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MODELING OF THE MATERIAL FLOW AT THE ECAE PROCESS USING THE CURRENT LINES METHOD

Nicolae CĂNĂNĂU
"Dunărea de Jos" University of Galati
email: ncananau@yahoo.com

ABSTRACT

The Equal Channel Angular Extrusion is an important method of Sever Plastic Deformation, applied in aim the obtaining of nanostructured metallic materials. The cinematic field of the material may be described with various calculus methods. In this paper is developed a study of the flow field using the current lines theory.

KEYWORDS: advanced material, modeling, CAE, extrusion

1. Introduction

The metallic nanostructured materials have special properties, which justify the many researches applied for obtaining this advanced material. A very good method in the aim of working the nanostructured metallic materials is the ECAE.

This method consists in the extrusion of the material in the extrusion die [1, 2] with a constant cross section channel in an angle of 90^0, usually broken (Figure 1).

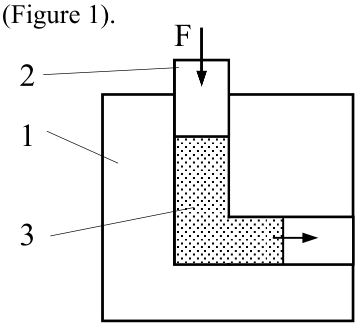
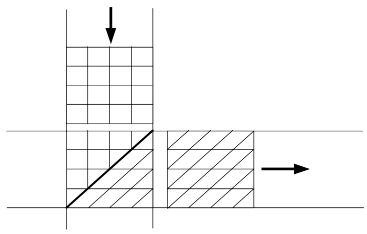


Fig. 1. *Scheme of the ECAE.*

The plunger 2 developed the force F and the material 3 flows in the channel of the extrusion die 1.

The plastic deformation process is localized in angular zone of the channel (Fig.2).



Fia. 2. *Deformation of lines net.*

In this location is developed a very intense plastic deformation by relative slide of the material layers, in condition of the compression stress state and great material plasticity. The plastic deformation degree is important.

For greater deformation intensity, the ECAE process may be repeated in a number of cycles [3, 4].

The equivalent strain, for a single extrusion, may be evaluated with the expression [5]:

$$\varepsilon_{eq} = \frac{1}{\sqrt{3}}\left[2\cot\left(\frac{\varphi}{2}+\frac{\psi}{2}\right)+\psi\,\mathrm{cosec}\left(\frac{\varphi}{2}+\frac{\psi}{2}\right)\right] \quad (1)$$

The geometrical factors are defined in Figure 3.

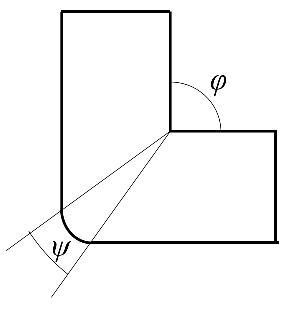


Fig. 3. *Geometrical factors of ECAE.*

For ensuring the continuity of the material flow the channel of die is worked with a cylindrical surface.

For this reason the passing zone, practically, is centered at the interior peak of the channel under the anter angle ψ (Fig.4).

The evolution of the material flow at the ECAE deformation process may be defined using the flow line method.

2. Definition of the flow line

The deformation volume is composed by three zones: first is the uniform zone at the entry of the channel, the second is the deformation zone, the material deformed by shear, the third is the uniform zone at the exit of the channel (Fig.4).

In these conditions the equation of the flow line is defined in the three zones as following.

In the first zone **D$_1$** the material particle flows in the direction of the Oy axis.

The equation of the flow line is:

$$x = x_0 \qquad (2)$$

In this relation x_0 is the Lagrange factor of the flow line and its value belongs to the real numbers domain, between 0 and a.

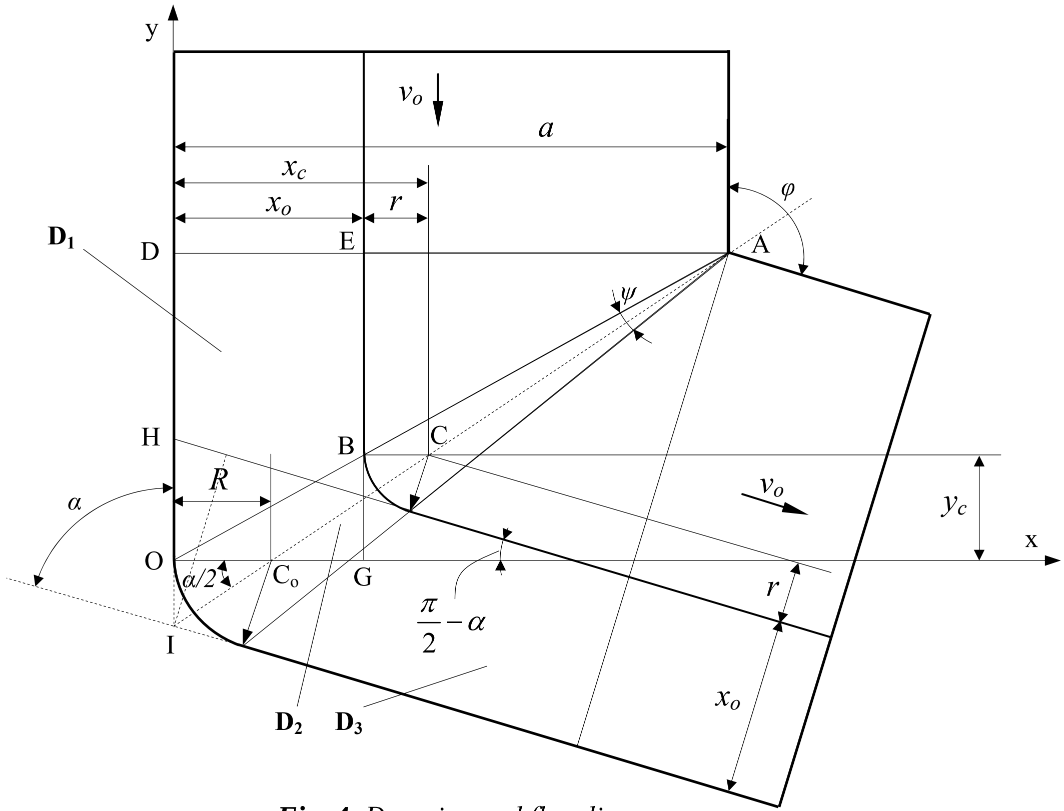


Fig. 4. *Domains and flow line.*

In the second zone **D$_2$** the elementary material particle goes from the first zone to the third zone through the circle arch *BF*.

The circle arch is determined by the construction of extrusion channel. In the practical conditions, at the passing from the vertical direction at the inclined direction, the channel has a cylindrical crossing zone.

We denote r_0 the radius of this cylindrical zone. The characteristics of the circle arch of the flow line are the radius *r* and the centre point *C*. It is evident the relation:

$$x_c = x_0 + r \qquad (3)$$

The coordinate y_c is defined by condition:

$$y_c = BG = EG - EB$$

but we have:

$$EG = OD = a \cdot \mathrm{tg}\left(\frac{\alpha}{2} - \frac{\psi}{2}\right)$$

and:

$$EB = AE \cdot \mathrm{tg}\left(\frac{\alpha}{2} - \frac{\psi}{2}\right) = (a - x_o) \cdot \mathrm{tg}\left(\frac{\alpha}{2} - \frac{\psi}{2}\right)$$

Results:

$$y_c = x_o \cdot \mathrm{tg}\left(\frac{\alpha}{2} - \frac{\psi}{2}\right) \qquad (4)$$

The angle ψ in the relation (4) is defined in function of the geometrical factors *a, r_0,* and α through the relation:

$$\mathrm{ctg}\frac{\psi}{2} = \frac{\frac{a}{R} - \sin^2\frac{\alpha}{2}}{\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}} \qquad (5)$$

The radius of the circle arch is defined from the condition:

$$\frac{BC}{OC_o} = \frac{AB}{AO} = \frac{AE}{AD}$$

Results: $r = R\dfrac{a - x_o}{a}$ (6)

The equation of the circle arch is:

$$(x - x_c)^2 + (y - y_c)^2 = r^2 \quad (7)$$

for $x \in [x_o, x_o + r(1 - \cos\alpha)]$

In the third zone the elementary particle goes a trajectory inclined at the angle *α* according to the Oy direction. The equation of flow line in the third domain is defined through:

$$y = -x \cdot \mathrm{ctg}\alpha + \frac{x_o}{\sin\alpha} - R \cdot \mathrm{tg}\frac{\alpha}{2} \quad (8)$$

3. Establishing the speed field

We consider v_0 the speed in the domain **D₁**.
Thus in the first domain we have:

$$v_x = 0, \; v_y = v_0 \quad (9)$$

In the third domain the speed field is defined by the expressions:

$$v_x = v_o \cdot \sin\alpha$$
$$v_y = v_o \cdot \cos\alpha \quad (10)$$

In the second domain we use the equation of the definition of flow line:

$$\frac{dx}{v_x} = \frac{dy}{v_y} \quad (11)$$

We differentiate the equation (7) and obtain:

$$\frac{dx}{dy} = -\frac{y - y_c}{x - x_c},$$

and we have:

$$\frac{v_x}{v_y} = -\frac{y - y_c}{x - x_c} \quad (12)$$

From the continuity condition we have:

$$v_x^2 + v_y^2 = v_o^2 \quad (13)$$

And, consequently it results:

$$v_x = v_o\frac{y - y_c}{r}$$
$$v_y = v_o\frac{x - x_c}{r} \quad (14)$$

for $x \in [x_o, x_o + r(1 - \cos\alpha)]$

4. Establishing the strain rate field

The components of the strain rate tensor are defined by the equations:

$$\dot{\varepsilon}_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right), \quad i, j = x, y \quad (15)$$

The detailed forms of these equations, in the conditions of the ECAE process are:

$$\dot{\varepsilon}_{xx} = \frac{\partial v_x}{\partial x}, \; \dot{\varepsilon}_{yy} = \frac{\partial v_y}{\partial y}, \; \dot{\varepsilon}_{xy} = \frac{1}{2}\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right) \quad (16)$$

From the continuity condition we have the relation:

$$\dot{\varepsilon}_{xx} = -\dot{\varepsilon}_{yy}$$

In the domains **D₁** and **D₃** the field of the strain rate field is defined by the components:

$$\dot{\varepsilon}_{xx} = -\dot{\varepsilon}_{yy} = \dot{\varepsilon}_{xy} = 0 \quad (17)$$

In the domain **D₂** we apply the conditions (16) of the equations (14) and obtain:

$$\dot{\varepsilon}_{xx} = -v_o\frac{(x - x_c)(y - y_c)}{\left[(x - x_c)^2 + (y - y_c)^2\right]^{\frac{3}{2}}} \quad (18)$$

$$\dot{\varepsilon}_{yy} = v_o\frac{(x - x_c)(y - y_c)}{\left[(x - x_c)^2 + (y - y_c)^2\right]^{\frac{3}{2}}} \quad (19)$$

$$\dot{\varepsilon}_{xy} = \frac{1}{2}v_o\frac{1}{\left[(x - x_c)^2 + (y - y_c)^2\right]^{\frac{1}{2}}} \quad (20)$$

The strain rate intensity is defined by the expression:

$$\bar{\dot{\varepsilon}} = \sqrt{\frac{2}{3}\left(\dot{\varepsilon}_{xx}^2 + \dot{\varepsilon}_{yy}^2 + 2\dot{\varepsilon}_{xy}^2\right)} \quad (21)$$

We consider relations (18), (19) and (20) in the equation (21) and, after some transformations, we obtain:

$$\bar{\dot{\varepsilon}} = \frac{2v_0}{\sqrt{3}r^3}\sqrt{4(y - y_c)^2\left[r^2 - (y - y_c)^2\right] + r^4} \quad (22)$$

This equation is an invariant of the strain rate tensor and represents the global effect of the deformation process.

5. Establishing the strain field

As a result of using the strain-rate, we can define the strain field. For the calculus of the strain rate components, we can use the general expression:

$$\varepsilon_{ij} = \int_0^t \dot{\varepsilon}_{ij} \cdot dt \qquad (23)$$

And for establishing of the strain intensity field we can use the expression:

$$\overline{\varepsilon} = \int_0^t \dot{\overline{\varepsilon}} \cdot dt \qquad (24)$$

The differential of the time must be defined in function of the extrusion speed and the differential of the coordinate *y*, respectively, thus we have:

$$dt = \frac{dy}{v_0} \qquad (25)$$

And coupling the expression (22) with the expression (25) we obtain:

$$\overline{\varepsilon} = \frac{2v_0}{\sqrt{3}r^3} \int_{y_\alpha}^{y_c} \sqrt{4(y - y_c)^2 \left[r^2 - (y - y_c)^2\right] + r^4}\, dy \qquad (26)$$

In the relation (26) we have:

$$y_\alpha = y_c \sin\alpha$$

Thus we can establish the strain field in the deformation zone.

Analyzing the expressions (18), (19) and (20) we observe that in zone of the entry of the extrusion die the deformations are null, also in the exit zone. The deformation is localized in the passing zone and consists in the crossing of material elements from the entry direction to the exit direction by a shear process (Fig.5).

***Table 1.** Numerical results*

Xo	Xmax	R	(α-Ψ)/2	Xc	Yc	X	Y	ε̇xx	ε̇xy	ε̇	Δd	Δt	Δε
0	2	2	38.66	2	0	0	0	0	1.25	1.4434	1.2649	0.1638	0.2820
						0.4	-1.2	-1.2	1.25	2.0008	0.5657	0.0722	0.1444
						0.8	-1.6	-1.2	1.25	2.0008	0.4629	0.0590	0.1118
						1.2	-1.833	-0.9165	1.25	1.7898	0.4196	0.0536	0.0896
						1.6	-1.9596	0.4899	1.25	1.5503	0.402	0.0512	0.0766
						2	-2	0	1.25	1.4434			0.7044
2	3.6	1.6	38.66	3.6	1.6	2	1.6	0	1.5625	1.8042	1.024	0.1328	0.2858
						2.32	0.64	-1.5	1.5625	2.501	0.4525	0.0578	0.1446
						2.64	0.32	-1.5	1.5625	2.501	0.3703	0.0472	0.1118
						2.96	0.1336	-1.1456	1.5625	2.2372	0.3357	0.0428	0.0896
						3.28	0.0323	-0.6124	1.5625	1.9378	0.3216	0.0410	0.0766
						3.6	0	0	1.5625	1.8042			0.7084
4	5.2	1.2	38.66	5.2	3.2	4	3.2	0	2.0833	2.4056	0.7589	0.0984	0.2824
						4.24	2.48	-2	2.0833	3.3347	0.3394	0.0434	0.1448
						4.48	2.24	-2	2.0833	3.3347	0.3463	0.0442	0.1420
						4.72	1.9904	-1.68	2.0833	3.0903	0.2424	0.0310	0.0880
						4.96	2.0242	-0.8165	2.0833	2.5837	0.2412	0.0308	0.0768
						5.2	2	0	2.0833	2.4056			0.7340
6	6.8	0.8	38.66	6.8	4.8	6	4.8	0	3.125	3.6084	0.506	0.0656	0.2594
						6.16	4.32	-2.9063	3.125	4.9278	0.2263	0.0290	0.1440
						6.32	4.16	-1.3179	3.125	5.0021	0.1852	0.0236	0.1118
						6.48	4.0668	-2.2913	3.125	4.4745	0.1678	0.0214	0.0894
						6.64	4.0162	-1.2247	3.125	3.8756	0.1608	0.0206	0.0770
						6.8	4	0	3.125	3.6084			0.6816
8	8.4	0.4	38.66	8.4	6.4	8	6.4	0	6.25	7.2169	0.253	0.0328	0.2824
						8.08	6.16	-6	6.25	10.0042	0.1131	0.0144	0.1446
						8.16	6.08	-6	6.25	10.0042	0.0926	0.0118	0.1118
						8.24	6.0334	-4.5825	6.25	8.9489	0.0832	0.0106	0.0884
						8.32	6.0106	-2.4338	6.25	7.7448	0.0816	0.0104	0.0778
						8.4	6	0	6.25	7.2169			0.7150

Consequently, in the domain **D₂** is developed great intensity of strain.

We consider $R=2$mm, $a=10$mm, $\alpha=\pi/2$, $v_0=5$mm/s and five flow lines and five points at the length of each flow line.

Using the methodology described above we obtain the numerical results showed in Table 1.

Under the graphical form, the results are represented in Figures 5, 6, 7, 8 and 9.

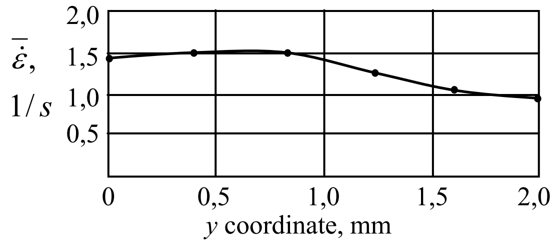


Fig. 5. *Variation of strain rate along the flow line for $x_0=0$.*

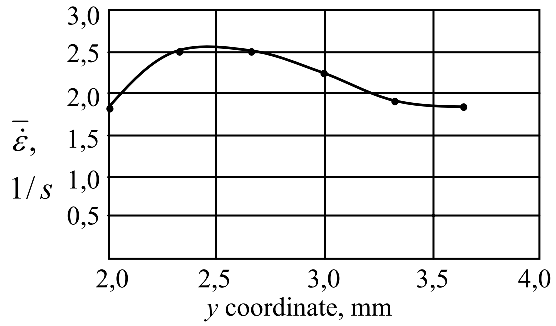


Fig. 6. *Variation of strain rate along the flow line for $x_0=2$.*

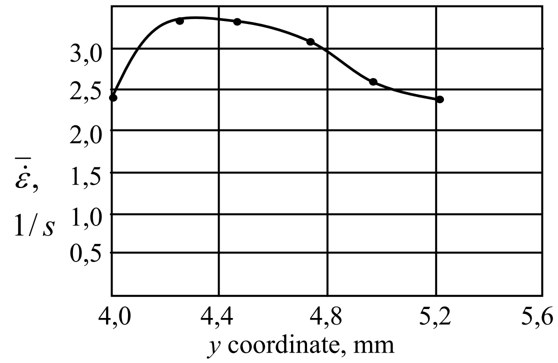


Fig. 7. *Variation of strain rate along the flow line for $x_0=4$.*

The strain intensity is calculated by numerical integration of the equation (24).

In this aim, we considered the continuity condition and we used the equation:

$$\bar{\varepsilon}_i = \frac{\bar{\dot{\varepsilon}}_{i-1} + \bar{\dot{\varepsilon}}_i}{2} \cdot \Delta t_i \qquad (27)$$

Δt_i is the time necessary for the passing of material particle of the Δl_i sequence along the flow line.

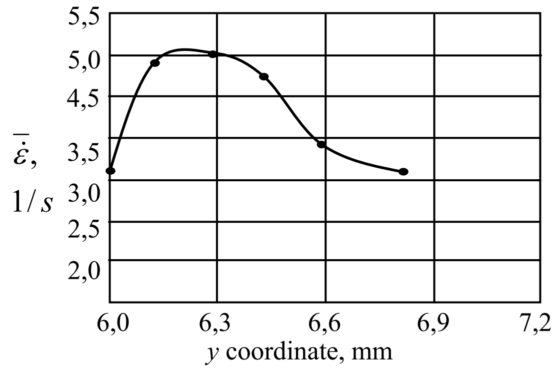


Fig. 8. *Variation of strain rate along the flow line for $x_0=6$.*

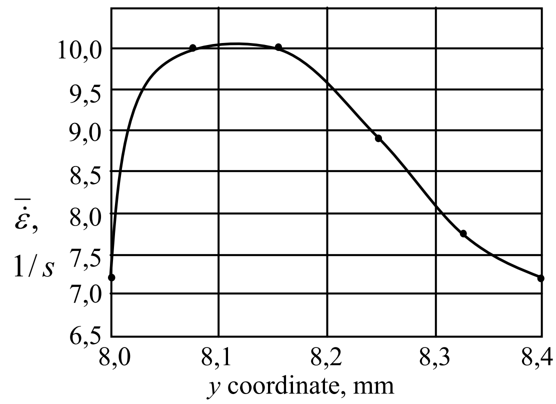


Fig. 9. *Variation of strain rate along the flow line for $x_0=8$.*

This factor is calculated with the equation:

$$\Delta t_i = \frac{4R}{v_0} \cdot \frac{\arcsin\left(\frac{\Delta d_i}{2R}\right)}{180} \qquad (28)$$

where

$$\Delta d_i = \sqrt{\Delta x_i^2 + \Delta y_i^2} \qquad (29)$$

The numerical values are written in Table 1. The distribution of the values of partial strain intensity is similar for each flow line.

The graphic of the variation of the partial strain intensity, along the flow line, is showed in Figure 10.

The partial strain rate is great at the entrance of the material in the second deformation domain. Then the value of the partial strain rate decreases.

The cumulate strain rate intensity is showed in Figure 11.

The cumulate strain rate increases and reaches the value of 0.352 – 0.367, a relatively great value.

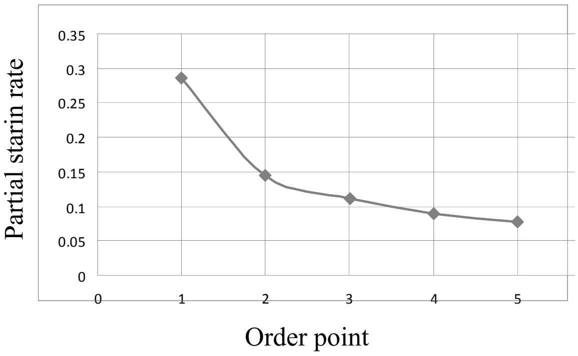


Fig. 10. *Variation of the partial strain rate intensity.*

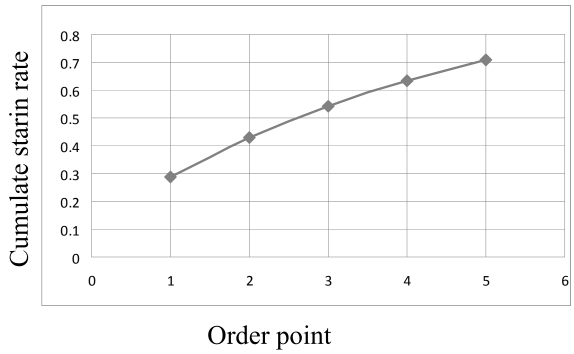


Fig. 11. *Variation of the cumulate strain rate intensity.*

6. Conclusions

The flow lines method allows the solving of the plastic deformation processes, such as the ECAE method.

The ECAE method permits intensive deformation process what leads to severe plastic deformation with a very fine structure.

The paper systematizes the basis aspects for application of the flow line method in case of the equal channel angular extrusion.

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