iy}-\dot{\phi}_x\sum_{1}^{16}c_{iy}z_i+ \\ +Y\sum_{1}^{16}k_{iy}-\phi_x\sum_{1}^{16}k_{iy}z_i=0 \\ J_x\ddot{\phi}_x-\dot{Y}\sum_{1}^{16}z_ic_{iy}+\dot{\phi}_x\sum_{1}^{16}\left(c_{iy}z_i^2+c_{iz}y_i^2\right)- \\ -Y\sum_{1}^{16}z_ik_{iy}+\phi_x\sum_{1}^{16}\left(k_{iy}z_i^2+k_{iz}y_i^2\right)=-e_yF \end{cases} \qquad (3)$$

Translation axis OZ

$$m\ddot{Z}+\dot{Z}\sum_{1}^{16}c_{iz}+Z\sum_{1}^{16}k_{iz}=-F_z \qquad (4)$$

Rotation axis OZ

$$\begin{aligned} &J_z\ddot{\phi}_z+\dot{\phi}_z\sum_{1}^{16}\left(c_{ix}y_i^2+2c_{iy}x_i^2\right)+ \\ &+\phi_z\sum_{1}^{16}\left(k_{ix}y_i^2+2k_{iy}x_i^2\right)=0 \end{aligned} \qquad (5)$$

Of the four coupled modes of motion will be studied in this paper only the proper motion in the vertical direction OZ. The bridge deck section is impulsively loaded by passing a four-axle truck weighing 41 tons over an obstacle with height h = 40mm, at a speed of 20 km/h. When passing a truck over the obstacle it results a force of impulsive application as shown in fig. 4, and it should be noted that impulsive force was considered as a rectangular function.

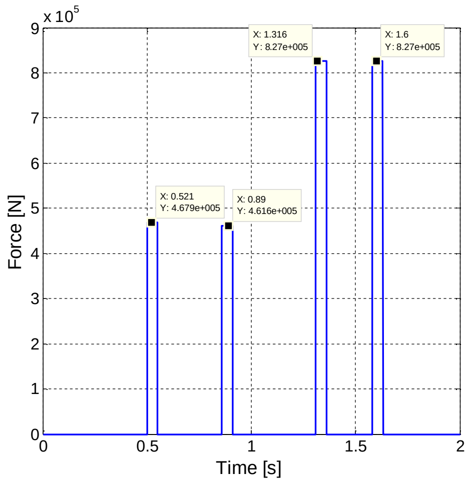

Fig. 4 Four rectangular pulse trains

3. Analysis of kinematics and energy parameters of the dynamic response of the deck

As I mentioned at the beginning of the paper, the dynamic analysis will be performed with the following assumptions [3]:

- elastic and viscous - linear forces
- elastic and viscous forces - nonlinear type:

$$F_e = k_{iz}(Z + sign(Z)\beta_{i1}Z^2 + \beta_{i2}Z^3);$$

$$F_v = c_{iz}(\dot{Z} + sign(\dot{Z})\gamma_{i1}\dot{Z}^2 + \gamma_{i2}\dot{Z}^3).$$

where: F_e - elastic force of an element; F_v - damping force on a single item; β_{i1}, β_{i2}, γ_{i1}, γ_{i2} - coefficients.

3.1 The mathematical model for the linear case

In this case, the equation of motion is as follows:

$$m\ddot{Z} + \dot{Z}\sum_{1}^{16} c_{iz} + Z\sum_{1}^{16} k_{iz} = -F_z \qquad (6)$$

where: m - mass of the deck; Z - moving on vertical direction; c_{iz} - damped coefficient of bearing on vertical direction; k_{iz} - coefficient of elasticity; F_z - vertical direction force application.

3.2 Mathematical model for nonlinear case

In this case, the equation of motion is as follows:

$$m\ddot{Z} + \dot{Z}\sum_{1}^{16} c_{iz}(\dot{Z} + sign(\dot{Z})\gamma_{i1}\dot{Z}^2 + \gamma_{i2}\dot{Z}^3) + $$
$$+ Z\sum_{1}^{16} k_{iz}(Z + sign(Z)\beta_{i1}Z^2 + \beta_{i2}Z^3) = -F_z \qquad (7)$$

Based on the differential equation of motion (7) were plotted and analyzed the following parameters, specific vibration deck supported on viscoelastic systems:

1. time response of the kinematic parameters of vibration
2. frequency response of the kinematic parameters of vibration
3. energy dissipated by viscous friction
4. motion trajectory
5. power spectral density

Solving mathematical model was made through the program MATLAB version R2008a, assuming the following numerical values of coefficients of equation of motion $k_{iz}=650\cdot10^6$N/m; $c_{iz}=2.5\cdot10^6$ Ns/m; $m=992\cdot10^3$kg; $\beta_{i1}=31250$ m^{-1}, $\beta_{i2}=1875\cdot10^5$ m^{-2}, $\gamma_{i1}=31.25$m/(Ns), $\gamma_{i2}=437.5$ $m^2/(Ns)^2$. In figures 7-20 are presented graphical representations of kinematic and energy parameters of the vibration of the bridge deck.

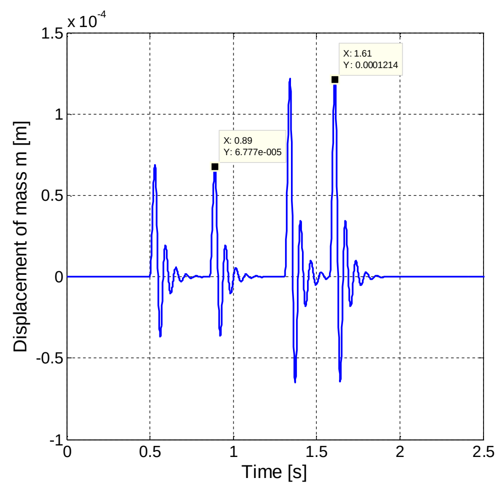

Fig. 5 Displacement on mass m: linear case

The representation in time of mass displacement, fig. 5, 6, shows a decrease in amplitude of this parameter for the nonlinear forces type case. In the spectral representation of displacement mass *m*, in the nonlinear case is observed a broadening of the dominant spectral components band towards the higher value. This change may adversely affect the structural integrity of the building because of the phenomenon of resonance.

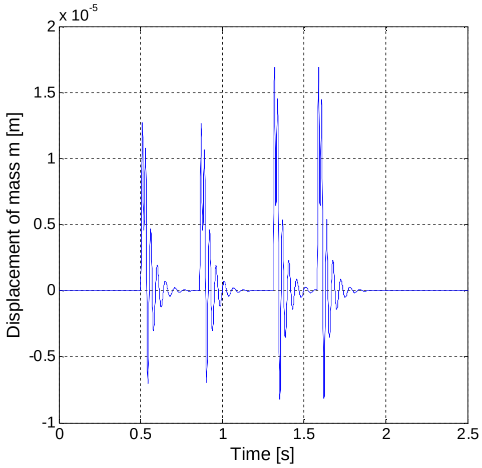

Fig. 6 Displacement on mass m: nonlinear case

If the linear case, spectral components of the movement was centered around 15 Hz, in nonlinear case, the dominant spectral components is up to the value of 44 Hz, fig. 7, 8.

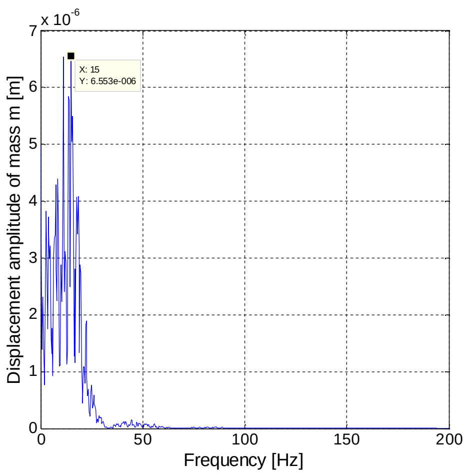

Fig. 7 Spectral representation – linear case

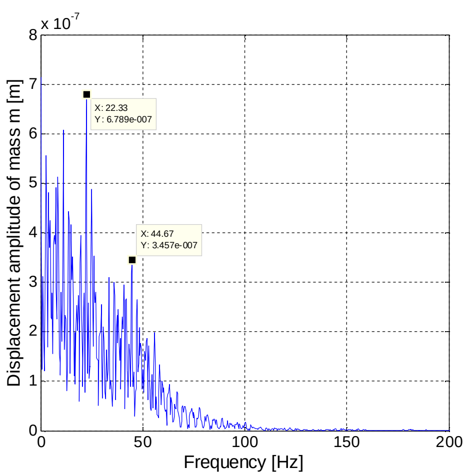

Fig. 8 Spectral representation – nonlinear case

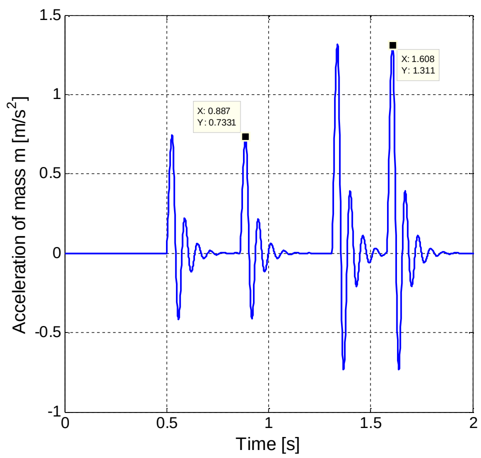

Fig. 9 Acceleration of mass m – linear case

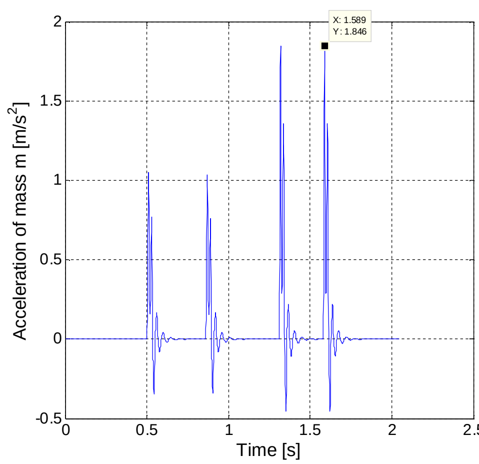

Fig. 10 Acceleration of mass m – nonlinear case

Acceleration signal in the linear case shows an increase in amplitude to the value of $1.84 m/s^2$, against the value of $1.31 m/s^2$ corresponding to the linear case.

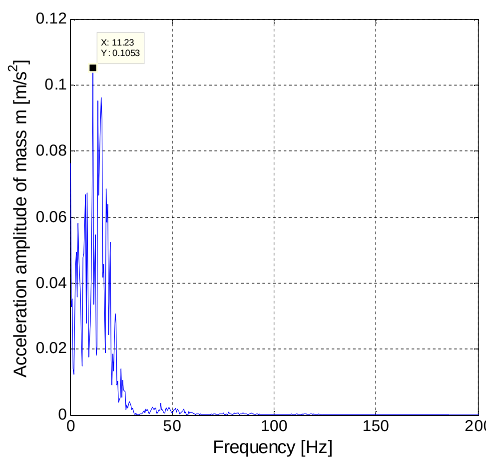

Figure 11 Spectral representation – linear case

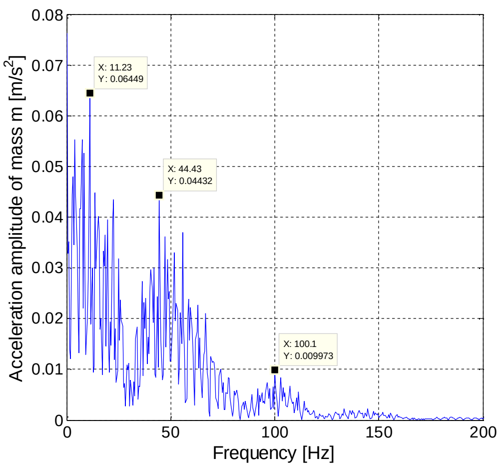

Fig. 12 Spectral representation – nonlinear case

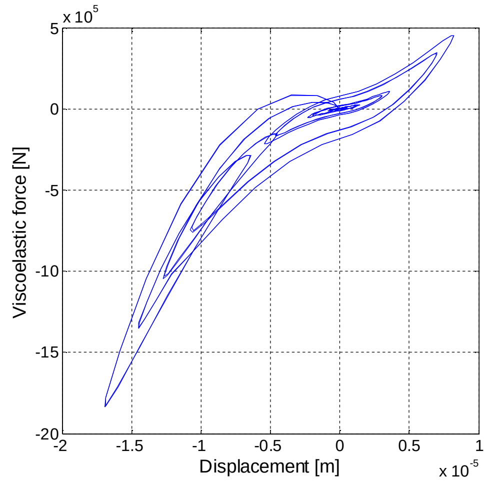

Fig. 14 Hysteresis loops: nonlinear case, W=29.5 J

As the displacement, acceleration frequency representation shows a broadening of the dominant band spectral components to values of 100 Hz. fig. 11, 12.

Hysteresis loops are graphical representations of the symmetry axis of elasticity coefficients, the energy dissipated in the same time being different, fig. 13, 14. In the nonlinear case it could be observed a significant reducing power dissipation which means that a significant amount of energy remains in the system. The remaining energy in the system, if it has significant value, may lead to the appearance of damage in bridge structure.

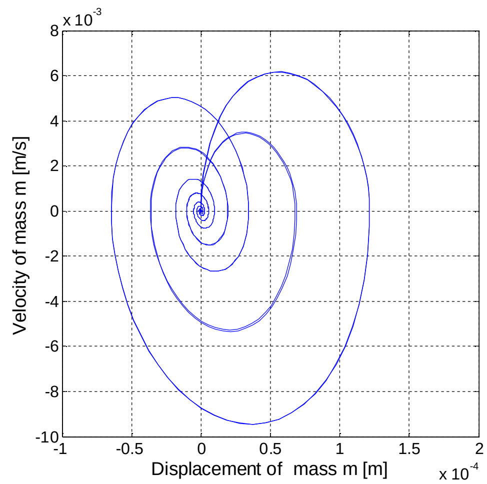

Fig. 15 Phase plane representation: linear case

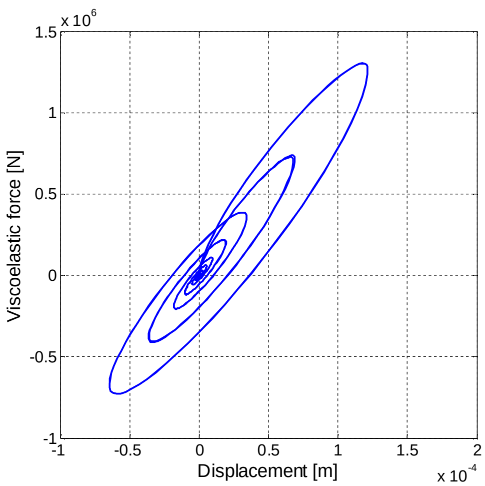

Fig. 13 Hysteresis loop: linear case, W=256 J

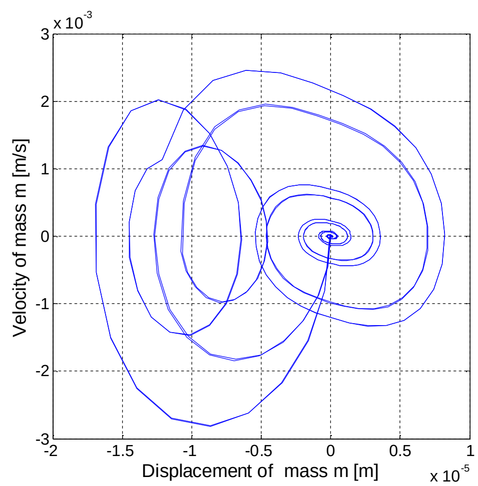

Fig. 16 Phase plane representation: nonlinear case

Phase plane representation, figure 15, 16, shows a motion around an apparently attractor point for the nonlinear case.

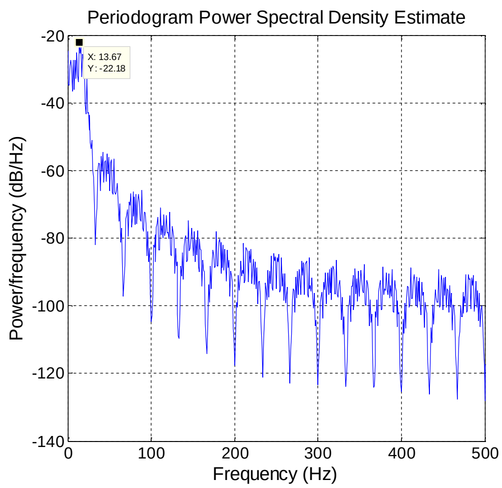

Fig. 17 Periodogram: linear case

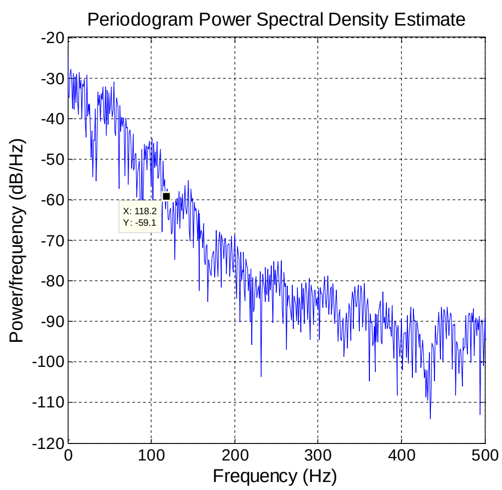

Fig. 18 Periodogram: nonlinear case

The power spectral density graphics for the two analyzed cases, fig. 19, 20, reveal that in nonlinear case a significant value of mean power of the signal is carried by spectral components in the range frequency (0 ÷ 118) Hz, while in the linear case this interval is (0 ÷ 30) Hz.

4. CONCLUSION

Systems isolators based on rubber, of the bridge elements are intensely stressed by traffic. Natural aging of rubber and dynamic stresses are factors that cause abnormal function of isolation systems dynamics that is why they must be replaced. The time of replacement is calculated by applying the methodology described in this document and theoretically demonstrated. Based on this method it can be developed a methodology able to diagnose the structural integrity of concrete bridges and viaducts.

Acknowledgments

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