

## ASSESSMENT OF THE INFLUENCE OF BREAKAGE STRENGTH ROCKS BY COX METHOD

Mihaela TODERAȘ, Ciprian DANCIU

University of Petroșani, Faculty of Mines, 20 Universitatii, Petrosani-332006, Romania  
e-mail: toderasmihaela@yahoo.com, danciu\_ciprian@yahoo.com

### ABSTRACT

*Analyzed according to the theory of continuous mechanics, medium rock massif is a natural environment difficult to know, so the forecasts referring to its behaviour are approximate and uncertain. Such an assertion becomes a certitude, even resulting from reality; consequences of the four fundamental features of massif are multiple and involve really difficult issues in mining field and generally, in the constructions field. The concept and the way to assess the stability of underground construction involves understanding its phenomenological evolution and doubtless the character of rocks behaviour in massif. Based on the principle scheme of rocks behaviour way to failure, we propose a statistical analysis method of results obtained by laboratory tests using Cox regression. This method offers the possibility to provide quantitatively and qualitatively the time when the breakage occurs. The method enables us to establish the link between a certain factor through which can forecast rock breakage and the ability to resist; the method can be developed and applied on a massif scale and also in the stability analysis of underground works.*

KEYWORDS: strength, hazard, breakage, regression, salt, event, time

### 1. Introduction

Viewed as a whole, the rock has a complex and nonlinear behavior and often with manifestation of dilatancy phenomenon under the action loads. Assessment of rocks behavior under load to fracture involves determining the mechanical and deformation characteristics and establishing the characteristic points on stress - deformation curve. From the principal schema of rocks behavior to breakage, it is found the existence of five areas and four characteristics stress levels on  $\sigma - \varepsilon$  characteristics curves (see Figure 1) [17, 18]:

- **I<sup>st</sup> area**, initial area of characteristic curves, starting from  $t = 0$  when the applied stress increases from  $\sigma = \sigma_0 = 0$  to a value  $\sigma = 0.05 \sigma_{rc}$ ; this latter stress defines actually the linearity limit of longitudinal (axial) deformations,  $\sigma_{oel}$  on  $\sigma - \varepsilon_1$  curves. Such area may be present or not, its existence depending on rock's porosity, initial fractures density and fractures geometry, this statement resulting from the interpretation of experimental data;
- **II<sup>nd</sup> area** starts for  $\sigma > 0.05 \sigma_{rc}$ , a continuous decrease of the sample volume being registered (it is assumed that porosity was surpassed and all

fractures were already closed), and the rock acting linearly – homogeneously – elastically. Such linear – elastic behaviour ends at certain  $\sigma$  value, when the linearity limit of longitudinal deformations is reached, this corresponds to  $\sigma_{if} \cong (0.3 - 0.4) \sigma_{rc}$  from that begins the fracturing initiation, that is when fracturing propagation process starts to develop [17];

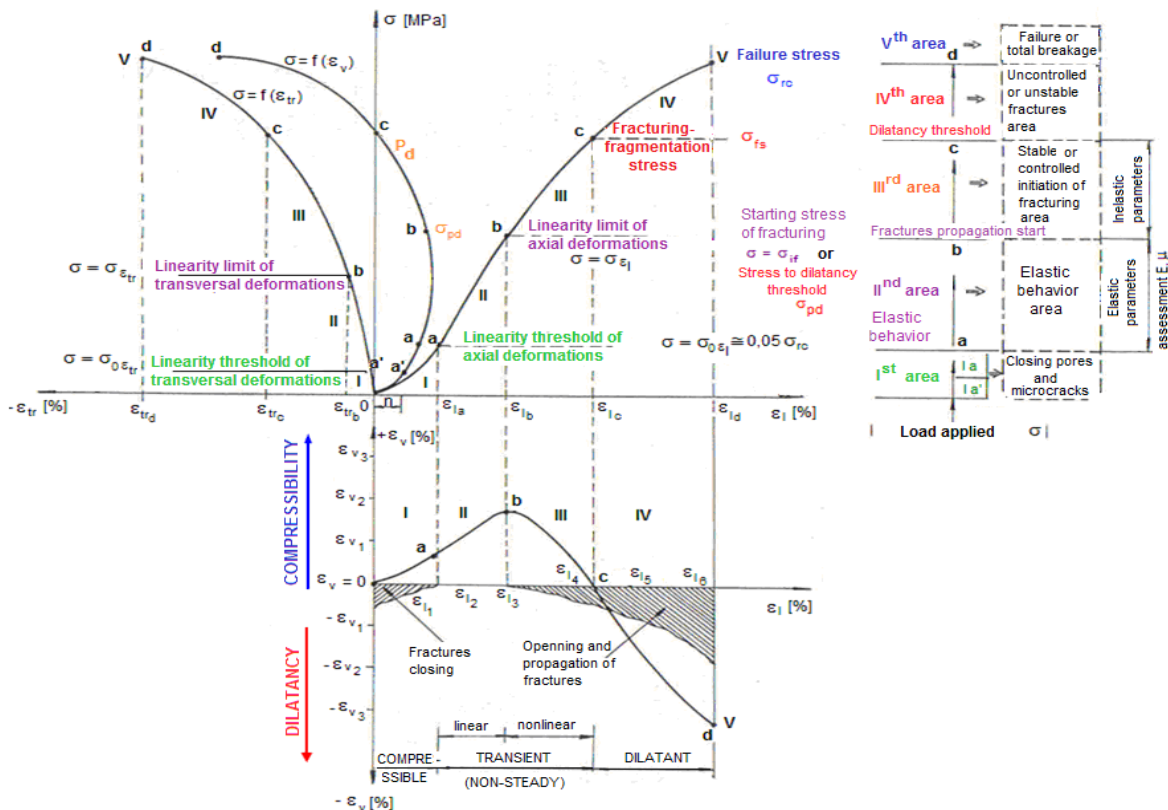
- **III<sup>rd</sup> area**, stable or controlled initiation of fracturing, represents the beginning of dilatancy  $\sigma > \sigma_{if}$ . In this area Toderaș (1999) considers that the dilatancy reflects the increasing of the axial fractures, such as parallel fractures to maximal applied load direction; that fracturing process, together with stresses  $\sigma > \sigma_{if}$  do not lead to diminishing the rock's strength. So, we can appreciate that fortuitous stable axial structural faults are not dangerous, having not significant effects on failure strength of studied rock;
- **IV<sup>th</sup> area** becomes present for a stress level  $\sigma > \sigma_{fd}$ , fracturing – destruction stress, when global volumetric deformation  $\varepsilon_v$  equals zero ( $\varepsilon_v = 0$ ). Unstable, uncontrolled fracturing starts simultaneously with dilatancy intensification. Generally, according to [5] there takes place levels ranging between  $(0.70 - 0.90) \sigma_{rc}$ . At this

level of stress, the longitudinal and transversal deformations will be far away from linearity. Sliding is the major mechanism acting when such longitudinal and cross-sectional deformations appear. Hallbauer mentioned that this area is characterized by strong and significant structural changes of the specimens and the density of micro-fracturing increases seven times. Actually, such a stress level detains a specific significance: it can be employed in assessment of long time strength of rocks [1, 17, 18]. Consequently, load increases over the fracturing – destruction stress value,  $\sigma > \sigma_{fd}$ , are inducing a temporary effect of deformation – hardening, which cannot be restored only through rheological and which leads to rock destruction, that cannot be tolerated under permanent load;

▪ **V<sup>th</sup> area** is worldwide employed in failure strength evaluation at compressive charges and in assessing critical failure value of rock or starting point of post-critical behaviour area of rocks.

These four stress levels were identified on stress – deformation curves, figure 1, are: (1)  $\sigma_0$  overcoming the perpendicular on loads direction porosity and micro-fractures stress which we were assimilated to the settling stress ( $\sigma_{asez}$ ); (2) fracturing starting stress  $\sigma_{if}$  assimilated by all researchers [1, 17] to the stress of dilatancy threshold  $\sigma_d^*$ ; (3) fracturing – fragmentation stress  $\sigma_{fs}$  and (4) failure stress or fracture strength at uniaxial compression  $\sigma_{rc}$ .

Breakage stress of rock is a function of the conditions in which the test is carried out and therefore, it constitutes an inherent property of the rock in question [17, 18].



**Fig. 1.** Nomogram for analysis – evaluation of rocks behaviour at deformation under uniaxial compressive load (after M. Toderaş, 1999)

Only the stress levels  $\sigma_{if}$  and  $\sigma_{fs}$  are independent parameters of test conditions and specimen volume, and  $\sigma_{fs}$  stress can be considered as a true resistance of rock in the case of monoaxial compressive load, because the load above this stress level leads to destruction of the rock and that rock cannot tolerate it under a permanent load;  $\sigma_{if}$ , as a stress that in fact defines the dilatancy threshold.

## 2. Cox proportional hazards regression in the analysis of rocks breakage

Cox regression or of proportional hazard was developed for the first time in the medicine and biosciences [1-4, 11, 14], but it seems that it began to be applied on a fairly large scale in engineering, mainly in the study of rocks breakage time. This

regression is also called of "proportional hazard" since it is based on the assumption that if the hazard may vary over time, then the hazard functions' ratio remains constant, namely the two hazard functions are proportional to each other [6]. We consider that this method of probabilistic analysis can be developed and also applied in the stability - reliability analysis of underground workings. In the engineering field, Cox regression may show if and in what way a characteristic of an analyzed rock may influence the occurrence of breakage; if one of the rock characteristic parameters leads to the manifestation of breakage phenomenon, by Cox regression it can establish the action intensity of this parameter and the direction of its action; by means of this regression there is the possibility to determine the time until which the breakage occurred (the action of a parameter, in the sense that it can extend or reduce breakage time) [19].

Cox regression presents importance regarding the existence and the link between a factor that can predict the breakage of rock and the rock capacity to resist and not to break, in other words, the existence and importance of correlation between a prognostic factor and resistance capacity of rock (the holding time in that domain where might exist anytime the potential possibility of breakage). What is specific to Cox regression is the fact that the response variable is a time dependent event and therefore, can be applied of probabilistic analysis of occurrence of breakage rocks. Through such regression we can achieve a multivariate analysis, which involves determining the relative influences or effects of different causes (or factors) on a single event, namely the breaking rocks time.

The importance of effect that rock parameter or characteristic has on holding time to breakage can be highlighted by means of hazard ratio (or risk ratio) (HR), established on the basis of risk (hazard).

From the point of view of geo-mechanic, we will define the hazard as an instantaneous risk of a rock to register a pre-established event, even though until that moment it opposed the event occurrence; in our case, we are talking about instantaneous risk of rock breakage submitted to a load around breaking strength (here we speak specially about the IV<sup>th</sup> on the nomogram for analysis – evaluation of rocks behaviour, where the cracking becomes unstable and uncontrolled and the dilatancy phenomenon intensifies too, Figure 1). The higher the hazard is, the higher the breakage risk is; but here we consider that other factors could also interfere here, and that could determine another event that could be produced (for example a growth of temperature or humidity, in case the analyzed sample is a salt one). Hazard ratio of a class or type of rock compared to another kind of rock represents the difference between hazards of two

types of rocks, mathematically considered as subjects of study.

### ***2.1. Analysis method of hazards***

There are known different methods of analyzing the occurrence risk of breakage, but most often used one is analysis method or mode of breakage time distribution, respectively Kaplan – Meier method [19, 20]. By means of this analysis there can be obtained more regression by which can be expressed the relationship between continuous or multiple variables and time until the appearance of breakage. The term of hazard can be defined in different modes, for different situations that are studied. We tried to define this term from the viewpoint of potential possibility of occurrence of rocks breakage situated under the action of a certain level of stress.

In the field of rock mechanics and implicitly of geo-mechanics (considering the events at scale of massif and in order to analyze the stability of underground workings), the term of hazard or risk, phenomenon that can occur at any time, represents the potential risk of occurrence and development of rocks breakage (occurrence of pre-established event) in a very short period of time, for any material system and implicitly of rocks too, that had the capacity (or a sufficiently high resistance, a sufficiently internal cohesion) to oppose (or to resist) to the damage of internal structure until that time.

As compared to the definition of risk factor given for the first time by Barlow (1963), we also define this notion from the viewpoint of rock mechanics. We consider a range of time whose lower limit is different from zero (time sufficiently low where breakage can appear and big enough in which a rock sample can resist until breakage); than the coefficient of risk is defined as the probability per unit time of a considered interval in which a rock sample did not break under the action of load which it has undergone, but will break at the time  $t$  belonging to this range. In other words, the risk coefficient can be determined as the ratio of the number of defects per unit time from the interval considered and the average specimens of the same type of rock from which no breakage was recorded at the time value corresponding to the middle of pre-established time range.

The average value of time until the breakage of rock sample does not occur represents the time at which the function of cumulative strength is equal to 0.5; to note is that percentage of 50 % of the cumulative strength function does not always correspond with the point that represents the time until which half of rock samples did not record the occurrence of breakage.

The hazard (the risk) and the probability that material system resists and does not register breakage are two interrelated concepts, the correlation of these being complex; through this there is the possibility to establish one parameter using the other. If the risk is constant for the whole period of study, it seems that this one is independent of the length of time where the material system (the rock) had enough internal cohesion to withstand, namely to oppose of the applied effort.

If the risk is constant throughout the study, it means that the risk of breakage is independent of the length of time that material system, in our case the rock, had sufficient internal cohesion, namely to oppose to the applied effort. For example, in case of uniaxial compressive loads of rock samples, after the effort applied exceeded the threshold of dilatancy or the fracturing - fragmentation stress, a rock with lower strength would have the same risk of breakage manifestation as a rock of higher strength. In case of creep tests, a rock sample mentioned under load, on stress degree of  $(0.7 - 0.85)\sigma_{rc}$ , for a period of 10 days would present the same risk of breakage in the next immediately moment (in a very short time) like a rock sample that was mentioned under load at the same stress degree for a period of 30 days. But we cannot exclude the hypothesis where the risk of breakage occurrence and development increases with increasing of time in which the rock presents sufficient strength; authors' opinion is that, in such situations, it could be a similar process to work hardening of rock under load, namely hardening.

An approach as close to reality as possible regarding the constancy of hazard is to eliminate any possible assumption and determine this characteristic based on the results of the laboratory tests (or measurements in situ, if we analyze the situation of stability of rock contours around underground works); an assumption regarding this parameter would not be fair and would involve significant errors. Therefore, for a given type of rock it would be necessary that relative hazard be considered as a particular combination of the values of variables with significant weight at a certain moment of time. The relative hazard is defined as the ratio of hazard for combining those variables at a given time and the hazard for the same time of a hypothetical sample (ideal model) whose values are all equal to 0 for predictive variables [8-11, 16]. Such a phenomenon is known as "basic hazard" and it is a virtual notion, namely in case of rocks we could assume that such a sample would exist, but the "ideal" term in this situation will eliminate a number of parameters or factors (as variables) that have significant influence in the development and manifestation of breakage. Whereas the calculation errors of the probability of breakage and respectively of risk of breakage

occurrence to be as small as possible, it should be analyzed each time at least 30 samples of the same rock type.

## 2.2. Cox's proportional hazard model

Cox's model expresses the function of instantaneous breakage risk of a rock,  $\lambda$ , depending on the time  $t$  and covariables  $X_1, \dots, X_n$ , namely:

$$\lambda(t, X_1, \dots, X_n) = \lambda_0(t) \exp\left(\sum_{i=1}^n \beta_i X_i\right) \quad (1)$$

For a rock sample, function  $\lambda(t, X_1, \dots, X_n)$  can correspond to the instantaneous risk of breakage at the moment  $t$ , knowing that at time  $t' < t$  breakage does not occur. The term  $\lambda_0(t)$  is named *basic risk* (in fact is the same for all analyzed rock samples at a given time), it is time dependent and corresponds to instantaneous risk of breakage when all of covariables are null; what is important for us is the ratio of instantaneous risk of breakage of two samples of the same type of rock exposed to different risk factors. The second term on the right side of the relationship (1) is independent of time. From the relationship (1) results hypothesis of proportional risks, that in fact is an essential hypothesis of Cox's model. In other words, assuming two rock samples that differ only by a single covariable, either this  $X_k$ , this covariable taking the values 0 and 1 for the two specimens (0 for the first specimen and 1 for the second), then whatever time  $t$ , a rock specimen had an instantaneous risk of breakage  $\exp(\beta_k)$  times higher compared to the other specimen considered; namely, for any time  $t$ , the risks ratio is independent of time; the term  $\exp(\beta_k)$  cannot be interpreted as a relative risk.

Rocks breakage could be analyzed by means of two models: Cox's proportional hazard model and Cox's proportional hazard model with time dependent covariates. Cox's proportional hazard model assumes establishing the strength function of rock defined by probability:

$$S(t) = P\{T > t\} \quad (2)$$

where:  $t$  is generally the time;  $T$  is the time until breakage.

Distribution of strength duration is:

$$F(t) = 1 - S(t) \quad (3)$$

where:  $f(t) = \frac{d}{dt} F(t)$  represents breakage rate of rock samples per unit of time.

From the point of view of mechanics rocks, we define the function of hazard as the risk of a rock to break away in a very short time  $dt$ , after a certain period of time  $T$  (assuming that until that moment the rock resisted); the function of hazard can be expressed according to the relationship:

$$\lambda(t) = P(t < T < t + dt) = \frac{f(t) dt}{S(t)} = -\frac{S'(t) dt}{S(t)} \quad (4)$$

Graphically, the strength curve of rock represents the cumulative strength curve with respect to time, and the derivative of strength curve will be the rate of occurrence of breakage in a short interval of time; this will be the hazard, namely a risk, and the ratio of hazards will be the relative risk. The hazard or risk can change in time, as meaning that it can increase or decrease, depending on certain factors (for example, the higher the time from appearance of cracks is, the higher the risk of breakage is).

Through regression or Cox model we can estimate the effect that in case of underground works, the applicability of consolidation method or improvement of rocks resistance characteristics on their contour, chosen in accordance with other independent variables (or factors) and so, we have the possibility to estimate the hazard (or the risk) of breakage or other event (or phenomenon) that would present interest, given their prognostic variables. In case that the analyzed rock samples (samples which were submitted to an improvement of the mechanical properties and the sample blank) are similar as regards the known variables that influence their strength capacity, the Cox model used for these variables will produce a more precise appraisal of the effect of increasing rock strength [19].

Proportional hazard model or Cox model is not based on any hypothesis regarding the distribution of strength, it is based on the assumption that the hazard is a function only of independent variables (predictive, covariates)  $Z_1, Z_2, \dots, Z_k$ :

$$h(t; Z_1, Z_2, \dots, Z_k) = h_0(t) \cdot \exp(b_1 \cdot Z_1 + b_2 \cdot Z_2 + \dots + b_k \cdot Z_k) \quad (5)$$

That by applying the logarithm, becomes:

$$\begin{aligned} \ln \left( \frac{h(t; Z_1, Z_2, \dots, Z_k)}{h_0(t)} \right) &= b_1 \cdot Z_1 + b_2 \cdot Z_2 \\ &+ \dots + b_k \cdot Z_k \\ \Rightarrow \ln \left( \frac{h(t; Z_1, Z_2, \dots, Z_k)}{h_0(t)} \right) &= \sum_{i=1}^k b_i \cdot Z_i \end{aligned} \quad (6)$$

From this relationship it can be found that this is a semi-parametric model [7, 12]. The parameter  $h(t, Z)$  could be considered as a Weibull type density distribution, because [20]:

$$h(t, Z) = \lambda \cdot p \cdot t^{p-1} \quad (7)$$

where:

$$\begin{aligned} \lambda &= \exp(b_1 \cdot Z_1 + b_2 \cdot Z_2 + \dots + b_k \cdot Z_k) \\ \text{or } \lambda &= \exp \left( \sum_{i=1}^k b_i \cdot Z_i \right) \\ h_0(t) &= p \cdot t^{p-1} \end{aligned} \quad (8)$$

$h_0(t)$  is named *basic hazard* and represents the hazard of a certain subject (specimen of rock) when all independent variables are equal to zero and, as it can be observed, it depends only on time  $t$ ; the exponential refers only to  $Z = (Z_1, Z_2, \dots, Z_k)$  and also to variables  $Z_i$  that are independent of the time  $t$  (examples of variables that are time independent in the field of rock mechanics, would be: genesis of analyzed rock, rock sample weight; ...). As shown in the scientific literature [8-10, 13, 15, 16], Cox's model is preferred to be used, since it appears that it would be one of the safest methods in many situations. The estimated hazards are positive, and if the term  $h_0(t)$  cannot be specified, then it is possible to determine the hazard rate, HR, given by relation:

$$HR = \frac{h(t, Z^*)}{h(t, Z)} \quad (9)$$

where:  $Z^* = (Z_1^*, Z_2^*, \dots, Z_k^*)$  and  $Z = (Z_1, Z_2, \dots, Z_k)$ .

To ease and simplify the interpretation [20], it is considered that  $HR \geq 1$ , which means that:

$$h(t, Z^*) \geq h(t, Z) \quad (10)$$

It is considered  $Z^*$  as a rock of  $I^{st}$  type (a population consisting of samples belonging to the  $I^{st}$  type of rock) having the higher risk of breakage (the hazard), respectively,  $Z$  representing a rock of  $II^{nd}$

type with lower risk of breakage (we can consider that for the  $\Pi^{\text{nd}}$  type it was applied a method of increasing rock strength). Taking into account the conditions:

- between  $h_0(t)$  and log-linear function of covariates, there must exist a multiplicative relationship, namely respecting the hypothesis of proportionality from the point of view of hazard;
- between hazard and independent variables there must exist a log-linear relation, we obtain the hazard rate as being:

$$\text{HR} = \frac{h_0(t) \cdot \exp\left(\sum_{i=1}^k \beta_i \cdot Z_i^*\right)}{h_0(t) \cdot \exp\left(\sum_{i=1}^k \beta_i \cdot Z_i\right)} \quad (11)$$

$$\text{HR} = \exp\left[\sum_{i=1}^k \beta_i \cdot (Z_i^* - Z_i)\right]$$

For example, if we consider the relation (6), we can use the predictive value of model by means of prognostic index (IP), defined as:

$$\text{IP} = b_1 \cdot Z_1 + b_2 \cdot Z_2 + \dots + b_k \cdot Z_k$$

$$\text{IP} = \sum_{i=1}^k b_i \cdot Z_i \quad (12)$$

Further, it can be established the function of strength or the function of maintaining the rock capacity to resist and not manifest the breakage:

$$S(t) = \exp[-H_0(t)]^{\exp(\text{IP})} \quad (13)$$

Where  $H_0(t)$  represents cumulate basic hazard [6, 20] and is a variable of scale function.

### 3. Estimating coefficients

To establish the coefficients  $\beta_i \mid i = 1, \dots, k$ , the maximum probability method is applied; the likelihood of a representative group of a population is actually the probability to observe intuitively this group, namely it aims at its maximizing [8, 9, 11-13, 20]. In this aim, we will consider a group of independent data, whose notations are:

- $X_i$  column vector of covariates of rock sample  $i$  (that could depend on the time);
- $\beta$  column vector of  $\beta_i$  coefficients;
- $t_i \mid i = 1, \dots, m$  breakage times;
- $d_i$  number of breakage at time  $t_i$ ;

- $D_i$  all breakages at time  $t_i$ ;
- $r_i$  number of rock samples with breakage risk at time  $t_i$ ;
- $R_i$  all samples with breakage risk at time  $t_i$ .

The probability that a sample will break at time  $t_i$  is:

$$p = \lambda_0(t_i) \exp(X_k^t \beta) dt \quad (14)$$

At time  $t_i$  the probability of all samples belonging to  $D_i$  will be [6]:

$$P_{t_i} = \frac{\exp(\beta^t)^{\sum_{k \in D_i} X_k}}{\left[\sum_{l \in R_i} \exp(\beta^t X_l)\right]^{d_i}} \quad (15)$$

Cox's probability function (partial) can be written as being [6]:

$$L(\beta) = \prod_{i=1}^m \frac{\exp(\beta^t)^{\sum_{k \in D_i} X_k}}{\left[\sum_{l \in R_i} \exp(\beta^t X_l)\right]^{d_i}} \quad (16)$$

This is reduced to solve the equation (16), so as  $L(\beta)$  is maximum; the value for which the function  $L$  reaches the maximum will represent the probability of the representative group considered.

### 4. Results and discussions

In the Laboratory of Geomechanics of the University of Petrosani it was studied a number of 37 salt cylindrical samples, having the fineness ratio  $\lambda \cong 1.5$ ; the tests were conducted at monoaxial compression, following also time at which the breakage of salt samples was manifested (Figure 2). Taking into account the number of samples submitted to the tests, we chose a number of 18 salt samples for which we realized the statistical analysis of the influence of the rock strength on the breakage time. Analysis can be done either by t-test or using the ANOVA techniques [1, 2, 10, 13] or by dispersion analysis, respectively analysis of variance.

t-test is somehow limited because of realizing only a comparison of the differences between two groups. T-test is a method by means is only done comparing differences between two groups, so it is reduced just to those situations where there are only two ways of the independent variables; if applied, this method in case of more independent variables, then statistical errors or the error of the set of comparison increase each time when are applied for each of the possible pairs of two ways.

On the other hand, if we have a larger number of independent variable levels, then it is necessary to apply other techniques so that the effect of independent variable on the dependent once will be expressed by a higher degree of fineness [10], for example the ANOVA technique. To apply this

technique, the first stage was to identify the possible method to be applied; available data are quantitative, thus the problem is reduced to the comparison of the averages of two populations (the time when the breakage occurs and the monoaxial compressive breaking strength of salt samples).

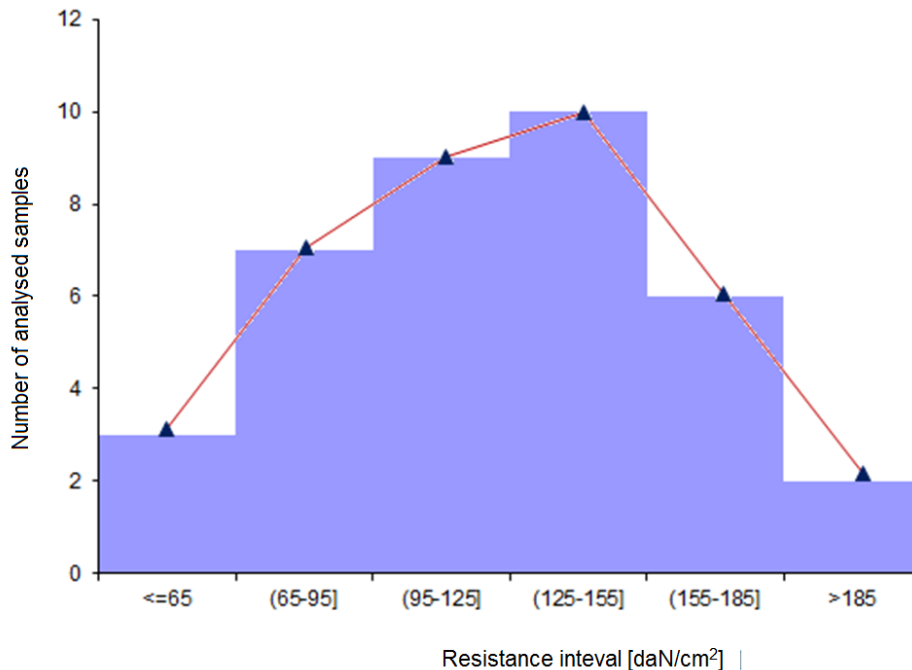


Fig. 2. Histogram variation of compressive strength of analyzed salt specimens

Table 1. t-Test: Paired Two Sample for Means

	Variable 1	Variable 2
Mean	159.4444	63.46035
Variance	21165.56	2935.414
Observations	18	18
Pearson Correlation	0.993908	
Hypothesized Mean Difference	0	
df	17	
t Stat	4.43462	
P(T<=t) one-tail	0.000182	
t Critical one-tail	1.739607	
P(T<=t) two-tail	0.000363	
t Critical two-tail	2.109816	

The fact that the analysis of these data proves that the two averages are different, means that we can apply the analysis of variance with one factor. The aim of the covariance analysis is to eliminate a number of external factors that cannot be identified, so that the effect of the independent variable can be clearly highlighted. Simple or unifactorial ANOVA analysis is a simple model considered to be a corresponding t-test for two independent specimens. Factorial ANOVA analysis is most often used, it is considered a more complex model and by this technique are tested the effects of several independent

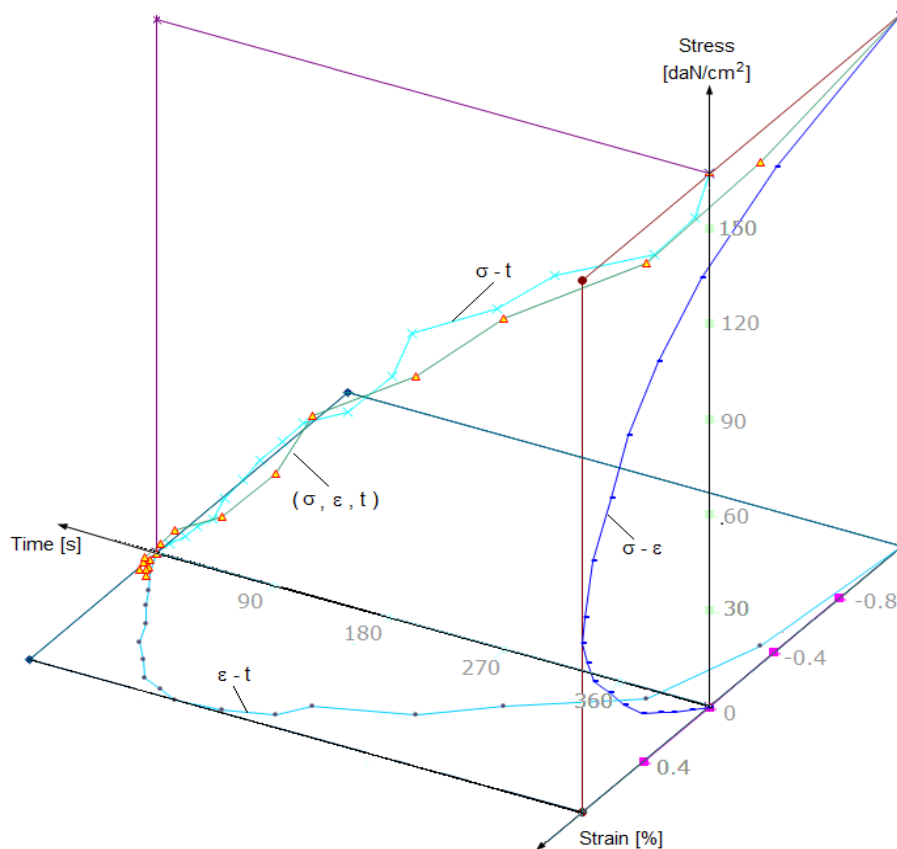
variables (or factors) on a dependent variable. In table 2 are presented the results obtained by statistical processing of experimental data, applying the ANOVA analysis of variance, simple or unifactorial (single factor), and in Table 3 are shown the results obtained by statistical processing applying the ANOVA factorial analysis of variance. In Table 2, in the area SUMMARY, are mentioned the data related to the two populations: number of units from each population, namely 18; the amount for each population; variance of population. It is observed that the smallest average and the highest dispersion were registered in terms of compressive breaking strength. In the area ANOVA from Table 2, is computed the statistics F for the unifactorial variance analysis, its value being 6.880762; in Table 3 the statistics F for variance factorial analysis is equal to  $F_{\text{calculat}} = 4.716186729$ . Because  $F_{\text{calculat}} = 6.880762 > F_{\text{crit}} = 4.130018$ , respectively  $F_{\text{calculat}} = 4.716186729 > F_{\text{crit}} = 2.271893$ , it follows that we reject the null hypothesis  $H_0$  and accept the alternative hypothesis  $H_1$  as a true one. Therefore, it can be said with a probability of 95 % that the compressive strength of salt samples influence significantly time variation until their breaking occurs.

**Table 2. Summary output and ANOVA analysis Single Factor**

SUMMARY						
Groups	Count	Sum	Average	Variance		
Time until breakage	18	2870	159.4444	21162.56		
Compressive breakage strength	18	1142.286	63.46035	2935.414		
ANOVA						
Source of Variation	SS (sum of squares)	df	MS (mean square)	F	P-value	F crit
Between Groups	82916.5129	1	82916.51	6.880762	0.012949	4.130018
Within Groups	409716.4878	34	12050.48			
Total	492633.0007	35				

**Table 3. Summary output and ANOVA Two-Factor Without Replication**

Source of variation	SS	df	MS	F	P-value	F crit
Rows	338039.9476	17	19884.7028	4.716186729	0.001292	2.271893
Columns	82916.51289	1	82916.51289	19.66585881	0.000363	4.451322
Error	71676.54019	17	4216.26707			
Total	492633.0007	35				



**Fig. 3. The curve of Cox's proportional hazard model**

For a confidence-associated range equal to 0.05 %, the hazard rate depends on the compressive breaking strength value of salt. Applying Cox's regression method, considering as interest variable

the value of compressive strength and the other constant variables, for the Cox's regression the hazard rate shows that the variation of this characteristic value leads to the change of hazard rate. The calculus



showed a hazard rate  $HR = 3.01 > 1$ , which means that the decreasing compressive breaking strength leads to increased risk of occurrence of breakage and reduces the time of manifestation of this phenomenon ( $p$ -value = 0.1234). In Figure 3 is shown the Cox curve obtained from processing of experimental results.

## 5. Conclusions

Cracking changes significantly certain properties of rocks cracking (for example, permeability to fluids, damping of acoustic waves); other properties may be less affected by this phenomenon (density, the modules of elasticity for stress that maintain the cracks closed). The variation of volume during a mechanical testing is an effective indicator of cracking; for example, dilatancy and microcracking lead to a significant increase of permeability of rock salt and consequently, the initial tightness of intact salt is reduced. From the macroscopic point of view, cracking is characterized by dilatancy due to micro-cracking and relative displacements of grains and fragments, namely an increase of permeability, and also presenting sensitivity to the change of loading rate.

The method of the classical analysis of time in which a rock resists and breakage does not occur cannot be used if we want to analyze which is the simultaneous effect of several variables on the time where there is no possibility for breakage to occur. The multiple regression method cannot be used directly, because the variable describes time until breakage has not normal distribution, but an exponential distribution or Weibull type due to the fact that in any case there is a possibility that the breakage occurs; for this analysis of time resistance, we often do not have complete information. Therefore, in order to carry out such analysis, we suggested using a regressive method, as Cox's proportional hazard model; the same analysis can be also carried out on the basis of Cox's proportional hazard model with time – dependent covariants. By applying the Cox model or proportional hazards regression we have the possibility to predict the occurrence of relative risk (potential risk) of rocks breakage based on several variables, risk (hazard) which may have a different variation, either in the sense that it can increase over time either it may decrease; if at times there is no variable or factor leading to an evolution towards rock breakage, then the risk of breakage is lower. Cox method offers information regarding the effect of certain factors (variables) - extrinsic or intrinsic - on the time resistance of rock after adjusting the other independent variables; this method offers the

possibility to assess the hazard or risk of rock breakage.

Salt presents a specific behavior under the effect of cracking phenomenon. Preexisting cracks or the ones caused by stresses could be developed and microscopically determine a change in volume, namely dilatancy; the cracking phenomena is irreversible. Dilatancy is a phenomenon that constitutes the effective indicator of the occurrence of cracking, leading to a significant increase of permeability of salt and consequently to the diminishment of its initial tightness; therefore, it can't be neglected in stability analysis of underground works executed in salt massive [18].

Through regression or Cox model we can estimate the effect that in case of underground works, the applicability of consolidation method or improvement of rocks resistance characteristics on their contour, chosen in accordance with other independent variables (or factors) and so, we have the possibility to estimate the hazard (or the risk) of breakage or other event (or phenomenon) that would present interest, given their prognostic variables. The higher the hazard is, the higher the risk of breakage is; other factors could also interfere here, and that could determine another event that could be produced (for example a growth of temperature or humidity, in case the analyzed sample is a salt one).

In horizontal plan (see Figure 3), the Cox curve projection expresses the strains variation corresponding to the breaking time of salt samples; the Cox curve projection in lateral plan expresses the strains variation corresponding to the appearance of breakage related to compressive strength. The results obtained by means of Cox proportional hazard model are in accordance with the nomogram for evaluation of behavior at deformation under load of rocks until the occurrence of breakage (see Figure 1) [17, 18].

## References

- [1]. **Armitage P., Berry G., Matthews J. N. S.**, *Statistical Methods in Medical Research* (4<sup>th</sup> Edition), Oxford: Blackwell Science, 2001.
- [2]. **Armitage P., Berry G.**, *Statistical Methods in Medical Research* (3<sup>rd</sup> Edition), Blackwell, 1994.
- [3]. **Barlow W. E.**, *Robust variance estimation for the case-cohort design*. *Biometrics.*, 50, p. 1064-1072, 1994.
- [4]. **Barlow W. E., Ichikawa L., Rosner D., Izumi S.**, *Analysis of case-cohort designs*, *Journal of Clinical Epidemiology*, 12, p. 1165-1172, 1999.
- [5]. **Bieniavski Z. T.**, *Mechanism of brittle fracture of rock*, *Int. J. of Rock Mech. and Min. Sci.*, no. 4, 1967.
- [6]. **Cox D. R.**, *Partial Likelihood*, *Biometrika*, vol. 62, p. 269-276, 1975.
- [7]. **Dennis E. Hinkle, William Wiersma, Stephen G. Jurs.**, *Applied Statistics for the Behavioural Sciences*, Boston, Published by Houghton Mifflin (Academic), 3<sup>rd</sup> Edition, p. 706, ISBN 10: 0395675553, 1994.
- [8]. **Gorunescu F., Gorunescu M.**, *Analiza exploratorie și procesarea datelor cu simulări în Matlab*, Editura Albastra.

- [9]. Gorunescu F., Gorunescu M., *Modele regresive*, math.ucv.ro/~gorunescu/courses/EDA/curs2EDA.pdf.
- [10]. Hinkle D. E., Wiersma W., Jurs S. G., *Applied statistics for the behavioral sciences* (3<sup>rd</sup>), Boston, USA, Houghton Mifflin Company, 1994.
- [11]. Lin D. Y., *Cox Regression Analysis of Multivariate Failure Time Data: The Marginal Approach*, Statistics in Medicine, vol. 13, p. 2233-2247, 1994.
- [12]. Keppel G., *Design and analysis: a researcher's handbook* (3rd ed.), Englewood Cliffs, USA: Prentice-Hall Inc, 1991.
- [13]. Kleinbaum D. G. et al., *Applied Regression Analysis and Other Multivariable Methods* (3<sup>rd</sup> Edition), Duxbury Press, 1998.
- [14]. Radu I., Miclea M., Albu M., Nemeş S., Moldovan O., Szamoskozi Ş., *Metodologie psihologică și analiza datelor*, Cluj-Napoca: Editura Sincron, 1993.
- [15]. Schoenfeld D., *Partial Residuals for The Proportionnal Hazards Regression Model*, Biometrika, vol. 69, p. 239-241, 1982.
- [16]. Spiekerman C. F., Lin D. Y., *Marginal Regression Models for Multivariate Failure Time Data*, Journal of the American Statistical Association, vol. 93, p. 1164-1175, 1998.
- [17]. Toderas M., *Mecanica rocilor, pământurilor și construcții subterane*, Editura Universitat, Petroșani, ISBN 978-973-741-381-9, 1167 p., vol. I, ISBN 978-973-741-382-6, vol. II, ISBN 978-973-741-383-3, 2014.
- [18]. Toderas M., *Cercetări și rezultate în stabilitatea lucrărilor miniere subterane*, Editura Universitat, Petroșani, ISBN 978-973-741-486-1, 343 pages, 2016.
- [19]. Toderas M., Danciu C., *Analysis Possibilities of Rocks Breakage by Means of Proportional Hazard Regression*, 7<sup>th</sup> International Multidisciplinary Symposium „Sustainable Development through Quality and Innovation in Engineering and Research” SIMPRO 2016, Universitatea din Petroșani, p. 299-302. ISSN-L 1842-4449; ISSN 2344-4754, 2016.
- [20]. Ying Z., Wei L. J., *The Kaplan-Meier Estimate for Dependent Failure Time Observations*, Journal of Multivariate Analysis, vol. 50, p. 17-29, 1994.