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ON BEHAVIOUR EVALUATION OF VIBRATORY SYSTEMS WITH UNCERTAIN PARAMETERS

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ABSTRACT

This study deals with the area of nonlinear and random mechanical vibration. It was supposed a simple classical vibratory system with translational motion, simulated as a single degree of freedom system. It was additionally considered that the main parameters characterizing inertia, dissipation and rigidity were affected by some uncertainties during the system evolution under the external dynamic load. The paper presents a method and a computational tool being able to provide useful information regarding the possible response of this system subjected by external harmonic loads. The results were comparatively analyzed with those obtained by the method of extremes identification of variation interval.

KEYWORDS: vibratory system, parameters uncertainties, dynamic response

1. INTRODUCTION

Modeling and simulation, in the area of mechanical systems, are very presented nowadays. Furthermore, this is known as computational dynamics, that consists by a group of methods and procedures, theoretically and scientifically proved, being able to provide useful tools for identify, modeling, simulate, analyze, and characterization of the specifically dynamic behavior of technical systems under various external and/or internal perturbations.

Vibratory systems can be modeled as single/multi degree of freedom lumped mass systems or, on the other hand, as a continuum mass system. For the models framed by the first category, the specific parameters that can characterize the system intrinsic properties are the mass (providing inertial characteristic), the damping (providing dissipation characteristic), and the rigidity (providing conservative characteristic).

Taking into account the known mechanical systems it can be assumed that some or all of these parameters may be affected by certain influences, which change, in a variable proportion, their initial values. For example, both the environmental temperature, and the aging, can affect the damping and the rigidity of the passive vibration isolators based on elastomeric materials. Changes of mass parameter can be occurred at belt-based transporters of granular non-cohesive materials.

Within such conditions, the evaluation of operational dynamic behaviour of these systems has to consider the potential range of variation for each parameter involved to the computational model. Thus, the necessity of an uncertainty analysis obviously results.

This study started from a single degree of freedom vibratory system, with lumped mass. It was supposed that each specific parameter was randomly affected by external factors. It was simulated and analyzed the system response based on two separated methods, within certain initial conditions and hypotheses, detailed into the second paragraph of the paper. The results were presented, discussed and comparative analyzed into the third paragraph. Paper also contains a conclusion and future direction paragraph.

2. PROBLEM BASICS AND WORK HYPOTHESES

The diagram of the proposed working model has a schematic representation within Fig. 1. Symbols on picture mean as follows: m, c, k respectively denote the mass, the damping coefficient and the rigidity, *F(t)* is the external dynamic perturbation, and *x(t)* is the system response (in terms of displacement). Dashed double arrows denote the fact that those parameters acquire uncertain values (but into a restricted or prior known range) during the operational cycle.

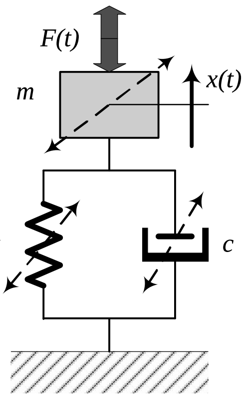

Fig.1. Schematic diagram of the SDoF model (see test for details)

It was initially stated that the external perturbation *F(t)* has a harmonic expression, charaterized by the force magnitude F_o and the pulsation ω

$$F(t) = F_o \sin(\omega\, t) \qquad (1)$$

The differential dynamic equation of the model in Fig. 1, ignoring the uncertainties of specific parameters, is

$$m\frac{d^2x(t)}{dt^2} + c\frac{d\ x(t)}{dt} + k\ x(t) = F(t) \qquad (2)$$

or, normalized by the mass *m* and using the short doted notation for derivatives

$$\ddot{x} + 2n\,\dot{x} + p^2\ x = q \qquad (3)$$

where (2*n*) denotes damping factor, p^2 denotes squared natural pulsation of the system, and *q* denotes the mass normalized external dynamic excitation.

Applying the Laplace operator to the Eqn. (2), in respect to the null initial conditions, results

$$\left(m\ s^2 + c\ s + k\right) X = F^* \qquad (4)$$

so that becomes very easy to evaluate the transfer function *H*(*s*) of the system. Hereby

$$H(s) = \frac{X}{F^*} = \frac{1}{m\ s^2 + c\ s + k} \qquad (5)$$

where

$$\begin{Bmatrix} X \\ F^* \end{Bmatrix} = \begin{Bmatrix} X(s) \\ F^*(s) \end{Bmatrix} = L\begin{Bmatrix} x(t) \\ F(t) \end{Bmatrix} = \int_{-\infty}^{\infty} \begin{Bmatrix} x(t) \\ F(t) \end{Bmatrix} e^{-st} dt \qquad (6)$$

and *s* denotes the complex variable $s = \sigma + j\omega$.

The evaluations of the system response were performed taking into account a few hypotheses. It was supposed that the changes of specific parameters does not have quick evolutions relative to the natural frequency of the system or the perturbation frequency. Thus, it does not involve a continuously transitory regime on system behaviour. It was adopted a computational example based on experimental setup of an SDoF system. This is characterized by the following values: *m* = 1 kg, *c* = 0.5 Ns/m, and *k* = 480 N/m. Hereby, the natural pulsation of the system, without damping, results *p* = 21.9089 rad/s and, respectively, the natural frequency is *f* = 3.4869 Hz. Taking into account the damping coefficient, the values of natural pulsations and frequency become *p* = 21.9075 rad/s and *f* = 3.4867 Hz respectively. Due to the main goal of this study, it was supposed that each parameter acquires ±20% changes of their average value.

3. RESULTS AND DISCUSSIONS

At this moment, it is possible to evaluate the ranges of each specific parameter as follows: $m \in [0.8 \quad 1.2]$ kg, $c \in [0.40 \quad 0.60]$ Ns/m and $k \in [384 \quad 576]$ N/m.

Supposing that the changes affect one single parameter for each computational test, the evaluation of the system response becomes very simple. But, in the case of simultaneously changes on two or three parameters, the evaluations become difficult. Taking into account the extreme and the average values of each parameter, the results of the evaluations were presented within Table 1 - in terms of natural pulsation [rad/s], respectively Table 2 - in terms of natural frequency [Hz].

Table 1. Discrete changes in natural pulsations

p for $\begin{Bmatrix} c_{\min} \\ c_{avg} \\ c_{\max} \end{Bmatrix}$	k_{min}	k_{avg}	k_{max}
m_{min}	21.9075 21.9067 21.9057	24.4936 24.4929 24.4920	26.8317 26.8310 26.8302
m_{avg}	19.5949 19.5943 19.5936	21.9080 21.9075 21.9068	23.9992 23.9987 23.9981
m_{max}	17.8878 17.8873 17.8868	19.9993 19.9989 19.9984	21.9083 21.9079 21.9075

Table 2. Discrete changes in natural frequencies

f for $\begin{Bmatrix} c_{\min} \\ c_{avg} \\ c_{\max} \end{Bmatrix}$	k_{min}	k_{avg}	k_{max}
m_{min}	3.4867 3.4866 3.4864	3.8983 3.8982 3.8980	4.2704 4.2703 4.2702
m_{avg}	3.1186 3.1185 3.1184	3.4868 3.4867 3.4866	3.8196 3.8195 3.8194
m_{max}	2.8469 2.8469 2.8468	3.1830 3.1829 3.1829	3.4868 3.4868 3.4867

Supposing the pseudo-pulsation of the basic unperturbed system (p = 21.9075 rad/s) and values in Table 1, obviously result the percentage changes of this parameter due to the uncertain within the system parameters. These values were presented in Table 3.

Table 3. Percentage changes in natural pulsations relative to the natural pseudo-pulsation of the basic system

$\frac{p}{p_o}$ [%] for $\begin{Bmatrix} c_{\min} \\ c_{avg} \\ c_{\max} \end{Bmatrix}$	k_{min}	k_{avg}	k_{max}
m_{min}	100 100 100	112 112 112	122 122 122
m_{avg}	89 89 89	100 100 100	110 110 110
m_{max}	82 82 82	91 91 91	100 100 100

Using the expression of transfer function, Eq.(5), and the facilities provided by the *Uncertain System Representation* within *Robust Control Toolbox* in Matlab©, it can be easily evaluate the system response (in terms of magnitude and phase of the pondered response) for the proposed dynamic model affected by uncertainties of main parameters.

The analyses were conducted step by step, successively considering one single uncertain parameter. The results were evaluated in terms of Bode Diagram for the transfer function. These were presented in Figure 2 – for uncertain mass parameter, Figure 3 – for uncertain damping parameter, and Figure 4 – for uncertain rigidity parameter.

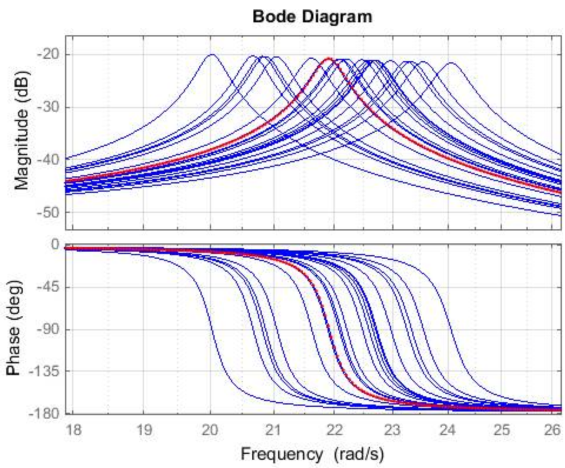

Fig.2. The Bode diagram of the transfer function for the case of mass affected by uncertainties. Red dotted line denotes the unaffected system

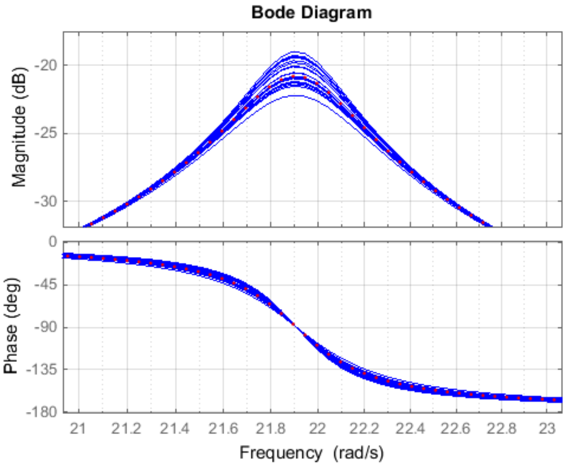

Fig.3. The Bode diagram of the transfer function for the case of damping affected by uncertainties. Red dotted line denotes the unaffected system

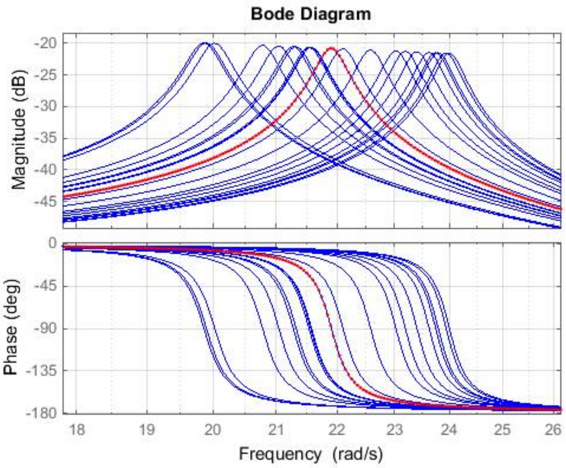

Fig.4. The Bode diagram of the transfer function for the case of rigidity affected by uncertainties. Red dotted line denotes the unaffected system

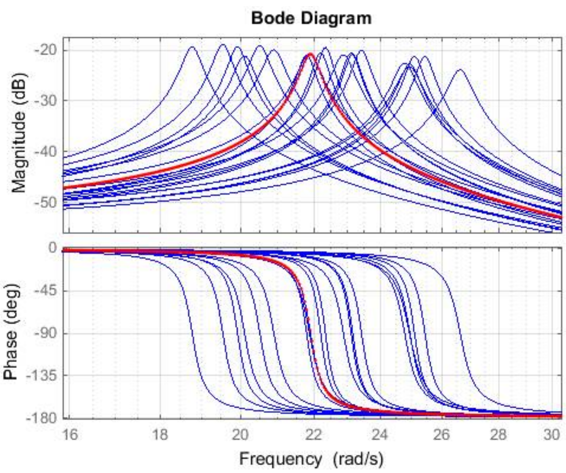

Fig.5. The Bode diagram of the transfer function for the case of all parameters affected by uncertainties. Red dotted line denotes the unaffected system

In addition, it was analyzed the case of the three parameters acquiring uncertain values within the initial provided ranges. The Bode Diagram for this case was depicted in Figure 5.

It has to be mentioned that, for each case within Figures 2...5, it was separately evaluated the situation of unaffected system and the correspondent results were depicted, in magnitude and phase terms respectively, using the red dotted line (all other results were depicted using blue continuous line). The graphs in Figures 2...5 were zoomed onto the frequency range of interest, thus that the analysis of characteristics changes can be facilely performed.

It has to taken into account that the computational analyses were done using randomly evaluations for the parameters of interest, which obviously acquire different values at each running step. Hereby, the histograms in Figure 6 present the linear distribution of the uncertain parameters *m*, *c* and *k*, grouped into 20 bins within the whole range of ±20% variation of the initial settled values. This is the configuration of the values used in Bode Diagrams evaluations presented in Figures 2...5.

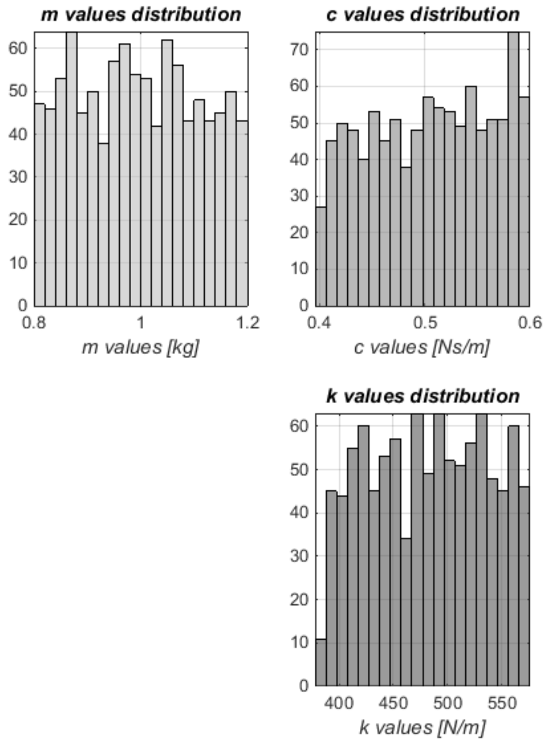

Fig.6. Histograms of values distribution for the mass *m*, damping *c* and rigidity *k* respectively

4. CONCLUSIONS

Comparative analysis between the discrete changes of pulsation, evaluated based on extreme values and presented in Table 1, and, respectively, the computational evaluations based on diagrams in Figures 2...5, reveals following aspects in terms of natural pulsation.

For mass uncertainties, the variation of pulsation yields 20...24.5 rad/s – for discrete evaluation, comparative to 20...24.1 rad/s – for computational evaluation.

For rigidity uncertainties, the variation of pulsation yields 19.6...24 rad/s – for discrete evaluation, comparative to 19.8...24 rad/s – for computational evaluation.

In case of damping uncertainties, the variation of pulsation obviously acquires very smooth changes, around the value of 21.9 rad/s for both types of evaluation (see the value of the pseudo-pulsation p = 21.9075 rad/s, for the initial unaffected system, previously presented in Paragraph 2).

Regarding the case of uncertainties influence on the whole set of parameters, the variation of pulsation yields 17.89...26.83 rad/s – for discrete evaluation, in the same time with 18.75...26.55 rad/s – for computational evaluation.

It is evident that the domains resulted by discrete evaluations are rather widely comparative to the computational method. However, it has to be consider that the differences are small, and the discrete method supposes only the extreme and average values of each interval, comparative to the second method, which involves a large number of bins. Hereby, clearly results the advantages of computational method for evaluation of the system characteristics. Nevertheless, this last method additionally facilitates the future evaluations such as impulse or step responses, Nyquist-based stability analysis, gaining the pole-zeros map.

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