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Is there any rational transformation for the relativistic force?

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Abstract
A simple thought experiment is carried out through which it is shown that the accepted Lorentz transformation for force results in irrational anomalies in the transverse direction. A spring, in its equilibrium state, is set in motion and considered to pass under a contracted spring located in the lab frame of reference. Applying the traditional Lorentz transformation, it is demonstrated that the final lengths of the springs, as they meet each other, are measured differently from the viewpoint of two inertial observers.

Keywords: Special relativity; Reference frames; Force; Spring; Spring constant; Paradox; Transverse direction

1. INTRODUCTION

Finding a comprehensive definition of force in the special relativity theory (SRT) mainly began with the *Trouton-Noble experiment*, [1,2] and the thought experiment known as the *Right-angle lever* or *Lewis-Tolman paradox*. [3] So far, too many resolutions to these problems have been proposed all of which seemingly validate the principle of relativity and the absence of any absolute rest frame. [4-6] The by-product of these attempts is the Lorentz transformation for the force which, in a specific case, asserts that transverse force vectors acting on a moving object are reduced, whereas the longitudinal ones are left unchanged as measured in the lab frame. [7,8] Besides, as it is interpreted from the Galilean postulate of relativity, length intervals perpendicular to the direction of travel are left unchanged as measured in both the rest and lab frames of reference.

In this article, we investigate the behavior of two simple springs by applying the traditional Lorentz transformation for force from the viewpoint of inertial observers away from any gravitational field. Indeed, we show that these observers disagree in measuring a displacement perpendicular to the motion, which is, oddly enough, contrary to the first postulate of relativity as well as the Lorentz transformation in the relativistic kinematics. In fact, it is shown that there is an inconsistency between dynamics and kinematics in relativity.

2. THE SPRING PARADOX

Assume a very long vehicle, equipped with a thin piston (in a cylinder) and a simple spring S_1 in its equilibrium state, starts a high-speed travel at a constant velocity v relative to the lab frame. The body of the vehicle is pressed by a contracted spring S_2 hanging from the ceiling of the laboratory. The spring S_2 has a tiny wheel that can easily slide/roll over the frictionless body of the vehicle. [See Fig. 1-(a).] Both S_1 & S_2 have a similar spring constant k_0 and a free length y_0 in their rest frames. Inasmuch as all surfaces are frictionless, the vehicle can easily pass under S_2 notwithstanding its downward force that presses the vehicle to the ground. Although the springs have similar constants in their rest frames, it is assumed that each spring constant, due to the reasons stated further on in the text, is measured differently by the observer who detects a relative velocity for the spring. Now we tend to calculate the displacement of S_1 as measured by the observers in the lab and the vehicle’s rest frames:

In the Lab Frame:

According to Fig. 1-(b&c) and using Hooke’s law for springs, we have:

$$F_1' = F_2 \to k_0' \Delta y_1' = k_0 \Delta y_2 . \tag{1}$$

Because S_2 had contracted prior to reaching the piston, according to Fig. 1-(c), the displacement of S_2 is calculated to be:

$$d_0 - y_0 + \Delta y_1' = y_0 - \Delta y_2 \to \Delta y_2 = 2y_0 - d_0 - \Delta y_1' . \tag{2}$$

Substituting Eq. (2) into Eq. (1), we get:

$$\Delta y_1' = \frac{k_0}{k_0' + k_0}\left(2y_0 - d_0\right) . \tag{3}$$

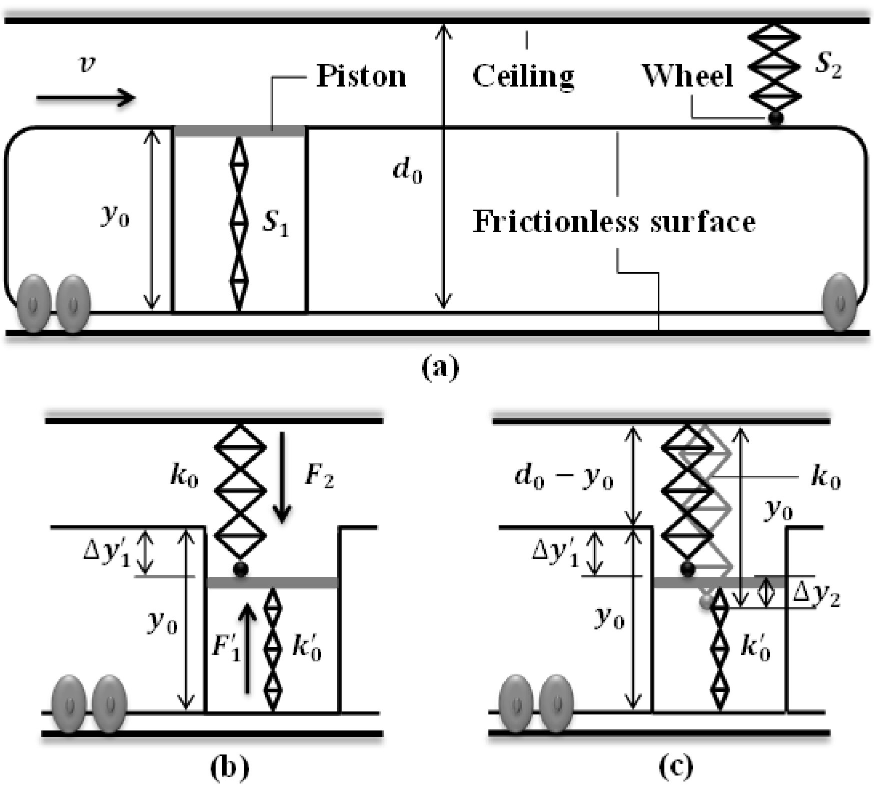

Fig. 1. The vehicle and springs as viewed by the observer in the lab frame. (a) The contracted spring S_2 moves over the frictionless body of the vehicle. (b) As S_2 reaches the piston, it exerts a great force of F_2, whereas S_1 can exert a small force of F_1' upward. Therefore, S_2 is stretched out, and when $F_2 = F_1'$, the final displacement of S_1 is $\Delta y_1'$. (c) With respect to the fact that S_2 had already contracted, its displacement is calculated to be Δy_2. The gray spring shows S_2 with a free length of y_0.

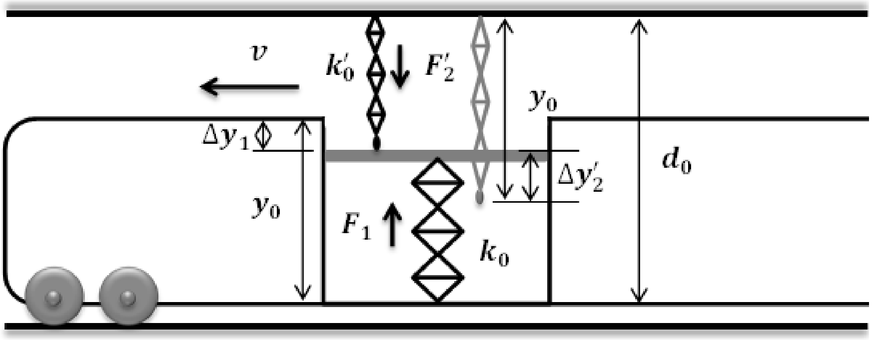

Fig. 2. The vehicle and springs as viewed by the observer in the vehicle's rest frame. When S_2 reaches the piston, it exerts a small force of F_2', whereas S_1 is capable of exerting a great force of F_1 upward for small vertical displacements. Therefore, S_2 is slightly stretched out, and when $F_2' = F_1$, the final displacement of S_1 is Δy_1. With respect to the fact that S_2 had already contracted, its displacement is calculated to be $\Delta y_2'$. The gray spring shows S_2 with a free length of y_0.

Recall that d_0 is the distance between the ceiling and the lower surface of the vehicle, and $y_0 < d_0 < 2y_0$.

In the Vehicle's Rest Frame:

According to Fig. 2, the displacement of S_1 can similarly be calculated by changing $F_2 \to F_2'$, $F_1' \to F_1$, $k_0 \to k_0'$, $\Delta y_1' \to \Delta y_1$, $k_0' \to k_0$. Therefore, repeating the calculations above, we obtain:

$$\Delta y_1 = \frac{k_0'}{k_0 + k_0'}\left(2y_0 - d_0\right). \tag{4}$$

If relativity excludes any paradox, it is anticipated that $\Delta y_1' = \Delta y_1$ and $\Delta y_2' = \Delta y_2$. This deduction is due to the fact that, according to SRT, lengths in the transverse direction – direction perpendicular to the velocity vector, that is – are left unchanged from the viewpoint of the moving observer. Nonetheless, the traditional Lorentz transformation for force predicts that the transverse force vectors acting on a moving object (our vehicle) are measured smaller in the lab frame. [7,9] This means that the force exerted by S_1 in the vehicle's rest frame (F_1) is reduced as measured by the lab observer (F_1'), and we have:

$$F_1' = \alpha_v F_1, \tag{5}$$

where $\alpha_v = \sqrt{1 - v^2/c^2}$. Moreover, based on the symmetry in SRT, the observer in the vehicle's rest frame can claim that the same happens for S_2, i.e., the force exerted by S_2 has a smaller magnitude measured by the observer in the vehicle's rest frame (F_2') compared to that measured by the lab observer (F_2), and we have:

$$F_2' = \alpha_v F_2. \tag{6}$$

Considering Eq. (5) together with $\Delta y_1' = \Delta y_1$, we get:

$$\begin{cases} F_1' = \alpha_v F_1 \\ \Delta y_1' = \Delta y_1 \end{cases} \xrightarrow{F_1' = k_0' \Delta y_1' \ \& \ F_1 = k_0 \Delta y_1} k_0' = \alpha_v k_0. \tag{7}$$

Moreover, one can use Eq. (6) along with $\Delta y_2' = \Delta y_2$ to re-obtain Eq. (7). Recall that this equation is compatible with the result obtained by O. Gron. [10] Therefore, if we substitute Eq. (7) into Eqs. (3&4), we can deduce:

$$\Delta y_1' = \frac{1}{1+\alpha_v}\left(2y_0 - d_0\right), \tag{8}$$

$$\Delta y_1 = \frac{\alpha_v}{1+\alpha_v}\left(2y_0 - d_0\right). \tag{9}$$

Or $\Delta y_1 \neq \Delta y_1'$, which is a paradoxical result with the reasoning stated earlier. Remember that we assumed that the maximum velocity of the piston or the wheel attached to S_2, when the piston is accelerated downward due to the force exerted by S_2, is small compared to the speed of light, and hence, no additional relativistic corrections are needed for the springs.

As a possible resolution to the paradox, however, one can claim that when S_2 meets the right edge of the piston, it takes time for the *force signals* to reach S_1 (or the midpoint of the piston), and thus this delay may upset our easy approach to the problem. It is worthwhile to mention that the delay of signals has nothing to do with our example. That is if we consider the piston as a Born-rigid body, the velocity of the force signals (light speed) is always greater than the velocity of the moving wheel, and the signals can reach S_1 prior to the time when S_2 reaches the other end (left edge) of the piston. For simplicity, one can imagine that the radius of the piston is infinitely large so that the Lorentz contraction at any arbitrary speed does not affect the radius length. In this case, from the viewpoint of both observers, there is considerable time for S_2 to act on the piston and the spring (S_1) beneath it.

For better perceiving this odd paradox, one can consider the velocity of the vehicle to be very close to that of light ($v \approx c$ or $\alpha_v \approx 0$), and k_0 to have a great value. In this case, S_2 falls into the cylinder and finally hits its left-hand interior side as seen by the lab observer, whereas the observer in the vehicle's rest frame claims that S_2 easily passes over the piston without affecting S_1 and survives falling into the cylinder. In order to prove, it suffices to use $\alpha_v = 0$ in Eqs. (8&9) and get $\Delta y_1' = 2y_0 - d_0$ & $\Delta y_1 = 0$.

Moreover, in the cases where the speed is a significant portion of the speed of light the experiment can be carried out in a way that the oscillatory situation of the system is negligible. That is to say, from the standpoint of the lab observer, S_2 is always *strong* enough to contract the *weak* S_1 (moving at $v \approx c$) into its minimum length (maximum displacement) with all its loops stuck onto each other. It suffices to assume that the final hitting of the springs occurs under *inelastic collision* conditions so that any oscillations are rapidly damped out and the interaction leads to a static situation. On the other hand, the moving observer, contrary to the lab observer, asserts that the *weak* S_2 cannot tangibly affect the *strong* S_1, but rather it can only make oscillations with infinitesimally small amplitudes near the surface of the vehicle. Therefore, the paradox is still valid. Furthermore, even in the cases where there are considerable oscillations, one can introduce some additional friction into the system which does not allow the system to remain in an oscillating state from the standpoints of both observers.

3. CONCLUSIONS

According to the special theory of relativity, length intervals are invariant perpendicular to the direction of motion. By applying the Lorentz transformation for force, indeed, we demonstrated the *incorrectness* of this statement, unless there is a fundamental amendment to the force transformation in special relativity!

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