



## DIMENSIONLESS NUMBERS IN THE ANALYSIS OF HYDRODYNAMIC INSTABILITY OF INTERFACE STEEL - SLAG AT MICROSCALE IN REFINING PROCESSES

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### ABSTRACT

*In evaluation of the local hydrodynamics of interface, in steel-slag refining processes influenced by solutal effects, adequate scales of length and time satisfying certain conditions are necessary to get a correct and suggestive image of the weight of factors possible to be used in industrial technologies. These could be obtained based on already established common dimensionless numbers but carefully applied in a specific manner, taking into consideration particularities of the established interface and the main involved process. Based of the general philosophy of building dimensionless artificial conglomerates, a new dimensionless group  $N_i = Ma \cdot Bo$  is proposed in this paper as a consequence of needs to evaluate particularly the effects of capillarity actions at interface, especially of solutal origin, in presence of other physico-chemical and physical actions, especially when the effects of capillarity prevail over all others.*

**KEYWORDS:** steel refining, hydrodynamic instability of interface, length scale, time scale, dimensionless numbers.

### 1. Introduction

Despite the fact that capillarity is recognized as an important phenomenon in hydrodynamics of interface, only extremely rare and more than prudent approaches of this subject are present in scientific and technical papers dealing with process at interfaces in metallurgical systems steel-slag. Therefore in the following paragraphs there is a short introduction in the specific problems and aspects of this class of systems and this represents an attempt of the author to open in a certain measure new doors in the fundamentals of some new metallurgical processes of refining based on the advanced knowledge. As a main approach, specific dimensionless numbers are presented and comments are made on the connections with the hydrodynamics of interface steel-slag.

Solutocapillarity plays an important and particular role in the hydrodynamics of interface between liquid phases of technological interest in steel refining under slags.

The amplitude of the local fluctuations of concentrations at interface between steel and slag is limited compared to the possible local fluctuations of temperature in other systems or when strong gradients of temperature are imposed externally by special techniques or devices. Supplementary, the depths of the layers of steel and slag are of a certain established

extend because of the final purpose of the applied treatments which is to provide a certain appreciable high amount of refined steel, presenting a high level of purity concerning harmful elements. Oxygen, sulphur and phosphorus are the most frequently target elements of treatments applied to steels using slags. Several problems in analyzing the influence exerted by the presence of surface active solute in the liquid phases composing the refining system steel-slag are analyzed in a simplified manner due to their low specific concentrations [1][2].

At low levels of concentrations of surface active solutes, their influence on density is insignificant, also the corresponding aspects of buoyancy due to variations of density. Up to values of maximal concentrations of 2 mass% sulphur in slags and 0.02% mass% sulphur in steel, which are specific to desulphurization process at industrial scale, there are not reported sensible variations of density in the respective liquid phases, compared to the densities of the same liquids in absence of sulphur. Therefore it is possible that aspects of buoyancy could be totally neglected.

The solutocapillarity due to surface tension gradient produced by perturbation of concentration is a factor of instability while the surface tension acts to stabilize the interface; viscosity, diffusion and gravitation are also stabilizing actions. Viscous



diffusion acts to dampen the concentration fluctuations and the associated fluid flow. The gravity acts physically to flatten the interface and thus to stabilize the deformational perturbations. The presence of a heavier liquid phase below a lighter one (as it is the case of steel under slag) is also a stabilizing factor.

If it is intended to put in evidence the absolute role of the destabilizing factor–solutocapillarity in promoting flow, it is necessary that other destabilizing factors or factors damping the solutocapillarity effects to be minimized at certain level, below a certain threshold value of the involved parameter. This could be performed selecting or imposing values of different parameters in a physically acceptable range and leading finally to an adequate time scale. Several general remarks regarding properly named operations in scaling and in its preliminary stages and considerations are useful by their close relation with the technological and physical reality and utility. In this sense, in an such analysis principle of in acts a certain incertitude; imposing too restrictive conditions to obtain favorable conditions of prevalence of a certain physical action, compared to another, not only that this could not serve too much, but it could block the utility of the respective scale. For example, the characteristic time scale which is dependent upon the characteristic length at different powers. These negative aspects could be totally avoided by a correct scaling operation based on a correct meaning of the processes and on an adequate selection of the dimensionless numbers used in these cases. Further going there is also the possibility to introduce a new dimensionless number taking into account the need to evaluate the simultaneous complex action of many several forces involved in interfacial flow and dynamics as it is more and more encountered in literature.

## 2. Dimensionless numbers involved in the interfacial hydrodynamic instability

### 2.1 Principles and particularities

Technological systems in steel refining under slags could be included in the class of complex process of multi-phase flow, where a fluid interface separates immiscible phases, presenting a certain surface tension ( $\sigma_{steel}, \sigma_{slag}$ ) and leading to certain interface tension ( $\sigma_i$ ). The fluid interface is deformable, it has an unlimited extend, reported to the dimensions of the interfacial layer thickness and it is unbounded at wall, that means the interface is in the free-slip condition. This is generally valid if the friction at interface could be neglected or included in other quantified physical factors. Therefore, there is a force balance at slag-steel interface and this could be

decomposed into normal components (normal stress and pressure) and a tangential component (shear stress). Boundary conditions at interface lead to the following physical considerations [3]:

1. jump in fluid pressure is balanced by interface tension;

2. jump in fluid stress is balanced by surface gradient of interface tension (gradient along the interface)  $\nabla_S \sigma_i$ .

Macroscopic interfacial no-slip conditions break when at microscale, processes due to surface tension gradient, become relevant as magnitude. Analysis of this class of processes could be performed in conditions of the existence of a deformable interface but which doesn't deform because of the small weight of factors leading to deformations. This situation could be encountered at least in the incipient state of considered flow, or during time intervals where required conditions are accomplished. This is the case when physical actions producing deformations are not dominant in comparison to others. It results that, in conditions of adequate scaling of length and time of different actions, computations of parameters regarding the resulting particular and specific flow are realistic and possible to be made.

The known dimensionless numbers are limited as possibilities to give a good image of the physical reality in the problems mentioned in title. Several of them use a characteristic length with a certain specificity which differs from a phenomenon to another (it is well known that there are different independent scales of length). In fact, the main problem in the analysis of local instability produced by different actions, including the mass transfer of a surface active element, consists in a correct and specific scaling of length, velocity and time. It is known that every physical or physicochemical action is characterized by scales of length, time and velocity which are almost independent among them, but when actions are simultaneous, the fastest action has the largest probability to occur. If it is analyzed the occurrence of a certain physical or a physicochemical action it is necessary to scale the time and the conditions when it is the dominant action with a certain high degree of certitude. This is usually made establishing the condition when the time scale is shorter with at least an order of magnitude, compared to other concurrent actions. From this condition a characteristic length scale results, and in certain conditions, also a velocity scale results. Other times it is possible to start the scaling from velocity, instead of time and then the characteristic length and time result on this base.

This kind of analysis is often the single way to get some information on the possible behavior of a system and to perform a simulation concerning the behavior of the considered system. In the system



steel-slag at liquid phases temperature it is obvious the difficulty of any direct observation. Some enough valuable data could be obtained using the rapid solidification method of samples containing portions of interface steel-slag.

Many dimensionless numbers represent ratios of two physical actions (forces) or of two families of actions. This level of synthetic representations despite extremely useful, does not satisfy the actual necessities, mainly consisting in the enlargement of the set of actions evaluated in such manner.

At small time and length scales, different phenomena leading to thermodynamic equilibrium exhibit strong particularities affecting the hydrodynamics in a dramatic manner. Local fluctuations of temperature and concentration at the interface or at free open surface alter locally the surface tension and if these local inhomogeneities are continual and persistent enough time, they could give a spontaneous flow (Marangoni effect). Therefore it is necessary always to relate the analyzed processes to adequate scales of time and of length. In such conditions the surface tension gradients  $\partial\sigma/\partial x$  contribute to a force proportional to the thermal coefficient  $|\partial\sigma/\partial T|$  or to the concentration coefficient  $|\partial\sigma/\partial c|$  of interface tension. In this paper only the influence of concentration will be considered, based on the physical reality consisting in much more values of the thermal diffusion and conductivity in slag and steels, compared to their mass equivalents.

## 2.2. Dimensionless numbers derived from the normal component of the free slip boundary condition

The normal component of the free-slip boundary conditions gives by scaling the **Laplace number (La)**; it represents a ratio of surface tension and inertia forces to the viscous forces, expressing the momentum-transport (especially dissipation) inside a fluid. **La** dimensionless number is linked to the free convection inside of immiscible fluids and is defined by the relation:

$$La = \frac{\sigma \cdot \rho \cdot L}{\mu^2} = \frac{\sigma \cdot L}{\rho \cdot \nu^2} \quad (1)$$

where:  $\sigma$  – is surface tension;  $\rho$  – is density;  $L$  – is characteristic length;  $\mu$  – is dynamic viscosity;  $\nu$  – is kinematic viscosity, also called momentum diffusivity.

Another form is Ohnesorge number  $Oh = La^{-2}$ .

**Bond dimensionless number (Bo)**, also called the static **Bo** number in opposition with another form called dynamic Bond dimensionless number is given by the following ratio [5]:

$$Bo = \frac{\rho \cdot a \cdot d^2}{\sigma} \quad (2)$$

Where:  $\rho$  – is the density, or the density difference between fluids;  $a$  – is the acceleration of the body force.

Mainly **Bo** dimensionless number could be derived from the general dimensionless number  $N$  introducing gravitation scaling of the pressure  $P$ .

It quantifies the importance of body forces over the surface tension forces, connected with the role of equilibrium capillarity ( $\sigma$ ). When the acceleration  $a$  is gravity acceleration  $g$ , The **Bo** number is the most common comparison of gravity and surface tension effects, usually used to find the characteristic length scale in complex scaling problems.:

$$Bo = \frac{\rho \cdot g \cdot d^2}{\sigma} \quad (3)$$

Surface tension dominates the flow at small values of  $d$ , this being the single parameter of a given system where the density, and gravity acceleration are constant and there is a certain variability of surface tension upon the concentration of a surface active solute. At interface steel-slag the competition between stabilizing factors of interface is from far in favor of surface tension if the length scale is small enough. A value  $Bo < 1$  indicates that surface tension dominates. The problem which is still persistent is to establish the length scale  $d$ . At  $Bo = 1$  the capillary length scale is obtained:

$$L_c = (\sigma / \rho \cdot g)^{1/2} \quad (4)$$

At an interface between two immiscible superposed liquids of density  $\rho_{top} = \rho_1$  and  $\rho_{bottom} = \rho_2$  the capillary length is obtained replacing  $\rho$  by  $\Delta\rho = \rho_2 - \rho_1$ :

$$L_c = (\sigma / \Delta\rho \cdot g)^{1/2} \quad (5)$$

If the interface tension gradient is persistent enough time and is continual on interface, it could be taken into account also in the relation giving the capillary action of the interface tension gradient of solutal origin:

$$L_c = (\Delta\sigma / \Delta\rho \cdot g)^{1/2} \quad (6)$$

At lengths  $L \leq L_c$  the capillarity prevails over gravity. This limit extend of action of capillarity must be corrected, taking into consideration the values where the capillarity prevails over other competing actions.

Other forms expressing the ratio between gravity forces and surface tension are also used in the form of other dimensionless numbers.

Influences exerted by the solutal effects, reflected in the interfacial tension coefficient, using the Rayleigh number and the dynamic Bond ( $Bo_d$ ) number, where the relative importance of buoyancy is



evaluated, are difficult to be put into evidence. Despite the fact that the Rayleigh dimensionless number usually refers to thermal effects on buoyancy due to the modifications of concentration, it can also refer to the solutal effects but only related to the modifications of the density due to this kind of effects.

**Capillary dimensionless number ( $Ca$ )**, also called as **crispation dimensionless number ( $C$ )** [6] is specific to situations where the scale of stresses in presence of surface tension is a viscous one.

It is given by the relation:

$$Ca = \sigma / \mu \cdot V \quad (7)$$

Where  $V$  – is the velocity scale.

If the velocity scale is given by the mass diffusion (D-isothermal mass diffusion coefficient), in a layer of depth  $d$ :

$$V = D / d \quad (8)$$

Which gives:

$$Ca = \frac{\sigma \cdot d}{\mu \cdot D} = Bo \cdot Ga \quad (9)$$

Another form using the same notation is the inverse form:

$$Ca = \frac{\mu \cdot D}{\sigma \cdot d} = Bo / Ga \quad (10)$$

If the velocity scale is Marangoni velocity  $V_{Ma}$ , the capillary number can be written in the form:

$$Ca = \frac{(-\partial\sigma/\partial C) \cdot \beta \cdot d}{\sigma} \quad (11)$$

In fluid dynamics, the  $Ca$  number in the form of rel.(10) expresses the relative effect of viscous forces versus surface tension acting across an interface between a liquid and a gas, or between two immiscible liquids. When  $Ca < 10^{-5}$  it is possible to neglect the effects of viscous forces compared to surface tension and this is specific to many liquids.

The dimensionless number  $Ca$  is useful but needs again a specified value of the characteristic length  $d$  which remains also to be established at an adequate magnitude, corresponding to necessities.

The inversely written is convenient in treating the  $Ca$  and  $Ga$  in the same way in the stress balance at boundaries.

**Weber dimensionless number ( $We$ )** [7] characterizes the relative importance of the deformability of an interface, giving a measure of the relative importance of the inertia of fluids compared to its surface tension, according to the relation:

$$We = \frac{\Delta\rho \cdot V^2 \cdot d}{\sigma} \quad (12)$$

Where:  $\rho$ - is the density of the fluid;  $V$ - is the fluid velocity;  $d$ -is the characteristic length;  $\sigma$ - is the surface tension. In conditions of terrestrial gravity  $g$ ,

replacing  $V^2 = g \cdot d^2$ , the dimensionless number  $We$  can be written as:

$$We = \frac{\Delta\rho \cdot g \cdot d^2}{\sigma} \quad (13)$$

The characteristic length in the case of a drop is its diameter. When inertial effects dominate over viscous effects, the stress could be expressed in dimensionless form based on the inertial scale.  $We$  number, which expresses the relative influence of inertia and hydrodynamic pressure related to the capillary pressure, it is particularly useful in analyzing fluid flow where there is an interface between two different fluids, especially for multiphase flows presenting curved surfaces, as it is the case of drop or bubble migration with significant surface deformation.

Apart from the above dimensionless numbers there are others expressing also relations between important physical actions involved in hydrodynamic stability/instability of interfaces between fluids. These numbers could be obtained as composites of some already established dimensionless numbers. **Galilei dimensionless number ( $Ga$ )** is one of them and results as the ratio of gravity forces divided by viscous forces[8], as ratio of dimensionless numbers  $Bo$  and  $Ga$ , or more simply forming a dimensionless group of quantities  $g$ ,  $\nu$ ,  $D$  related to the same characteristic length scale  $d$ :

$$Ga = \frac{Bo}{Ca} = \frac{g \cdot d^3}{\nu \cdot D} \quad (14)$$

In terms of characteristic time scales, the dimensionless number  $Ga$  could be written as it follows:

$$Ga = \frac{\tau_{diff} \cdot \tau_{visc}}{\tau_{grav}^2} \quad (15)$$

Where:

$\tau_{diff} = d^2 / D$  – is the time scale of mass diffusion;  $\tau_{visc} = d^2 / \nu$  – is the time scale of viscosity;  $\tau_{grav} = (d/g)^{1/2}$  – is the gravity time scale .

The Galilei number is used more frequently in viscous flow and thermal expansion calculations, for example to describe fluid film flow over walls. By extension it could be used for the case of fluid flow at interfaces on a side or the other side of interface. In these cases the mass diffusion coefficient of the considered solute and the kinematic viscosity of the considered fluid will be taken into account in calculations.

The gravity time scale considered  $\tau_{grav}$  represents in this case the time needed for a body to travel a distance  $d$  under the gravity acceleration  $g$ .

Considering the Boussinesq approximation and using the definitions of Rayleigh dimensionless number and Galilei dimensionless number, a parameter called Boussinesq parameter could be computed:

$$Ra/Ga = \alpha \cdot \Delta T \quad (16)$$

From here it follows that

$$\tau_{grav} / \tau_{buoy} = (\alpha \cdot \Delta T)^{1/2}$$

Further, other useful expressions of  $Ga$  number will be presented in the context of the present paper.

### 2.3. Dimensionless numbers derived from the tangential component of the free slip boundary condition

The Marangoni dimensionless number ( $Ma$ ) is linked to the Marangoni effect (sometimes also called the Gibbs-Marangoni effect) which consists in the mass transfer along an interface due to surface tension gradient. The surface tension gradient can be caused by a concentration gradient or by a temperature gradient. The presence of a gradient in surface tension will naturally cause the liquid to flow away from regions of low surface tension. The Marangoni number for the solutal case, valid for isothermal conditions, is the ratio of capillary force due to the surface tension gradient of solutal origin to the viscous drag in the flow. It can be defined in several ways but an accepted form of larger utility is the following [9]:

$$Ma = \frac{(-\partial\sigma/\partial C) \cdot \Delta C \cdot L}{\mu \cdot D} \quad (17)$$

Where:  $\partial\sigma/\partial C$  is the concentration coefficient of the surface tension or of the interfacial tension, related to the surface active solute  $C$ , in  $N \cdot m^{-1} \cdot (mass\%)^{-1}$  or in  $N \cdot m^{-1} \cdot (mole\ fraction)^{-1}$ ;  $\Delta C$  is a characteristic concentration difference across the liquid layer of slag or, after case, along its surface, expressed in terms of solute ( $C$ ) content, in mass% or mole fraction, or a characteristic concentration of the liquid phase ( $C$ )<sub>0</sub>;  $L$  is a characteristic length, in m;  $\rho$  is the density of the phase, in  $kg/m^3$ ;  $D$  is the mass diffusion coefficient of solute,  $m^2/s$ ;  $\mu = \rho \cdot \nu$  is the dynamic viscosity of the considered phase, in Pa·s;  $\nu$  is the kinematic viscosity of the considered phase,  $m^2/s$ .

Dealing with solutal surfactant problems the corresponding Marangoni dimensionless number is also called elasticity number and is generally defined accordingly for diffusion and adsorption processes. If there are adsorbed surfactants, the product  $(\partial\sigma/\partial C)C_0$  can be taken as a coefficient of elasticity in compression of the surfactant monolayer, where  $C$  denotes the excess surfactant surface concentration and  $C_0$  is a reference value.

In the solutal Marangoni number can be identified a group presenting dimensions of velocity, called Marangoni velocity:

$$V_{Ma} = \frac{(-\partial\sigma/\partial C) \cdot \Delta C \cdot (d/L)}{\mu} \quad (18)$$

Where  $L$  is the horizontal, lateral extent of the liquid layer and  $d$  is the layer depth where there is a difference of concentration  $\Delta C$ . The ratio  $d/L$  represents the coefficient of slenderness of the liquid system and is used mostly in liquid bridges. If it is considered  $d=L$ , which is acceptable for continuity reasons and equal conditions of comparison, it results:

$$V_{Ma} = \frac{(-\partial\sigma/\partial C) \cdot \Delta C}{\mu} \quad (19)$$

This is the case of many situations encountered in interfacial mass transfer of surface active solute soluble in immiscible liquid including the systems steel-slugs. In order to specify this and to keep the importance of dimension scale, if the solutal Marangoni effect is present on a depth  $d$  of the layer, a parameter can be written:

$$\beta = \Delta C / d \quad (20)$$

Or generally, using cartesian coordinate  $z$  instead of  $d$ , this became the slope of concentration profile:

$$\beta = \frac{\partial C}{\partial z} \quad (20')$$

The Marangoni dimensionless number reaches the form:

$$Ma = \frac{(-\partial\sigma/\partial C) \cdot \beta \cdot d^2}{\mu \cdot D} \quad (21)$$

In this form the Marangoni number represents the ratio between the capillary force due to the surface tension gradient and the viscous drag in the flow.

It is interesting to show that noting:

$$\gamma_{\sigma(C)} = \frac{\partial\sigma(C)/\partial C}{\sigma(C)} \quad (22)$$

The dimensionless solutal Marangoni number could be written using the form of  $Ca$  dimensionless number from rel. (10) in the following form:

$$Ma = \gamma_{\sigma(C)} \cdot \Delta C \cdot Ca^{-1} \quad (23)$$

Using the form of  $Ca$  dimensionless number from rel. (9), the following form is obtained:

$$Ma = \gamma_{\sigma(C)} \cdot \Delta C \cdot Ca \quad (24)$$

This last relation remembers formally, in a certain measure, to  $Ra = \alpha \cdot \Delta T \cdot Ga$  from rel.(16) which refers to buoyancy effect.

The characteristic time scale of the capillarity due to solutal gradient of surface tension is given by:

$$\tau_{Ma} = \left( \frac{\rho \cdot d^3}{|\partial\sigma(C)/\partial C| \cdot \Delta C} \right)^{1/2} \quad (25)$$



The time scale associated with capillary forces acting at a deformable interface (Laplace force in the normal stress boundary condition) is the following:

$$\tau_{cap} = \left( \frac{\rho \cdot d^3}{\sigma} \right)^{1/2} \quad (26)$$

It results the ratio:

$$\frac{\tau_{cap}^2}{\tau_{Ma}^2} = \frac{|\partial\sigma(C)/\partial C| \cdot \Delta C}{\sigma(C)} = \gamma_{\sigma(C)} \cdot \Delta C \quad (27)$$

### 3. Other possible groups proposed to evaluate particularly the effects of capillarity actions at interface in presence of other physico-chemical and physical actions

A dimensional number called Marangoni coefficient  $N_{Ma}$ , in  $[m^{-2}]$  was introduced and argued by D. Agble and T.A.Mendes-Tatsis[10] based on a phenomenological approach as having the ability to express, in a quantitative manner, the influence of factors that initiate or inhibit Marangoni convection, when surfactant transfers from an aqueous phase A into an organic phase B. High values of  $N_{Ma}$  are obtained in conditions that promote Marangoni convection and smaller values when conditions that inhibit Marangoni convection are prevalent.

It presents a high numerical sensitivity and was proven as being able to predict stability in 29 cases from 30 studied cases, when  $N_{Ma} > 10^{-9} m^2$ . According to its definition it represents a ratio between the surface tension and its associated effects which produce surface motion and the viscous and related effects that restrict the surface movements:

$$N_{Ma} \propto \frac{\left( \frac{d\gamma}{\gamma} \right) d\gamma}{\mu_B D_{AB} \left( RMM_{surfactant} / RMM_{phase} \right)} \cdot \frac{C_{surfactant}}{\Gamma} \quad (28)$$

There is a certain touch of tautology at the numerator in the expression of the Marangoni coefficient  $N_{Ma}$  established according to [10], which seems to be due mainly because of the non-linear dependence of the surface tension upon the concentration of the surface active and transferable solute.

Starting from these aspects, but following the philosophy used in forming the Marangoni

coefficient, interfacial instability versus stabilization actions could also be well characterized by a new dimensionless number ( $Ni$ ) introduced in the following paragraphs in this paper for the case of complex effects of capillarity force at a unique length scale in depth and along the interface. It could be derived combining the specific actions involved in the Marangoni dimensionless number and in the Bond dimensionless number (or Weber dimensionless number according to the case), rearranging the specific actions in order to put into evidence the ratio between destabilizing effect of capillarity force (solutal, thermal or both two) due to the gradient of the surface tension, in the form containing the concentration coefficient of the surface/interface tension or the temperature coefficient of the surface/interface tension and the stabilizing effect of interface tension force.

Unfortunately, the total difference from the other dimensionless numbers, made impossible the nomination of this new proposed dimensionless number in other way than it was made in this paper. Otherwise, it is known that nomination of dimensionless numbers using the same basic symbols, even when using sub-scripts, already introduces frequently strong confusions with negative effects in understanding and handling in complex relations. When it is written only for soluto-capillarity effect, the new proposed dimensionless number symbolized ( $Ni$ ) by the author of the present paper, reach the form:

$$Ni = Ma \cdot Bo = \frac{\text{solutocapillarity force}}{\text{viscous forces}} \cdot \frac{\text{gravity force}}{\text{interface tension force}} = \frac{\text{solutocapillarity force}}{\text{interface tension force}} \cdot \frac{\text{gravity force}}{\text{viscous forces}} \quad (29)$$

$$Ni = Ma_{solutal} \cdot Bo = \gamma_{\sigma(C)} \cdot \Delta C \cdot Ga \quad (30)$$

Replacing the solutocapillarity effect by the thermocapillarity effect an equivalent form could be obtained.

$$Ni = Ma_{thermal} \cdot Bo = \gamma_{\sigma(T)} \cdot \Delta T \cdot Ga \quad (31)$$

In terms of the corresponding time scales of the included physico-chemical actions, the dimensionless number  $Ni$  reaches the following form:

$$Ni = \frac{\tau_{cap}^2 \cdot \tau_{visc} \cdot \tau_{diff}}{\tau_{Ma}^2 \cdot \tau_{grav}^2} \quad (32)$$

The new dimensionless number could quantify better, the contributions of stabilizing actions versus destabilizing actions at interface between immiscible liquids in conditions when the action of capillarity force (solutal or thermal) prevails over the other physico-chemical or pure physical actions.



The new proposed dimensionless number  $Ni$  addresses sharp interfaces, evaluating separately in each phase the possibilities of instability to occur in established conditions. This is possible if a convenient length scale is established for all physico-chemical actions. Also, it allows a computation of the major factor relevant in industrial technology – the characteristic concentration ( $\Delta C$ ) or temperature ( $\Delta T$ ) which could vary or could be modified by different internal or external processes or by different intended actions. This dimensionless number, in the mentioned conditions, leads to a more suggestive comparison between the thermal and solutal capillarity effect because it contains simultaneously both the interfacial tension and the concentration coefficient of this quantity, or after the case the thermal coefficient.

This kind of treating specific problems based on new proposed dimensionless groups is not an isolated one especially when particular cases of flow must be evaluated.

As an example is the dimensionless number Bejan in fluid mechanics and heat transfer representing the dimensionless pressure drop along a channel of length  $L$  and playing the same role in forced convection as Rayleigh dimensionless number in natural convection [11].

#### 4. Conclusions

Various dimensionless numbers derived from the normal component of the free slip boundary condition present a limited capability to express quantitative aspects at interface involved in hydrodynamic aspects at interface with interfacial tension, because they are simple ratios of separate physical actions. Nevertheless they are able to give a characteristic length scale and time scale if physical factors involved in their computations are well evaluated.

Marangoni dimensionless number, despite useful as proved until now in many cases, presents the major difficulties in obtaining results of high degree of validity and generality because it does not include the effects due to the presence of gravity as state and of interfacial tension, both exerting stabilizing effects.

Proposal of the dimensional number under the form of Marangoni coefficient, besides the fact it is

unusual compared to the requirement and benefits of the dimensionless analysis, it is difficult to be used in other systems than those aqueous-organic, and it takes into account neither the whole panel of physico-chemical actions present at interface, nor a whatever characteristic length scale.

The new proposed dimensionless group  $Ni = Ma \cdot Bo$  and its different expressions showed in this paper, exhibits many and improved possibilities to evaluate the occurrence of Marangoni flow and convection.

This new dimensionless number, in one of its computation forms contains the Galilei dimensionless number which introduces a convenient characteristic length scales allowing keeping the interface flat. Also it contains the coefficient expressing the ratio between the product of the characteristic concentration and the concentration coefficient of interfacial tension and the interfacial tension as in the dimensional Marangoni coefficient.

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