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THEORETICAL STUDY REGARDING THE FLOW CONDITIONS FOR ALLOYS THROUGH THIN CHANNELS

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ABSTRACT

A common tool used to demonstrate the phenomenon of movement of liquids through thin channels is the capillary tube. When the lower end of the tube is placed vertically in a liquid (such as water), it forms a concave film. Surface tension pulls the liquid column up until there is a sufficient mass of liquid for gravitational forces to overcome intermolecular forces. The contact length (around the edge) between the top of the liquid column and the tube is proportional to the diameter of the tube, while the weight of the liquid column is proportional to the square of the tube's diameter, so a narrow tube will draw a liquid column higher than a wide tube. This paper wants to bring theoretical arguments to the possible flowing limits for alloys through thin channels.

KEYWORDS: thin channel, flow, limit conditions, capillarity, surface tension

1. Properties of metal melt

Metal melt properties can be grouped as follows:

-technological properties (fluidity or flow capacity, viscosity, density, surface tension);

-physical properties;

-thermodynamic properties.

Properties such as density, viscosity, surface tension, electrical conductivity are sensitive to structural features; they provide us with important information related to the melt structure.

1.1. Fluidity

Metal melt fluidity is one of the properties, which is related to the shape of the resulting product from solidification. Fluidity is defined as:

-flow capacity of the alloy-channels and cavity shape, expressed by the filling time, flow velocity, the length travelled in channels and filled cavities;

-filling capacity of the cavity shape, reproducing the finest details of configuration.

Factors influencing fluidity are:

-material properties shape;

-conductivity material form;

-diffusivity, heat storage capacity;

-cavity shape geometry, hydraulic pressure, external pressure applied to the alloy;

- viscosity, way of crystallization, purity alloy, casting temperature, specific heat, latent heat of crystallization, solidification range, thermal conductivity, surface tension, which are intrinsic properties of the alloy;

-casting conditions.

The biggest influence on the fluidity has the solidification range of the alloy. When solidification range is higher, then flow is lower. Pure metals, eutectic alloys and chemical compounds have the biggest fluidity.

Alloy flows as long is in liquid phase and a period after solid phase occurred, see Fig 1.

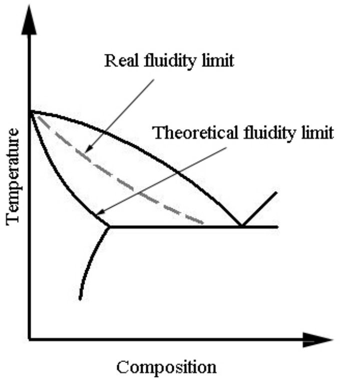

Fig. 1. *Temperature range up to the alloy solidification runs.*

Casting temperature affects fluidity of the alloy in the sense that higher temperatures mean higher fluidity. When specific heat and density of the alloy it’s higher, then the flow will be higher.

Thermal diffusivity of alloy (a=λ/C$_p$ρ) influences fluidity in the sense of her decreasing. Latent solidification heat (melting) affects the flow upwards.

Viscosity determines the flow velocity, which is defined by the dimensionless Reynolds criterion:

$$R_e = \frac{V \cdot d}{\nu} = \frac{V \cdot d \cdot \rho}{\eta} \qquad (1)$$

where: v is the linear flow velocity; d – hydraulic diameter; ν - cinematic viscosity, η - dynamic viscosity.

The influence of viscosity on fluidity resulting from the equation of laminar flow and alloy fluidity, being inversely proportional to the cinematic viscosity

$(f = \frac{1}{\nu\rho})$.

Surface tension influences fluidity by moistening or not moistening the alloy flowing channel. Alloys moistening the wall form have greater ability to fill the cavity mould (better fluidity).

The influence of material form is manifested by its ability to take heat from the liquid alloy and to liberate it.

When the ability to absorb heat transferred form the alloy is higher, then the flow decreases faster. If the coefficient of heat accumulation of the form is higher ($b_f = \sqrt{\lambda_f \cdot c_{p \cdot f} \cdot \rho_f}$), then fluidity will be lower.

1.2. Density

At binary alloys consisting of components having similar physic-chemical properties the Vegard law can be applied:

$$\rho_{aliaj} = \rho_1 \cdot x_1 + \rho_2 \cdot x_2 \qquad (2)$$

where: ρ_1 and ρ_2 are the component densities 1 and 2, x_1 and x_2 atomic fractions of components 1 and 2.

In case of alloys components with differing physical - chemical properties, the Vegard law can not be used to determine alloy density because significant deviations occur.

Fraction between density and temperature of the metal is defined as the density temperature coefficient (dρ / dT). For most elements, temperature coefficient of density increases linearly with the melting temperature.

Density of liquid metal is dependent on temperature, which is given by:

$$\rho_t = \rho_o + a \cdot t \qquad (3)$$

where: ρ_o and a are constants that depend on the alloy nature, T - temperature in ° C.

In case of transition metals with incomplete 3d layer, the density is given by:

$$\rho = \rho_{top}\left[1 - X'\left(T - T_{top}\right)\right] \qquad (4)$$

where ρ_{top} is the density of liquid metal at absolute temperature of melting (T$_{top}$); X' -effective temperature coefficient of density.

This factor differs from the actual temperature coefficient of density in a relatively small heating over T$_{top}$, is given by:

$$X' = \frac{1}{\rho} \cdot \left(\frac{\partial \rho}{\partial T}\right)_p \qquad (5)$$

1.3. Viscosity

Viscosity is the property of fluids to oppose deformation (movement) but which does not reduce their volume by developing unitary effort. This property is manifested only in fluids in motion and expresses the inner friction between fluid layers moving at different speeds.

The viscosity can be defined as follows: between the applied force (noted with ε) acting per unit area, which determines the relative displacement of two adjacent layers of fluid and the velocity gradient perpendicular to the applied force (dU / dZ) there is proportionality. Proportionality factor (η) is the coefficient of viscosity or dynamic viscosity.

$$\eta = \frac{\varepsilon}{dU / dZ} \qquad (6)$$

Relation (6) is the mathematical expression of Newton's law.

Stokes defined the relation between dynamic viscosity and density as the fluid cinematic viscosity:

$$\nu = \frac{\eta}{\rho} \qquad (7)$$

Cinematic viscosity of metals and alloys at casting temperature is almost smaller than the temperature of water at room temperature, thus explaining their high fluidity. A relation is established semi-empirically by Andrade for dynamic viscosity of liquid metal at melting point:

$$\eta = \frac{B\left(M \cdot T_{top}\right)^{\frac{1}{2}}}{V^{\frac{2}{3}}} \qquad (8)$$

where: B is a constant, M - atomic mass, V - atomic volume.

Dynamic viscosity varies with temperature for liquid metals and alloys. This dependence is expressed by the relationship:

$$\eta = \eta_O \cdot e^{-\frac{E_V}{R\cdot T}} \qquad (9)$$

where: η_O is the viscosity at reference temperature; E_V - activation energy of viscous flow.

Dynamic viscosity decreases with increasing of the temperature.

For binary liquid alloys, which are close to ideal alloys, alloy viscosity can be determined with relation:

$$\eta_{alloy} = \chi_1 \cdot \eta_1 + \chi_2 \cdot \eta_2 \qquad (10)$$

where: χ_1 *and* χ_2 are molar fractions of components 1 and 2; η_1 *and* η_2 - viscosity components.

Viscosity is influenced by the existing impurities in metals and alloys.

1.4. Superficial tension

Surface tension is one of the properties of metal melt, which depends on two other properties, cohesion and adhesion. Atomic-scale phenomena can be regarded as if atoms or molecules within a substance are spaced sufficiently for each of them to be surrounded by a field of forces, which form a system equivalent to zero. Forces are forces of cohesion, named “Van der Waals” forces.

To separate atoms or molecules of the same type, some others must consume a mechanical work; it named mechanical work of cohesion.

The force that opposes atoms separation or molecules of that substance reported at length shall be known as surface tension (σ). At the melt surface occurs a flux of collectivized electrons, which tend quit the melt, but positive ions pull them back into the melt.

Metal melt surface is covered by a thin layer of liquid with negative loads, under which at a certain depth of atomic radius order, it is located a layer with compensating positive electrical loads. The two layers form a double layer with a thickness of interaction radius of an atom order. This double layer acts as an electrical condenser as a barrier, preventing, at least partially, the outgoing of electrons molten metal.

An atom in the melt, under the double layer, at a distance greater than the range of inter-atomic forces is surrounded by a field of forces, which together with its own forces form a system equivalent to zero. It has an effect of mutual annihilation of the forces of interaction between atoms, which allows free movement, as there is cohesion between them.

The surface tension varies with the degree of dispersion and the specific surface of dispersed phase. Under specific surface we understand the relation between the surface phase and its volume. When the dispersion exceeds a certain limit, the surface tension starts to decrease, and when you reach the molecular dispersion of surface, the tension tends to zero.

A specific area can be obtained by dispersing phase or substance, or by inoculation in a liquid of very fine particles.

2. Obtaining ceramic-metal materials by impregnating

One method of obtaining ceramic-metal composites is to impregnate the porous ceramics with molten metal.

Although this process has been less studied and applied, it presents a higher interest. The process is based on the penetration principle of molten metal under the action of capillary forces.

The impregnation method finds many applications that can be infiltrated in the ceramic base mass of several types of molten metals and alloys.

The impregnation mechanism is governed by the relation of possible interaction between the solid phase with high melting point (ceramic shell) and liquid metal phase.

In this sense the following issues are highlighted:

-chemical reactivity, dissolving in metal of ceramic parts, with two consequences, the chemical transformation of metal and destruction of ceramic shell;

-moistening of ceramic material by molten metal.

For moistening, there are three possible cases:

Case 1: lack of solid surfaces moistening by liquid and lack of solubility of an element in the other;

Case 2: good moistening, but without solubility;

Case 3: good moistening and limited solubility of solid in liquid.

Case 1: contact angles and interfacial tensions are very high and the capillary action tends to reject the metal from blanks of ceramic shell. In this case it can be used an external force to force molten metal flow in capillary spaces or in some cases the problem can be solved by interfacial additives which tend to decrease moistening angle.

Case 2: solubility of the refractory metal oxide is practically zero, but the contact angle is very low (below 90°). In this case capillary forces are sufficient to induct metal penetration in ceramic structure.

Case 3: the diffusion phenomena and fluid flow creating a new act in accord with capillary forces. The liquid phase enters the capillaries, it tends to advance in areas where solid particles are in contact (inter-granularly boundaries). At extremes, the ceramic shell can be disaggregated but solid liquid phase saturation at the domain limit solid solution before moistening, avoids excessive transport of material. Dissolved substance remains in solution or precipitates on existing solids particles or even precipitates separately in the liquid phase of the solidified metal. This case is more complicated because in addition to capillary phenomena occur the diffusion of liquid or viscous creep processes. Thus, temperature and time become very important factors to control the impregnation process.

Classical methods of impregnation with molten metal are:

-total immersion - ceramic shell (matrix) is totally immersed in the metal bath;

-capillary-immersion - ceramic shell is suspended - partly immersed in the metal bath;

- through contact - ceramic shell is attached to solid objects falling gradually and melting by heating.

To obtain adequate impregnation are frequently applied external forces such as: supply by drop, compressing the fluid, centrifugal force or vacuum aspiration.

Composite material consisting of ceramic and metal is called cermet. A cermet is designed to have optimal properties of both components, such as high temperature resistance and hardness (like ceramic) and resistance to plastic deformation (taken from metal).

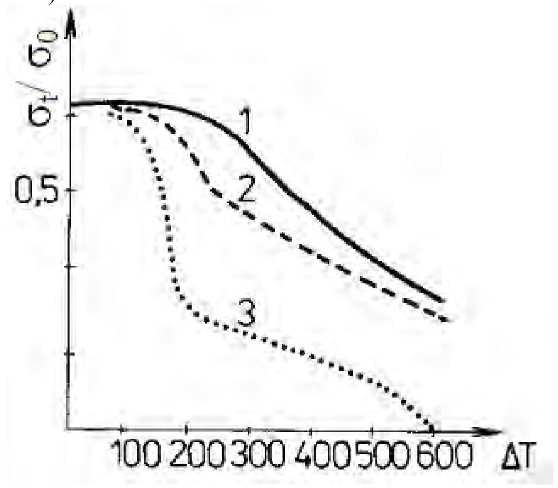

Fig. 2.*Resistance variation at thermal shock for various cermet.*
1 – cermet Al_2O_3-Cr, granulary diameter of Cr=1 μm; 2 – cermet Al_2O_3-Cr, granulary diameter of Cr=25 μm; 3 – Al_2O_3.

Figure 2 shows the variation ratio of bending strength after thermal shock (σ_t) divided by the initial resistance to bending (σ_0) function by the value of thermal shock (ΔT).

3. Flow through porous media

Complete description of the process of isothermal flow through a porous medium can be done by considering a set of three equations:

- motion equation, which expresses the conservation of momentum;

- continuity equation, expressing mass conservation;

- state equation, providing a relation between state parameters ρ = f (P, T).

Motion equation through porous media is known as Darcy's law, taking the form:

$$V_f = \frac{\rho g}{\eta} k_p \frac{h_1 - h_2}{L} \qquad (11)$$

where: V_f - is the filtration velocity; ρ –fluid density; g - gravitational acceleration; k_p- permeability coefficient defined by relation $k_p = k\frac{\eta}{\rho g}$; k - hydraulic coefficient of conductivity or filtration coefficient; η - dynamic viscosity of the fluid; $\frac{h_1 - h_2}{L}$ - hydraulic gradient, L - cylindrical length of porous body.

If environment is homogeneously porous, k and k_p are constant.

Continuity equation in fluid flow through porous medium has the general form:

$$\frac{\partial mg}{\partial t} + \text{div}\,(\rho v_f) = 0 \qquad (12)$$

4. Movement of liquid through porous medium

Highlighting fluid movement thru porous medium due to capillarity effect is highlighted by the introduction of a capillary tube in a liquid, which moistens the capillary wall (see Fig. 3).

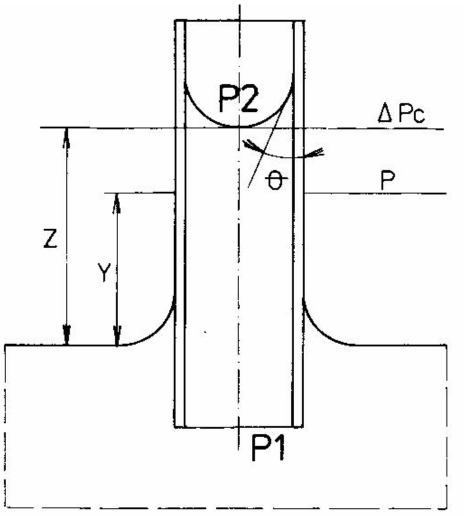

Fig. 3. *Ascending of the moistening liquid in a capillary tube.*

The ascendant movement of the liquid in the capillary is without external action until a given height (z). This is determined by lowering the free energy of the system to thermodynamic equilibrium, which causes a decrease in pressure between the free surface of liquid in the tank and the pressure from the free surface of liquid in the capillary.

Between liquid and its vapour there is an interface. Increasing the interface is consuming mechanical work.

If the movement of the liquid in the capillary tube is carried out isothermal (dT = 0):

$$dF = - P \cdot dV + \gamma \cdot dA \qquad (13)$$

where γ is the surface tension.

Term - P · dV corresponds changing free energy between homogeneous phases participating in the case, and the term $\gamma \cdot dA$ represents free energy changes at system interface.

The change of the total system volume is given by the change in volume of each phase $dV = dV_1 - dV_2$. Given the state parameters of the two phases (P_1 and V_1 in the liquid phase and P_2 and V_2 în vapour phase), the term - P.dV of equation (13) becomes:

$$- P \cdot dV = - P_1 \cdot d\,V_1 - P_2 \cdot dV_2 \qquad (14)$$

And so relation (13) becomes:

$$dF = - P_1 \cdot d\,V_1 - P_2 \cdot dV_2 + \gamma \cdot Da \qquad (15)$$

If the process is isochoric, then:

$dV = dV_1 + dV_2 = 0$, so $dV_1 = - dV_2$ and therefore:

$$dF = - P_1 \cdot dV_1 + P_2 \cdot dV_2 + \gamma \cdot dA \qquad (16)$$

At equilibrium dF = 0 and results:

$$P_1 = P_2 + \gamma \cdot \frac{dA}{dV_1} \qquad (17)$$

The difference between the two pressures, liquid pressure (P_1) and vapour pressure (P_2) is called capillary pressure.

Capillarity effect can be expressed as the maximum height of capillary rise of liquid (h_{max}). Neglecting friction losses, maximum height results from the condition of equilibrium, i.e. at the end of ascent, the pressure difference will be equal to the hydrostatic pressure of capillary liquid column, i.e.:

$$\Delta P_c = \frac{2 \cdot \gamma \cdot \cos\theta}{r} = \rho_l \cdot g \cdot h_{max} \qquad (18)$$

From where:

$$h_{max} = \frac{2 \cdot \gamma \cdot \cos\theta}{r \cdot \rho_l \cdot g} \qquad (19)$$

ρ_l is the fluid density.

In a more general case is considered a fluid motion in an inclined cylindrical capillary (see Fig. 4).

Balance of forces will be given by:

$$\Delta P_c - \rho_l \cdot g \cdot z \cdot \sin\alpha - \Delta P_f = 0 \qquad (20)$$

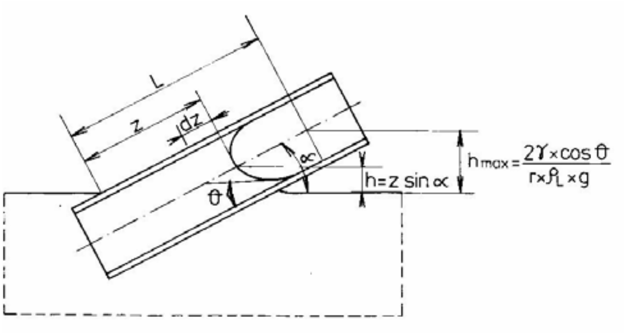

Fig. 4. *Ascending of a liquid in an inclined cylindrical capillary*

Where (ΔP_f) is a current value corresponding to the pressure drop length (z) and current velocity ($v = d_z/d_\tau$).

Pressure drop caused by friction can be expressed by Fanning-Darcy equation:

$$\Delta P_f = c_f \,.\, \frac{z}{d} \,.\, \frac{\rho_l \cdot v^2}{2} \qquad (21)$$

Liquid ascending through capillary is possible only if the fluid pressure (P_1) occurs in the capillary. Pressure differences ($P_1 - \rho_l \cdot g \cdot y$) are equalizing in the liquid during transport. It results that in isolated cylindrical capillary liquid movement can be considered a process of equalization (balancing), such as thermal conductivity or diffusion. Under these conditions mass flow transport (M_m) can be expressed by the relation:

$$M_m = S.\chi_c \, \frac{d(P - \rho_l \cdot g \cdot \gamma)}{dy} \,.\, \rho_l \qquad (22)$$

where: M_m is the mass flow of liquid through the capillary in kg/s; S - cross-sectional area of capillary in m; χ_c - liquid capillary conductivity coefficient in m^2/s.

Relation (22) shows that the height (y) Fig. 3, mass flow (M_m) is moved upwards as long as $P > \rho \,.\, g \,.\, y$.

Fluid pressure distribution is linear, so we can write:

$$\frac{P - \rho_l \cdot g \cdot \gamma}{y} = \frac{\Delta P_c - \rho_l \cdot g \cdot z}{z} = \frac{d(P - \rho_l \cdot g \cdot \gamma)}{dy} \qquad (23)$$

And therefore the mass flow will be:

$$M_m = S.\chi_c \, \frac{\Delta P_c - \rho_l \cdot g \cdot z}{z} \,.\, \rho_l \qquad (24)$$

We taking into account the definition of mass flow:

$$M_m = s \cdot \rho_l \cdot \frac{dz}{d\tau} \qquad (25)$$

By equality of relations (24) and (25) follows:

$$\frac{dz}{d\tau} = \chi_c \cdot \frac{\Delta P_c - \rho_l \cdot g \cdot z}{z} \qquad (26)$$

Comparing the above relations, we obtain the physical significance of capillary conductivity coefficient of the liquid:

$$\chi_c = \frac{r^2}{8\eta_l} \qquad (27)$$

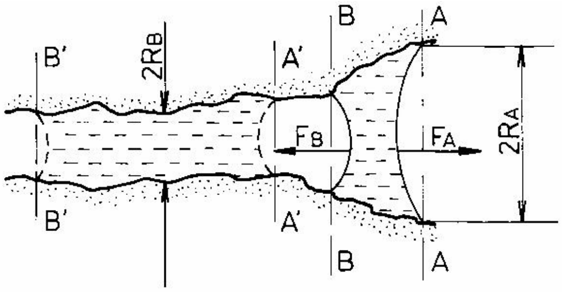

***Fig. 5.** Representation of a capillary pore with variable radius $r_A > r_B$.*

Capillary conductivity coefficient is a physical quantity that depends on the radius of the capillary and on friction forces in the capillary fluid. Porous bodies do not always have cylindrical pores with the same radius (r), in most cases these pores have variable radius.

Considering a variable radius pore (see fig. 5), capillary depression created between the section (A) and section (B), makes the liquid move in position (A'B'). For section (A) and (B) have relations:

$$P_A = P_1 + \frac{2 \cdot \gamma \cdot \cos\theta}{r_A} < P_B = P_2 + \frac{2 \cdot \gamma \cdot \cos\theta}{r_B} \qquad (28)$$

because $r_A > r_B$.

Pressures in the two sections can be expressed as the ratio between force and area, i.e.:

$$P_A = \frac{F_A}{S_A} = \frac{F_A}{\pi \cdot r^2_A} \qquad (29)$$

$$P_A = \frac{F_B}{S_B} = \frac{F_B}{\pi \cdot r^2_B} \qquad (30)$$

The F_A and F_B are the acting forces on the surfaces S_A and S_B, they are opposite forces. From equation (28) inequalities become:

$$P_A < P_B \text{ sau } \frac{F_A}{S_A} < \frac{F_B}{S_B} \qquad (31)$$

But as $S_A = \pi \cdot r_A > S_B = \pi \cdot r_B$, it results that $F_A > F_B$. Resulting forces $\Delta F = F_A + F_B$ cause fluid to move into position (A 'B').

5. Conclusions

Theoretical investigations of binary liquid alloys show consistent changes in surface tension and viscosity with the increasing percentage of a component. Viscosity (η) and surface tension (σ') of system Ag-Sn and Sn-Zn decreases with increasing percentage of tin in its composition.

The capillary pressure acts to stabilize, but pressure convection acts otherwise. Stability criterion for a constant flow is based on a higher gradient of capillary pressure. This criterion defines a critical point. This critical point corresponds to maximum flow.

A constant flow is possible only in the sub critical zone (below the critical point).

Level value of a liquid by capillary flow is appreciable to a minimum values capillary radius (r) for example: a tube with a diameter of 2 m, water will ascend a little, approx. 0.014 mm. But at a diameter of 2 cm, the water would ascend for about 1.4 mm and for a 0.2 mm diameter, the water would ascend for 140 mm.

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