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A GENERAL FIXED POINT THEOREM FOR MAPPINGS SATISFYING AN IMPLICIT RELATION IN COMPLETE *Gp* - METRIC SPACES

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Abstract
The purpose of this paper is to prove a general fixed point theorem in complete *Gp* - metric spaces for mappings satisfying a new type of implicit relation. If *Gp* - metric is symmetric, we prove that the fixed point problem is well posed.

Keywords: fixed point, *Gp* - metric space, well posedness of fixed point problem, implicit relation.

1. INTRODUCTION

In [13], [14], Dhage introduced a new class of generalized metric spaces, named *D* - metric spaces. Mustafa and Sims [21], [22] proved that most of the claims concerning the fundamental topological structures on *D* - metric spaces are incorrect and introduced an appropriate notion of generalized metric space, named *G* - metric spaces. In fact, Mustafa, Sims and other authors [1], [17], [20] - [27], [38], [39] studied many fixed point results for self mappings in *G* - metric spaces under certain conditions.

In 1994, Matthews [19] introduced the concept of partial metric space as a part of the study of denotional semantics of dataflows and proved the Banach contraction principle in such spaces. Recently, in [2], [7], [11], [15], [16] and in other papers, some fixed point theorems under various contractive conditions in partial metric spaces are proved.

Quite recently, Zand and Nezhad [45] introduced a generalization and unification of *G* - metric space and partial metric space, named *Gp* - metric space. In [8], some fixed point theorems in *Gp* - metric spaces are proved. Other results are obtained in [9] and [10].

The notion of well posedness of fixed point problem generated more interest to several mathematicians, for example [12], [18], [37].

Several classical fixed point theorems and common fixed point theorems have been unified considering a general condition by an implicit relation in [28], [29] and in other papers. Recently, the method is used in the study of fixed points in metric spaces, symmetric spaces, quasi - metric spaces, *b* - metric spaces, ultra - metric spaces, Hilbert spaces, convex metric spaces, reflexive spaces, compact metric spaces, paracompact metric spaces, in two and three metric spaces, for single - valued mappings, hybrid pairs of mappings and set - valued mappings. Recently, the method has been used in the study of fixed points for mappings satisfying contractive/extensive conditions of integral type, in fuzzy metric spaces, probabilistic metric spaces and intuitionistic metric spaces. Also, the method allows the study of local and global properties of fixed point structures.

The study of fixed points for mappings in G - metric spaces satisfying implicit relations is initiated in [32] - [36] and in other papers. The study of fixed points for mappings satisfying an implicit relation in partial metric spaces is initiated in [44].

In [3] - [6], [30], [31] and in other papers, the authors studied well posedness of fixed point problem for mappings satisfying implicit relations.

The purpose of this paper is to prove a general fixed point theorem for mappings satisfying a new type of implicit relation in the *Gp* - metric spaces. We prove that for these mappings, if the *Gp* - metric is symmetric, then the fixed point problem is well posed.

2. PRELIMINARIES

Definition 1 ([45]) *Let X be a nonempty set. A function $G: X^3 \to \mathbb{R}_+$ is called a Gp - metric on X if the following conditions are satisfied:*

(Gp_1): $x = y = z$ *if* $Gp(x, y, z) = Gp(x, x, x) = Gp(y, y, y) = Gp(z, z, z)$;

(Gp_2): $0 \le Gp(x, x, x) \le Gp(x, x, y) \le Gp(x, y, z)$ *for all* $x, y, z \in X$, *with* $y \ne z$;

(Gp_3): $Gp(x, y, z) = Gp(y, z, x) = \ldots$ *(symmetry in all three variables);*

(Gp_4): $Gp(x, y, z) \le Gp(x, a, a) + Gp(a, y, z) - Gp(a, a, a)$ *for all* $x, y, z, a \in X$.

The pair (X, Gp) is called a Gp - metric space.

Definition 2 ([45]) *Let (X, Gp) be a Gp - metric space and $\{x_n\}$ be a sequence in X. A point $x \in X$ is said to be the limit of the sequence $\{x_n\}$ (we write $x_n \to x$ and we say that $\{x_n\}$ is Gp - convergent to x) if $\lim_{m,n \to \infty} Gp(x, x_n, x_m) = Gp(x, x, x)$.*

Theorem 3 ([8]) *Let (X, Gp) be a Gp - metric space. Then, for any $\{x_n\} \in X$ and $x \in X$, the following conditions are equivalent:*

a) $\{x_n\}$ *is* Gp *- convergent to* x;

b) $Gp(x_n, x_n, x) \to Gp(x, x, x)$ *as* $n \to \infty$;

c) $Gp(x_n, x, x) \to Gp(x, x, x)$ *as* $n \to \infty$.

Definition 4 ([45]) *Let (X, Gp) be a Gp - metric space.*

1) A sequence $\{x_n\}$ of X is called a Gp - Cauchy sequence if $\lim_{n,m \to \infty} Gp(x_n, x_m, x_m)$ exists and is finite.

2) A Gp - metric space is said to be Gp - complete if and only if every Gp - Cauchy sequence in X Gp - converges to $x \in X$ such that $\lim_{n,m \to \infty} Gp(x_n, x_m, x_m) = Gp(x, x, x)$.

Lemma 5 ([8]) *Let (X, Gp) be a Gp - metric space.*

1) If $Gp(x, y, z) = 0$, then $x = y = z$.

2) If $x \ne y$ then $Gp(x, x, y) > 0$.

Definition 6 ([45]) *A Gp - metric on X is said to be symmetric if $Gp(x, y, y) = Gp(y, x, x)$.*

In this case, (X, Gp) is said to be symmetric.

Lemma 7 ([45]) *Let (X, Gp) be a Gp - metric space. Then $Gp(x, x, y) \le 2Gp(x, y, y)$ for all $x, y \in X$.*

Lemma 8 ([8]) *Let (X, Gp) be a Gp - metric space and $\{x_n\}$ a sequence in X. Assume that $\{x_n\}$ is Gp - convergent to a point $x \in X$ with $Gp(x, x, x) = 0$. Then $\lim_{n \to \infty} Gp(x_n, y, y) = Gp(x, y, y)$ for all $y \in X$. Moreover, $\lim_{n,m \to \infty} Gp(x_n, x_m, x) = 0$.*

Lemma 9 *Let (X, Gp) be a Gp - metric space and $\{x_n\}$ is Gp - convergent to a point $x \in X$ with $Gp(x, x, x) = 0$. Then $\lim_{n \to \infty} Gp(x_n, x_n, y) = Gp(x, x, y)$ for all $y \in X$.*

Proof. By (Gp_4),

$$Gp(y,x,x) \le Gp(y,x_n,x_n) + Gp(x_n,x,x).$$

Hence,

$$Gp(y,x,x) - Gp(x_n,x,x) \le Gp(y,x_n,x_n) \le Gp(y,x,x) + Gp(x,x_n,x_n).$$

Letting n tends to infinity, by Theorem 3 we obtain

$$Gp(y,x,x) \le \lim_{n\to\infty} Gp(y,x_n,x_n) \le Gp(y,x,x).$$

Hence,

$$\lim_{n\to\infty} Gp(y,x_n,x_n) = Gp(y,x,x).$$

3. IMPLICIT RELATIONS

Let $\mathcal{F}_{Gp}$ be the set of all continuous functions $F:\mathbb{R}_+^6 \to \mathbb{R}$ satisfying

(F_1): F is non - increasing in variables t_3, t_4, t_5, t_6;

(F_2): There exists $h_1 \in [0,1)$ such that for all $u,v \ge 0$, $F(u,v,v,u,u+v,u) \le 0$ implies $u \le h_1 v$;

(F_3): There exists $h_2 \in [0,1)$ such that for all $t,t' \ge 0$, $F(t,t,t,t',t,t') \le 0$ implies $t \le h_2 t'$.

In the following examples, property (F_1) is obviously.

Example 10 $F(t_1,...,t_6) = t_1 - a_1t_2 - a_2t_3 - a_3t_4 - a_4t_5 - a_5t_6$, *where* $a_1,...,a_5 \ge 0$, a_3 *or* $a_5 \ne 0$ *and* $a_1 + a_2 + a_3 + 2a_4 + a_5 < 1$.

(F_2): *Let* $u,v \ge 0$ *and* $F(u,v,v,u,u+v,u) = u - a_1v - a_2v - a_3u - a_4(u+v) - a_5u \le 0$, *which implies* $u \le h_1 v$, *where* $0 \le h_1 = \dfrac{a_1 + a_2 + a_4}{1-(a_3 + a_4 + a_5)} < 1$.

(F_3): *Let* $t,t' \ge 0$ *and* $F(t,t,t,t',t,t') = t - a_1t - a_2t - a_3t' - a_4t - a_5t' \le 0$, *which implies* $t \le h_2 t'$, *where* $0 < h_2 = \dfrac{a_3 + a_5}{1-(a_1 + a_2 + a_4)} < 1$.

Example 11 $F(t_1,...,t_6) = t_1 - k\max\{t_2,t_3,t_4,t_5,t_6\}$, *where* $k \in \left[0,\dfrac{1}{2}\right)$.

(F_2): *Let* $u,v \ge 0$ *and* $F(u,v,v,u,u+v,u) = u - k\max\{u,v,u+v\} = u - k(u+v) \le 0$, *which implies* $u \le h_1 v$, *where* $0 \le h_1 = \dfrac{k}{1-k} < 1$.

(F_3): *Let* $t,t' \ge 0$ *and* $F(t,t,t,t',t,t') = t - k\max\{t,t'\} \le 0$. *If* $t > t'$, then $t(1-k) \le 0$, a contradiction. Hence, $t \le t'$, *which implies* $t \le h_2 t'$, *where* $0 < h_2 = k < 1$.

In the following, the proofs are similar and thus omitted.

Example 12 $F(t_1,...,t_6) = t_1^2 - t_1(at_2 + bt_3 + ct_4) - dt_5t_6$, *where* $a,b,c,d \ge 0$ *and* $a + b + c + 2d < 1$

Example 13 $F(t_1,...,t_6) = t_1^2 - a\max\{t_2^2, t_3^2, t_4^2\} - bt_5t_6$, *where* $a,b \ge 0$ *and* $a + 2b < 1$.

Example 14 $F(t_1,...,t_6) = t_1^3 - at_1^2t_2 - bt_1t_2^2 - ct_2t_3t_4 - dt_1t_5t_6$, *where* $a,b \ge 0$, $c,d > 0$ *and* $a + b + c + 2d < 1$.

Example 15 $F(t_1,...,t_6) = t_1 - at_2 - bt_3 - c\max\{2t_4, t_5 + t_6\}$, *where* $a > 0, b,c \ge 0$ *and* $a + b + 3c < 1$.

Example 16 $F(t_1,...,t_6)=t_1^2+\dfrac{t_1}{1+t_5+t_6}-(at_2^2+bt_3^2+ct_4^2)$, *where* $a,b\geq 0, c>0$ *and* $a+b+c<1$.

4. MAIN RESULTS

Theorem 17 *Let* (X,Gp) *be a* Gp *- metric space and* $T:X\to X$ *be a mapping such that:*

$$F(Gp(Tx,Tx,Ty),Gp(x,x,y),Gp(x,x,Tx),Gp(y,y,Ty),Gp(x,x,Ty),Gp(y,y,Tx))\leq 0 \qquad (1)$$

for all $x,y\in X$ *, where* F *satisfy property* (F_3)*. Then,* T *has at most a fixed point.*

Proof. Suppose that T has two distinct fixed points u and v. By (1) we have successively

$$F(Gp(Tu,Tu,Tv),Gp(u,u,v),Gp(u,u,Tu),Gp(v,v,Tv),Gp(u,u,Tv),Gp(v,v,Tu))\leq 0, \qquad (2)$$

$$F(Gp(u,u,v),Gp(u,u,v),Gp(u,u,u),Gp(v,v,v),Gp(u,u,v),Gp(v,v,u))\leq 0. \qquad (3)$$

By (Gp_2),

$$Gp(u,u,u)\leq Gp(u,u,v), \qquad (4)$$

$$Gp(v,v,v)\leq Gp(v,v,u). \qquad (5)$$

Then, by (F_1) we obtain

$$F(Gp(u,u,v),Gp(u,u,v),Gp(u,u,v),Gp(v,v,u),Gp(u,u,v),Gp(v,v,u))\leq 0, \qquad (6)$$

which implies by (F_3) that

$$Gp(u,u,v)\leq h_2 Gp(v,v,u). \qquad (7)$$

Similarly, by (1) we obtain

$$Gp(v,v,u)\leq h_2 Gp(u,u,v). \qquad (8)$$

Hence,

$$Gp(u,u,v)\leq h_2^2 Gp(u,u,v), \qquad (9)$$

which implies

$$Gp(u,u,v)(1-h_2^2)\leq 0. \quad (10)$$

Then,

$$Gp(u,u,v)=0$$

and by Lemma 5,

$$u=v.$$

Theorem 18 *Let* (X,Gp) *be a* Gp *- complete metric space and let* $T:X\to X$ *satisfying inequality (1), for all* $x,y\in X$ *, where* $F\in\mathcal{F}_{Gp}$*. Then,* T *has an unique fixed point.*

Proof. From (1), for $y=Tx$, we get

$$F(Gp(Tx,Tx,T^2x),Gp(x,x,Tx),Gp(x,x,Tx),Gp(Tx,Tx,T^2x),Gp(x,x,T^2x),Gp(Tx,Tx,Tx))\leq 0. \qquad (11)$$

By (Gp_4), we have

$$Gp(x,x,T^2x)\leq Gp(x,x,Tx)+Gp(Tx,Tx,T^2x)) \qquad (12)$$

and by (Gp_2), we have

$$Gp(Tx,Tx,Tx)\leq Gp(Tx,Tx,T^2x)). \qquad (13)$$

Then by (F_1) we obtain

$$F\left(\begin{array}{c}Gp(Tx,Tx,T^2x),Gp(x,x,Tx),Gp(x,x,Tx),Gp(Tx,Tx,T^2x),\\ Gp(x,x,Tx)+Gp(Tx,Tx,T^2x),Gp(Tx,Tx,T^2x)\end{array}\right)\leq 0, \qquad (14)$$

which implies by (F_2) that

$$Gp(Tx,Tx,T^2x)\leq h_1 Gp(x,x,Tx). \qquad (15)$$

Let $x_0 \in X$ be an arbitrary point and $x_n = Tx_{n-1}, n = 1,2,...$.

If $x_n = x_{n-1}$, then x_n is a fixed point of T. Suppose that $x_n \neq x_{n-1}$, for each $n \in \mathbb{N}$. Then by (15) we obtain

$$Gp(Tx_{n-1}, Tx_{n-1}, Tx_n) \leq h_1 Gp(x_{n-1}, x_{n-1}, Tx_{n-1}), \quad (16)$$

i.e.,

$$Gp(x_n, x_n, x_{n+1}) \leq h_1 Gp(x_{n-1}, x_{n-1}, Tx_{n-1}), \quad (17)$$

which implies

$$Gp(x_n, x_n, x_{n+1}) \leq h_1^n Gp(x_0, x_0, x_1). \quad (18)$$

Moreover, for $m, n \in \mathbb{N}$, $m > n$, from repeating use of (Gp_4) we obtain

$$Gp(x_n, x_n, x_m) \leq \sum_{j=n}^{m-1} h_1^j Gp(x_0, x_0, x_1) \leq \frac{h_1^n}{1-h_1^n} Gp(x_0, x_0, x_1). \quad (19)$$

By Lemma 7,

$$Gp(x_n, x_n, x_m) \leq 2Gp(x_n, x_n, x_m) \leq \frac{h_1^n}{1-h_1^n} Gp(x_0, x_0, x_1). \quad (20)$$

Consequently,

$$\lim_{n,m\to\infty} Gp(x_n, x_n, x_m) = 0 \quad (21)$$

and thus, $\{x_n\}$ is a Gp - Cauchy sequence. Since (X, Gp) is a Gp - complete metric space and by Theorem 3, there exists $z \in X$ such that

$$\lim_{n,m\to\infty} Gp(x_n, x_n, x_m) = \lim_{n\to\infty} Gp(z, x_n, x_n) = Gp(z, z, z) = 0 \quad (22)$$

we prove that z is a fixed point of T.

By (1) we have successively

$$F(Gp(Tx_n, Tx_n, Tz), Gp(x_n, x_n, z), Gp(x_n, x_n, Tx_n), Gp(z, z, Tz), Gp(x_n, x_n, Tz), Gp(z, z, Tx_n)) \leq 0, \quad (23)$$

$$F(Gp(x_{n+1}, x_{n+1}, Tz), Gp(x_n, x_n, z), Gp(x_n, x_n, x_{n+1}), Gp(z, z, Tz), Gp(x_n, x_n, Tz), Gp(z, z, x_{n+1})) \leq 0. \quad (24)$$

By Lemmas 8 and 9, Theorem 3 and (22), letting n tends to infinity we obtain

$$F(Gp(z, z, Tz), 0, 0, Gp(z, z, Tz), Gp(z, z, Tz), Gp(z, z, Tz)) \leq 0, \quad (26)$$

which implies by (F_2) that $Gp(z, z, Tz) = 0$. By Lemma 5, $z = Tz$. Hence, z is a fixed point of T. By Theorem 17, z is the unique fixed point of T.

From Theorem 18 and Example 11 we obtain

Corollary 19 *Let (X, Gp) be a Gp - complete metric space and $T : X \to X$ be a mapping such that*

$$Gp(Tx, Tx, Ty) \leq k \max\{Gp(x, x, y), Gp(x, x, Tx), Gp(y, y, Ty), Gp(x, x, Ty), Gp(y, y, Tx)\} \quad (27)$$

where $k \in \left[0, \frac{1}{2}\right)$, for all $x, y \in X$. Then T has an unique fixed point.

From Theorem 18 and Examples 10, 12-16 we obtain other results.

Example 20 *Let* $X = [0, \infty)$ and $Gp : X^3 \to \mathbb{R}$ *defined by* $Gp(x, y, z) = \max\{x, y, z\}$. *Then* (X, Gp) *is a* Gp *- complete metric space. Let* $T : X \to X$ *be defined by* $Tx = \frac{x}{x+3}$. *Without loss of generality, we assume that* $x \geq y$. *Then*

$$Gp(Tx, Tx, Ty) = \frac{x}{x+3} \leq \frac{x}{3} = \frac{1}{3} Gp(x, x, y) \leq kGp(x, x, y), \quad (28)$$

where $k \in \left[\frac{1}{3}, \frac{1}{2}\right)$ *which implies*

$$Gp(Tx, Tx, Ty) \leq k \max\{Gp(x, x, y), Gp(x, x, Tx), Gp(y, y, Ty), Gp(x, x, Ty), Gp(y, y, Tx)\}. \quad (29)$$

By Corollary 19, T *has an unique fixed point* $x = 0$. *Moreover,* $Gp(0,0,0) = 0$.

5. WELL POSEDNESS PROBLEM OF FIXED POINT IN *Gp* - METRIC SPACES

Definition 21 ([37]) *Let (X,d) be a metric space and $f:(X,d)\to(X,d)$ be a mapping. The fixed point problem of f is said to be well posed if:*

1) f has an unique fixed point x_0;

2) for any sequence $\{x_n\}\in X$ with $\lim_{n\to\infty} d(x_n, fx_n)=0$ we have $\lim_{n\to\infty} d(x_n,x_0)=0$.

Definition 22 *Let (X,Gp) be a Gp - metric space and let $T:X\to X$ be a self mapping. The fixed point problem of T is said to be well posed if:*

1) T has an unique fixed point x_0;

2) for any sequence $\{x_n\}\in X$ with $\lim_{n\to\infty} Gp(x_n,x_n,Tx_n)=0$ we have $\lim_{n\to\infty} Gp(x_0,x_0,x_n)=0$.

Definition 23 *A function $F:\mathbb{R}_+^6\to\mathbb{R}$ has property (F_q) if for $u,v,w\ge 0$ and $F(u,v,0,w,u,v)\le 0$, there exists $q\in(0,1)$ such that $u\le q\max\{v,w\}$.*

Remark 24 *All the functions F from Examples 10 - 16 have property (F_q).*

Theorem 25 *Let (X,Gp) be a Gp - symmetric space and let $T:X\to X$ be a mapping satisfying the conditions from Theorem 18 and F having property (F_q). Then the fixed point problem of T is well posed.*

Proof. By Theorem 18, T has an unique fixed point x_0. Let $\{x_n\}$ be a sequence in X such that $\lim_{n\to\infty} Gp(x_n,x_n,Tx_n)=0$. By (1) we have successively

$$F\begin{pmatrix} Gp(Tx_0,Tx_0,Tx_n), Gp(x_0,x_0,x_n), Gp(x_0,x_0,Tx_0), \\ Gp(x_n,x_n,Tx_n), Gp(x_0,x_0,Tx_n), Gp(x_n,x_n,Tx_0) \end{pmatrix}\le 0, \qquad (30)$$

$$F(Gp(x_0,x_0,Tx_n), Gp(x_0,x_0,x_n), 0, Gp(x_n,x_n,Tx_n), Gp(x_0,x_0,Tx_n), Gp(x_n,x_n,x_0))\le 0. \quad (31)$$

Since Gp is symmetric, $Gp(x_n,x_n,x_0)=Gp(x_0,x_0,x_n)$, hence

$$F(Gp(x_0,x_0,Tx_n), Gp(x_0,x_0,x_n), 0, Gp(x_n,x_n,Tx_n), Gp(x_0,x_0,Tx_n), Gp(x_0,x_0,x_n))\le 0. \quad (32)$$

Since F satisfy property (F_q) we have

$$\begin{aligned} Gp(x_0,x_0,Tx_n) &\le q\max\{Gp(x_0,x_0,x_n), Gp(x_n,x_n,Tx_n)\} \\ &\le q[Gp(x_0,x_0,x_n)+Gp(x_n,x_n,Tx_n)]. \end{aligned}$$

By (Gp_4),

$$\begin{aligned} Gp(x_0,x_0,x_n) &\le Gp(x_0,Tx_n,Tx_n)+Gp(Tx_n,x_0,x_0)\} \\ &\le Gp(x_n,Tx_n,Tx_n)+q[Gp(x_0,x_0,Tx_n)+Gp(x_n,x_n,Tx_n)], \end{aligned}$$

which implies

$$Gp(x_0,x_0,x_n)\le \frac{1+q}{1-q} Gp(x_n,x_n,,Tx_n). \qquad (33)$$

Hence,

$$\lim_{n\to\infty} Gp(x_0,x_0,x_n)=0 \qquad (34)$$

and the fixed point problem of T is well posed.

Corollary 26 *Let (X,Gp) be a Gp - symmetric space and let $T:X\to X$ be a function satisfying the conditions of Corollary 19. Then, the fixed point problem of T is well posed.*

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