THE ANNALS OF "DUNAREA DE JOS" UNIVERSITY OF GALATI.
FASCICLE IX. METALLURGY AND MATERIALS SCIENCE
$N^0$. 1 – 2007, ISSN 1453 – 083X

# EXPERIMENTAL RESEARCHES ON THE CONSTITUTIVE EQUATION OF CONCRETE STEEL WITH SUPERIOR CHARACTERISITICS

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## ABSTRACT

*The plastic deformation behavior is defined by the function of the deformation strength according to the strain, strain rate and temperature as the factor of the deformation process. The behavior law establishes by the experimental way, using the torsion test method. The paper shows the results of the researches for establishing of the equation of deformation behavior of steel destined of rolled wires for reinforced concrete.*

KEYWORDS: constitutive equation, torsion test, stress intensity, strain intensity

## 1. Introduction

The plastic deformation of a metallic material is described by the equation [1]:

$$\sigma = \sigma(\varepsilon, \dot{\varepsilon}, T) \quad (1)$$

in this equation $\sigma$ is the stress intensity in the really deformation conditions, $\varepsilon$ - strain intensity, $\dot{\varepsilon}$ - strain rate intensity, $T$ – temperature.

The knowledge of this equation of plastic deformation behavior is necessary for the evaluation, programming, modeling, simulation and optimization of the plastic deformation processes, by applying in the calculus program of the constitutive equation [1,2].

$$\dot{\varepsilon}_{ij} = \frac{2}{3} \cdot \frac{\dot{\varepsilon}_0}{\sigma_0} \cdot S_{ij} \quad (2)$$

in this equation $\dot{\varepsilon}_{ij}$ - is the component $ij$ of the strain rate tensor, $\dot{\varepsilon}_0$ - is the strain rate intensity in the really deformation conditions, $\sigma_0$ - the stress intensity, $S_{ij}$ - the component $ij$ of the deviator tensor of stress state. This equation is defined by:

$$S_{ij} = \sigma_{ij} - \delta_{ij} \cdot \sigma_m \quad (3)$$

in equation $\sigma_{ij}$ is the component $ij$ of the stress tensor, $\delta_{ij}$ – Kronecker's symbol, $\sigma_m$ – mean normal stress of the stress tensor.

In this paper it presents the results of researches effectuated for establishing of the equation of plastic deformation behavior of steel for wires destined to reinforced concrete.

## 2. Experimental conditions

The constitutive equation is established through experimental way using a torsion testing machine.

The researched material has the chemical composition rendered in table 1.

***Table 1.*** *Chemical composition of steel, [%]*

| C | Mn | Si | P | S | Cr | Ni |
|---|---|---|---|---|---|---|
| 0.18 | 1.23 | 0.35 | 0.37 | 0.035 | 0.21 | 0.15 |

The form of active zone of the sample is cylindrical and has the dimensions $(\phi 6 \pm 0.02) \times (36 \pm 0.1)$mm.

The torsion testing installation is equipped with: electro-hydraulic system for action of sample with the power of 5kW, the revolution is 1-2000 rpm, data acquisition system type Spider 8, heating system, maximum temperature of $1100^0$C and precision ±5 $^0$C [3,4]. The research program must cover a temperature area, according to the researched material and a domain of the strain rate values. A test corresponds at a certain strain rate value and certain temperature according to the established research program. In the aim of the testing we must regulation the revolution of hydraulic system, then it mounts the sample in the action device and we put in the function the heating system. Also it is put in the function the data acquisition system.

When the temperature of the sample becomes equal at the programmed temperature, the action system is coupled and the deforming process it is made until the tearing of the sample.

As result of the torsion test we obtain the torque diagram $M(t)_{\dot{\varepsilon},T}$ where *t* is the time, which may be transformed in strain. Thus we obtain the $M(\varepsilon)_{\dot{\varepsilon},T}$ diagram.

## 3. Experimental results

The research program consists in: research temperatures of 1023K, 1073K, 1123K, 1173K and the revolution of 25, 107, 400 rpm.

The torsion moment diagrams are rendered in the figure 1, 2, 3.

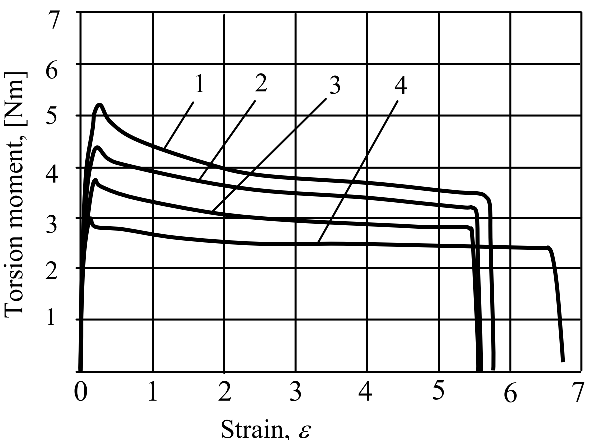


***Fig. 1.*** *The torsion moment diagram – strain for the revolution of 25rpm: 1-1023K, 2-1073K, 3-1123K, 4-1173K*

The analysis of the diagram shows that at the increasing of the strain rate the deformation resistance of material increases and its deformability decreases. At the increasing of the temperature the deformation resistance decreases and the deformability increases. At the temperature of 1073K it is manifest a trend of decreasing of the plasticity.

The function of the torsion moment is depended of the deformation degree (ε), strain rate ( $\dot{\varepsilon}$ ) and the temperature (T). The mathematical expression of the torque is:

$$dM = \frac{\partial M}{\partial \varepsilon} \cdot d\varepsilon + \frac{\partial M}{\partial \dot{\varepsilon}} \cdot d\dot{\varepsilon} + \frac{\partial M}{\partial T} \cdot dT \qquad (4)$$

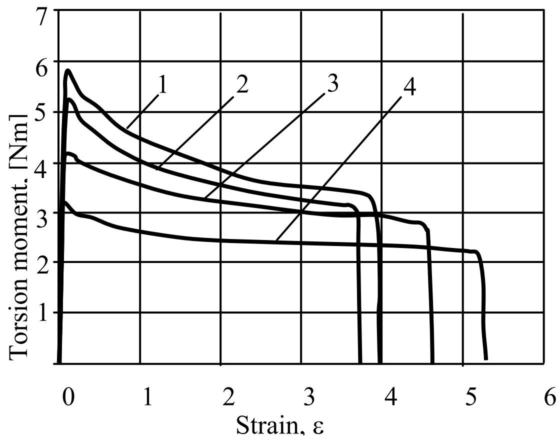


***Fig. 2.*** *The torsion moment diagram – strain for the revolution of 107rpm: 1-1023K, 2-1073K, 3-1123K, 4-1173K*

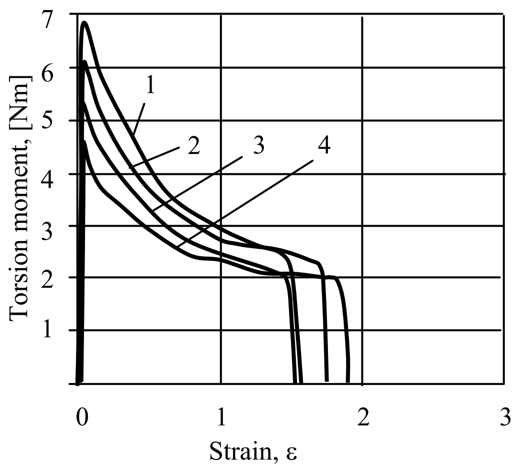


***Fig. 3.*** *The torsion moment diagram – strain for the revolution of 400rpm: 1-1023K, 2-1073K, 3-1123K, 4-1173K*

For the maximum values of the torque the expression (4) becomes:

$$dM_{max} = \frac{\partial M_{max}}{\partial \dot{\varepsilon}} \cdot d\dot{\varepsilon} + \frac{\partial M_{max}}{\partial T} \cdot dT \qquad (5)$$

Selecting the maximum values of the torque, which correspond at the research tests, according to the strain rate and temperatures values we obtain the diagrams rendered in figure 4.

The deformation strength of the metallic materials varies with the strain $\varepsilon$ by a hardening law (power or exponential law), with the strain rate $\dot{\varepsilon}$ by a power law and in function of temperature through an exponential law.

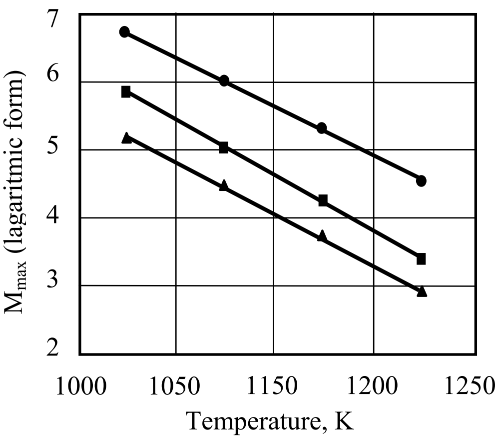


***Fig.4.*** *Torsion moment diagram in logarithmic coordinates: 1-25rpm, 2-107rpm, 3-400rpm*

The general mathematical expression of the torsion moment, frequently used for description of the function of the torsion moment has the following expression [2,4,5]:

$$M_{max} = A_2 \cdot \dot{\varepsilon}^m \cdot exp\left(\frac{m \cdot Q}{RT}\right) \quad (6)$$

In (5) *m* is the coefficient of the sensibility of deformation strength at the strain rate, Q is named the activation energy of deformation process, R – the ideal gas constant, T – temperature, in Kelvin, A – experimental constant.

We transformed the relation (6) in the linear form (we applied the logarithmic coordinates) and applied a regression calculus program with two independent variables and one dependent variable and we obtained the results rendered in table 2.

***Table 2.*** *Regression data at the equation (5)*

| | | | | |
|---|---|---|---|---|
| Standard Error of the Estimate = 6,56297855244627E-02 | | | | |
| Coefficient of Multiple Determination (R^2) = 0,9415813889 | | | | |
| **Regression Variable Results** | | | | |
| **Variable** | **Value** | **Standard Error** | **t-ratio** | **Prob(t)** |
| **a** | -2,299048475 | 0,405759417 | -5,666038497 | 0,00031 |
| **b** | 0,119960921 | 0,016731242 | 7,16987528 | 0,00005 |
| **c** | 4687,084251 | 484,3296201 | 9,677467692 | 0,0 |

The constants which are included in the expression (6) have the values:

$A_2$ = 9,964 ;
m = 0,119961 ;
Q = 325,858 kJ/mol

The mathematical expression of the maximum torsion moment is the following:

$$M_{max} = 9,964 \cdot \dot{\varepsilon}^{0,119961} \cdot exp\left(\frac{4687,08}{T}\right) \quad (7)$$

The determination of the function of the stress intensity is possible using the expression:

$$\sigma = \frac{\sqrt{3}}{2\pi R^3}\left(3M + \dot{\varepsilon}\frac{\partial M}{\partial \dot{\varepsilon}} + \varepsilon\frac{\partial M}{\partial \varepsilon}\right)$$

We admit a composed function for the hardening factor. The equivalent stress may be defined by equation [2,5]:

$$\overline{\sigma} = \begin{cases} A \cdot a \cdot \frac{\varepsilon}{(\varepsilon + \varepsilon_0)^p} \cdot \left(\dot{\overline{\varepsilon}}\right)^m \cdot exp\left(\frac{mQ}{RT}\right) & for\ \varepsilon \le \varepsilon_0 \\ \sigma^* \cdot exp(-n(\varepsilon - \varepsilon_0))\left(\dot{\overline{\varepsilon}}\right)^m \cdot exp\left(\frac{mQ}{RT}\right) & for\ \varepsilon > \varepsilon_0 \end{cases} \quad (8)$$

In this expression $\varepsilon_0$ is the value of the strain which corresponds at the maximum value of the torsion moment. This factor is, also, a function of the strain rate and temperature. The exponent *p* has the order 2.

The values of the constants end exponents of the relation (8) will be established using a calculus program for experimental data.

The experimental data for the strain $\varepsilon_0$ that corresponds at the maximum torque are rendered graphic in the figure 5.

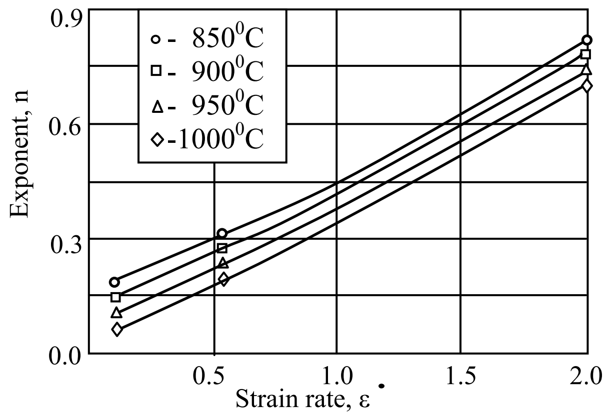


***Fig.5.*** *Values of the factor $\varepsilon_0$*

The formula of the factor $\varepsilon_0$ has the expression:

$$\varepsilon_0 = a_\varepsilon \cdot T^{b_\varepsilon} \cdot (c_\varepsilon)^{\dot{\varepsilon}} \quad (9)$$

Constants have the values:
$a_\varepsilon$ =3712423635
$b_\varepsilon$ =-3.445121355
$c_\varepsilon$ =0.4117625738

The coefficients of quality of statistic evaluation are:
Standard Error of the Estimate =
=1.73577573415809E-02
Coefficient of Multiple Determination
(R^2) = 0.9583618773
Proportion of Variance Explained =
= 95.83618773%

The experimental data for the exponent *n* are rendered graphic in the figure 6.

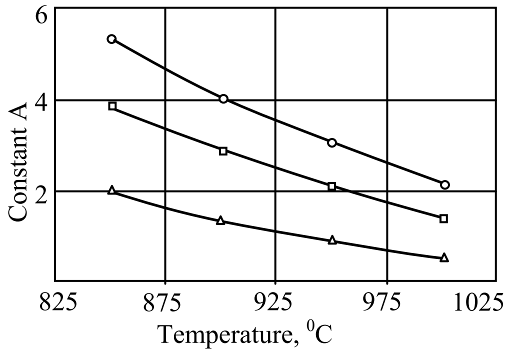


***Fig.6.*** *Values of the exponent n*

The formula of the factor $\varepsilon_0$ has the expression:

$$n = a_n \cdot T^{b_n} \cdot (c_n)^{\dot{\varepsilon}} \quad (10)$$

Constants have the values:
$a_n$ =1063.736881
$b_n$ =-1.255542663
$c_n$ =1.990376716

The coefficient of quality of statistic evaluation are:
Standard Error of the Estimate =
= 0.031135955009085
Coefficient of Multiple Determination
(R^2) = 0.9899384287
Proportion of Variance Explained =
=98.99384287%

The experimental data for the constant *A* are rendered graphic in the figure 7.

The formula of the factor $\varepsilon_0$ has the expression:

$$A = a_a \cdot T^{b_a} \cdot (c_a)^{\dot{\varepsilon}} \quad (11)$$

Constants have the values:
$a_A$ =1.106688815E+020
$b_A$ =-6.575749697
$c_A$ =0.5306206519

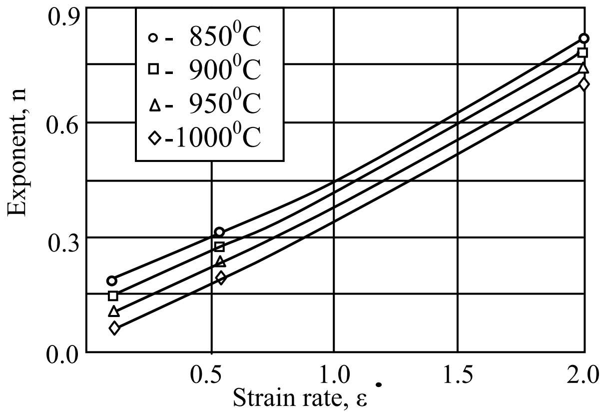


***Fig.7.*** *Values of the constant A.*

The coefficients of quality of statistic evaluation are:
Standard Error of the Estimate =
= 0.289355804175773
Coefficient of Multiple Determination
(R^2) = 0.9687668616
Proportion of Variance Explained =
= 96.87668616%

The factor *a* from the equation (8) has the value:
a=30,645

## 4. Conclusions

The knowledge of the constitutive equation of the material is necessary form the modeling, simulation and optimization of the plastic deformation process. The best method for establishing of constitutive equation is the torsion testing. Applying a research program at the torsion testing machine in the Plastic deformation laboratory at the Faculty of Metallurgy and materials science from *Dunarea de Jos* University of Galati it established the constitutive equation of steel for wires destined at the reinforcing of the concrete. The constitutive equation shows that the influence of strain rate is described of the power mathematical relation, the influence of the temperature is described by an exponential function. The strain has a complex influence described by a compose function.

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