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MATHEMATICAL MODELING OF THE HEAT TREATMENT PROCESS APPLIED TO ALLOY Al-Zn-Mg-Cu-4.5% Zn, USED IN AERONAUTICS

Marian NEACSU, Doru HANGANU
„Dunărea de Jos" University of Galaţi
e-mail: uscaeni@yahoo.com

ABSTRACT

This paper presents mathematical modeling based on the regression analysis method by active experiment of a thermal treatment process applied to an alloy belonging to Al-Zn system, which finds application in aeronautics.

Using the mathematical model determined on a statistical basis, we achieved the prediction of the mechanical properties studied by simulating the heat treatment process, more exactly by simulating the heat treatment parameter values within the experimental values.

The mathematical model equations obtained in this paper can also be used to achieve the optimal values of the treatment technological parameters in order to obtain the desired complex of properties with minimum costs.

KEYWORDS: mathematical modeling, aluminum alloy, heat treatment

1. Introduction

Mathematical modeling represents the transposing of a real physical process into a mathematised form.

Mathematical modeling as a process can be achieved in two stages as following: the first stage represents the stage when it is stated the way in which the mathematical model is used in order to obtain a series of predictions concerning both the input quantities, the value of the heat treatment process parameters, and the mechanical properties studied.

All the equations can be obtained mathematical model and optimize the heat treatment process studied. On the basis of the equations of the mathematical model obtained, we can also achieve the optimisation of the heat treatment process studied.

In this paper we develop a mathematical model of the heat treatment process applied to alloy Al-Zn-Mg-Cu containing 4.5% Zn, namely the regression analysis method by active experiment.

The regression analysis method by active experiment is a method which involves solving the problems of extreme values and requires determining the levels for of the independent input values u_1, u_2, ... u_k, where the objective function:

$$y = f(u_1, u_2, ..., u_k)$$

has extreme values (maximum and minimum) as well as the calculation of these values. [1]. Input values for the heat treatment process studied are: t- temperature heat treatment i.e. artificial aging temperature; τ - time of artificial aging.

Based on these input data, and using the mathematical model obtained, we studied the influence of input quantities on the researched mechanical properties: mechanical strength, yield strength, elongation at break, HB hardness, which represents output quantities.

The starting point randomly chosen has as coordinates in the factorial space, the basic levels u_{01}, u_{02} for the two input values.

There were also established the variation ranges of these input quantities, Δu_1, Δu_2.

By adding the variation range to the basic level, the superior level is attained, while by subtracting it, the lower level of the factor is obtained.

If x_i denotes the encoded value of the u_i factor resulted from the relationship:

$$x_i = \frac{u_i - u_{0i}}{\Delta u_i} \quad (1)$$

the upper level is coded +1, the lower level -1, and basic level 0. [1]

2. Experimental conditions

The aluminum alloy processed as illustrated in Figure 1 has a chemical composition shown in Table 1. As illustrated in Figure 1, the alloy was processed in a homogenization heat treatment at 480°C, followed by quenching in solution at 500°C for 2 hours and finally by artificial aging at five different temperatures (120, 140, 160, 180, 200°C) with five maintaining times (4, 8, 12, 16, 20 hours) for each of the five aging temperatures. The thermal processing considered for achieving mathematical modeling of this processes were determined as: 1 - artificial aging temperature - t [°C]; 2 – maintaining time - τ [h];

Table 1. *Chemical composition of the alloy heat treated*

Chemical element / Alloy	Zn	Mg	Cu	Si	Fe	Pb	Cr	Mn	Al
$AlZn_4,5Mg_1$	4.5	1.4	0.2	0.35	0.4	-	0.35	0.5	rest

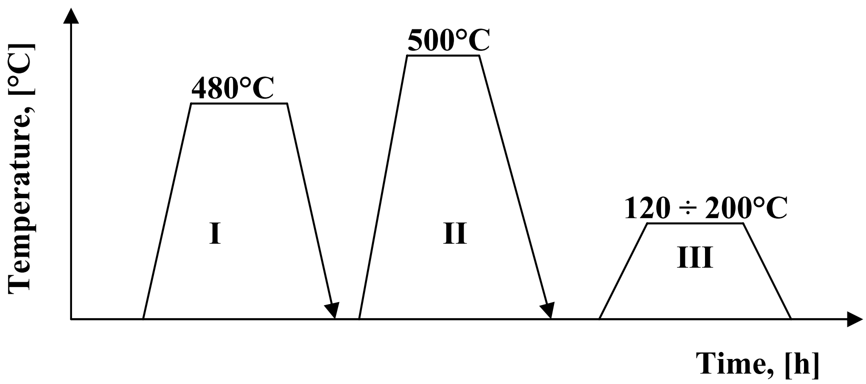

Fig. 1. *Technological scheme for achieving thermal treatment*
I - homogenization, II - implementing solution hardening, III - artificial aging

Table 2 shows the correlation between the different levels of factors expressed in natural values with those expressed in coded values for the two factors used in the heat treatment.

Table 2. *Correspondence between the factors expressed in natural units and those expressed in encoded units*

Factor	Process temperature		Process duration	
	Natural units, in °C	**Encoded values**	**Natural units,** in hours	**Encoded values**
Basic level	$u_{01} = 160$	$\frac{160-160}{40}=0$	$u_{02} = 12$	$\frac{12-12}{4}=0$
Variation interval	$\Delta u_1 = 40$	0	$\Delta u_2 = 8$	0
Superior level	$u_{1s} = 200$	$\frac{200-160}{40}=+1$	$u_{2s} = 20$	$\frac{20-12}{8}=+1$
Inferior level	$u_{1i} = 120$	$\frac{120-160}{40}=-1$	$u_{2i} = 4$	$\frac{12-20}{8}=-1$

For the coded representation of the experiment we used the following notations and symbols:

- x_1 - artificial aging temperature, t, °C;
- x_2 - retention time, τ [h];
- Y_1 - tensile strength, R_m [MPa];
- Y_2 - yield, $Rp_{0.2}$ [MPa];
- Y_3 - hardness, [HB];

The following relations are established between the natural and the coded values of the x_i factors:

$$x_1 = \frac{t - t_0}{\Delta t}; \qquad x_2 = \frac{\tau - \tau_0}{\Delta \tau}; \qquad (2)$$

A complete type 2^2 factorial experiment was achieved, as shown in Table 3.

Table 3. *Determination of 2^2 type factorial experiment matrix*

No. exp.	X_0	X_1	X_2	$X_1 X_2$	Y_1	Y_2	Y_3	Y_4
1	+1	+1	+1	+1	288	248	14.5	71
2	+1	-1	+1	-1	440	385	10.7	127
3	+1	+1	-1	-1	253	221	15.6	62
4	+1	-1	-1	+1	312	269	13.2	91

Considering that the model structure of I, is: [4]

$$Y_i = c_0 + \sum_{i=1}^{2} c_i \cdot x_i + \sum_{\substack{i=1 \\ j=1 \\ i \neq j}}^{2} c_{ij} x_i x_j \qquad (3)$$

Equation (3) is written in matrix form as follows:

$$Y = X \bullet C \qquad (4)$$

where: X is the matrix of experimental conditions

$$X = \begin{Vmatrix} x_{01} & x_{11} & x_{21} & \dots & x_{m1} \\ x_{02} & x_{12} & x_{22} & \dots & x_{m2} \\ x_{03} & x_{13} & x_{23} & \dots & x_{m3} \\ \dots & \dots & \dots & \dots & \dots \\ x_{0n} & x_{1n} & x_{2n} & \dots & x_{mn} \end{Vmatrix} \qquad (5)$$

where: m - number of terms of equation (3);
n - number of experiments considered;
C - column vector of coefficients but
C = [c_0, c_1, ..., c_n] T, where: T is the symbol matrix transposition
Y - Matrix of experimental results

$$Y = [Y_1, Y_2, \dots, Y_n]^T \qquad (6)$$
$$Y = [Y_1, Y_2, Y_3]^T$$

where: Y_1 = [288;440;253;312];
Y_2 = [248;385;221;269];
Y_3 = [14.5;10,7;15.6;13.2]
Y_4 = [71;127;62;91];

For this case, the linear function (3) is a particular form:

$$Y_i = c_0 + c_1 \cdot x_1 + c_2 \cdot x_2 + c_{12} \cdot x_1 \cdot x_2 \qquad (7)$$

Multiplying on the left both terms of the matrix equation by the unitary matrix:

$$\mathbf{E} = [X^T X]^{-1} \times X^T,$$

It follows:

$$\mathbf{C} = [X^T \times X]^{-1} [X^T \text{ x } Y] \qquad (8)$$

expression that represents the relationship for calculating the coefficients of the regression equation.

Using the values in Table 3, based on the relation (8) the coefficient of the first-order models are obtained as presented in Table 4.

Table 4. *Values of the coefficients of the order I models*

c_i \ Y_i	Y_1	Y_2	Y_3	Y_4
c_0	323.25	280.75	13.5	87.75
c_1	-52.75	-46.25	1.55	-21.25
c_2	40.75	35.75	-0.9	11.25
c_{12}	-23.25	-22.25	0.35	-6.75

Therefore, the equation of the first-order mathematical model (7), for each property separately, is:

$$Y_1 = 323.25 - 52.75 \cdot x_1 + 40.75 \cdot x_2 - 23.25 \cdot x_1 \cdot x_2; \qquad (9)$$
$$Y_2 = 280.75 - 46.25 \cdot x_1 + 35.75 \cdot x_2 - 22.25 \cdot x_1 \cdot x_2; \qquad (10)$$
$$Y_3 = 13.5 + 1.55 \cdot x_1 - 0.9 \cdot x_2 + 0.35 \cdot x_1 \cdot x_2 \qquad (11)$$
$$Y_4 = 87.75 - 21.25 \cdot x_1 + 11.25 \cdot x_2 - 6.75 \cdot x1 \cdot x_2; \qquad (12)$$

By replacing the xi variables with the relations (2) and by effectuating the respective calculations in the above equations, we obtain the following equations representing the expressions of the first order mathematical models for the four properties considered:

$$Y_1(t, \tau) = 333.625 - 0.446 \cdot t + 16.718 \cdot \tau - 0.072 \cdot t\,\tau; \qquad (13)$$
$$Y_2(t, \tau) = 278.625 - 0.04 \cdot t + 15.59\,\tau - 0.07 \cdot t\,\tau; \qquad (14)$$
$$Y_3(t, \tau) = 10.75 + 0.025 \cdot t - 0.287\,\tau + 0.001 \cdot t\,\tau \qquad (15)$$
$$Y_4(t, \tau) = 157.975 - 0.544 \cdot t + 1.231\,\tau + 0.001 \cdot t\,\tau; \qquad (16)$$

First-order mathematical models were verified statistically by using Fisher criterion to decide whether they can be used for studying the analyze process or it is necessary to determine the higher order models.

The testing has proved that all models are consistent with the experimental data.

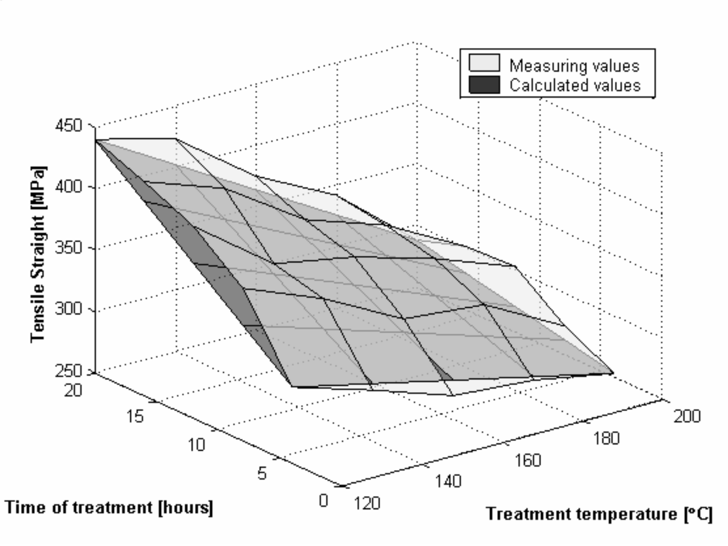

Fig. 2. *Breakthrough variation depending on the time and temperature of artificial aging*

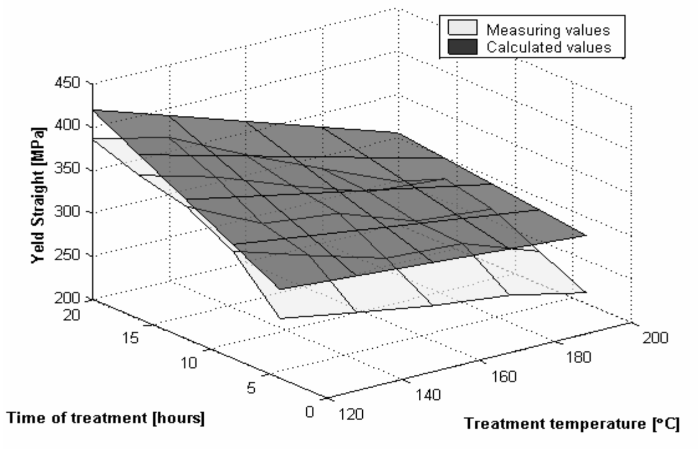

Fig. 3. *Variation of tensile yield stress function of time and artificial ageing temperature*

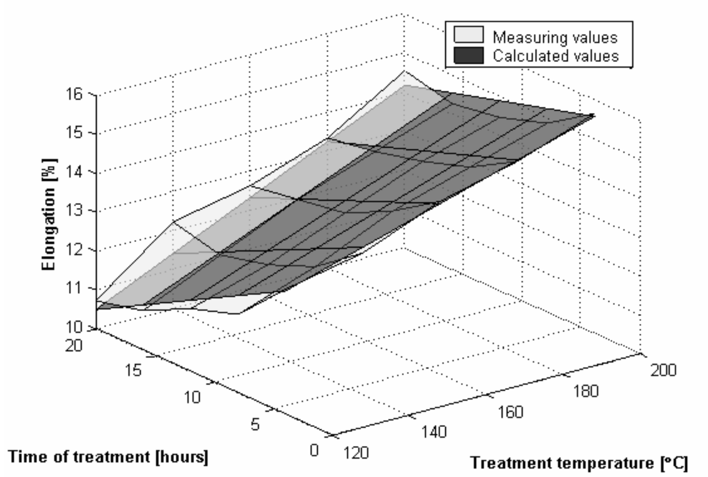

Fig. 4. *Change in elongation at break, depending on the time and temperature of artificial aging*

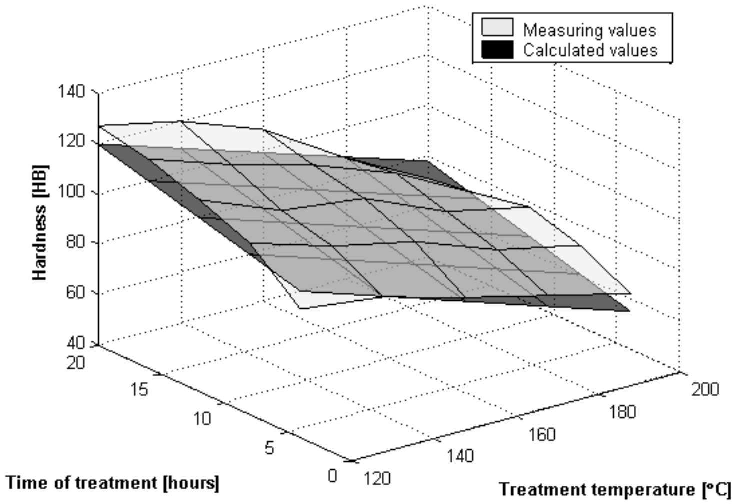

Fig. 5. *Variation of hardness function of time and artificial ageing temperature*

As shown in Figures 2 to 4, the values of the mechanical properties determined by calculations using mathematical models are close to the experimentally determined values. This is the proof that performed allows the simulation of the heat treatment process presented in Figure 1.

3. Conclusions

The simulation of heat treatment was based on the mathematical model obtained and presented in equations (13), (14), (15), (16). This simulation is based on the variation of the technological parameters values (t, τ) within the experimental limits:

-the duration of the artificial aging treatment is the factor with the greatest influence on the mechanical strength obtained, as it follows from the evaluation of equation (13), where the value of the coefficient of parameter τ is positive and of the highest value;

-also from the evaluation of equation (13) it results that as the artificial aging temperature increases, there is a decrease in the strength properties;

-the regression equations obtained show that increasing artificial aging temperature in the range considered, above the 180°C, leads to a decrease in tensile strength, yield strength and hardness;

-the mathematical model presented allows the calculation for the optimization of heat treatment process parameters in order to obtain the optimal complex of resistance properties, with minimal expenses.

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