THE USE OF SYSTEMS ANALYSIS IN ENVIRONMENTAL ENGINEERING

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ABSTRACT

The depletion of energy resources has been of primary concern in the `70s and `80s. In recent years, the decline of the environment due primarily to our energy-related activities has become severe and raises serious concern too. For this reason, method to analyze, improve and optimize energy-intensive systems have to deal not only with energy consumption and economics, but also with the pollution and degradation of the environment.

KEYWORDS: energy resources, pollution and degradation of the environment

1. Introduction

Linear programming is about making the most of limited resources. Specially, it deals with maximizing a linear function of variables subject to linear constraints. Applications range from economic planning and environmental management to the diet problem. The aim is to provide a simple introduction to the subject.

2. Linear programming

Linear programming, a specific class of mathematical problems, in which a linear function is maximized (or minimized) subject to given linear constraints. This problem class is broad enough to encompass many interesting and important applications, yet specific enough to be tractable even if the number of variables is large.

The general form of a linear program is

Maximize $c_1 x_1 + ... + c_n x_n$

Subject to
$$a_{11} x_1 + ... + a_{1n} x_n \le b_1,$$
$$..................................$$
$$a_{m1} x_1 + ... + a_{mn} x_n \le b_m,$$
$$x_1 \ge 0, ..., x_n \ge 0$$

Here $c_1, \dots, c_n$, $b_1, \dots, b_m$ and $a_{11}, \dots, a_{mn}$ are given numbers, and $x_1, \dots, x_n$ are variables whose values are to be determined, maximizing the given objective subject to the given constraints. There are *n* variables and *m* constraints, in addition to the nonnegativity restrictions on the variables. The constraints are called linear because they involve only linear functions of the variables. Quadratic terms such as x_1^2 or $x_1 \cdot x_2$ are not permitted. If minimization is desired instead of maximization, this can be accomplished by reversing the signs of $c_1, \dots, c_n$.

The most remarkable mathematical property of linear programs is the theory of duality.

The *dual* of the linear program given above is

Maximize $b_1 y_1 + ... + b_m y_m$

Subject to
$$a_{11} y_1 + ... + a_{m1} y_m \ge c_1,$$
$$..................................$$
$$a_{1n} y_1 + ... + a_{mn} y_m \ge c_n,$$
$$y_1 \ge 0, ..., y_m \ge 0$$

This is a linear program in the variables $y_1, \dots, y_m$. It is not hard to show that if $(x_1, \dots, x_n)$ is in the feasible region for the first linear program and $(y_1, \dots, y_m)$ is in the feasible region for the dual linear program, than the first objective function $c_1x_1+\dots+c_nx_n$ is less than or equal to the dual objective function $b_1y_1+\dots+b_my_m$. The remarkable fact is that these two objectives are always *equal* for any $(x_1,\dots,x_n)$ and $(y_1,\dots y_m)$ which are, respectively, optimal solutions for the two linear programs. This is of great practical importance for both the interpretation of the solutions of linear programs and the methods for calculating their solutions.

3. Debate on the economy aspects of an environmental problem

A production unit in the consumers goods field (aliments) is the unit that produces this kind of goods at the cost of 15 currency units, each product unit being worth/costing 3 such units. Unfortunately, in the production process, a waste-pollutant is generated: 2 waste units for each product unit. A part of a pollutant is left untreated and gets into an emissary (water course); another part is removed by treatment in a special station, the treatment cost for an unit of product being 1 currency unit. But the unit of polluted water treatment cannot treat more than 10 waste units, with a reduction by 80% of the level of pollutant. The requested tax for each waste unit discharged untreated into the water is 2 currency units and the environmental protection authority limits the waste quantity that can be discharged by the producer to 4 waste units.

The system manager wants to know the optimum level of finite objects production in the above-mentioned conditions.

For the problem equalization, we will use the following notations:

x_1 – the level of finite consumers goods' production;

x_2 – the quantity of pollutant that is left untreated and is discharged in the environment (by a water course);

y – objective function, for which the maximum is searched for;

y function is the difference between the gross benefit and a sum formed by the production costs, waste treatment costs and of the part that is discharged into the affluent.

The pollutant mass $2x_1 - x_2$ passes through the waste water treatment plant.

y function is given by the relation:

$$y = 15x_1 - 3x_1 - 1(2x_1 - x_2) - \\ - 2[x_2 + 0,2(2x_1 - x_2)] \qquad (1)$$

that means:

$$y(x_1, x_2) = 9,2x_1 - 0,6x_2 \qquad (2)$$

$$2x_1 - x_2 \le 10 \qquad (3)$$

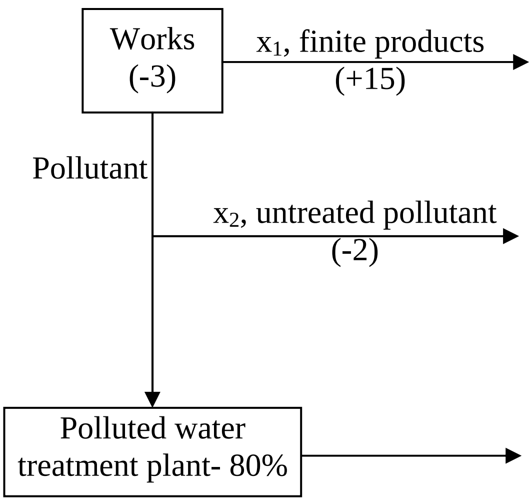


Figure 1. *– The scheme of the production unit*

$$0,4x_1 + 0,8x_2 \le 4\,; \; x_1 + 2x_2 \le 10 \quad (4)$$

$$2x_1 - x_2 \ge 0 \qquad (5)$$

Condition number (5) is the expression of the fact that in all the cases we have to possess sewerage for polluted water through the waste treatment plant. The scheme for this situation is:

4.The dual problem of linear programming

We will make the following notations:

$$2x_1 - x_2 = a \qquad (6)$$

$$x_1 + 2x_2 = b \qquad (7)$$

a and b units of measure can vary from a=10 and b=10, as regards their value.

The objective function y is considered in the optimal conditions, that means y_0. The issue that has to be taken into account is how to find the way in which y_0 depends on a and b parameters.

Considering the function $y_0(a,b)$ we can write:

$$dy_0 = \frac{\partial y_0}{\partial a} da + \frac{\partial y_0}{\partial b} db \qquad (8)$$

We will make the following notations:

$$(z_1)_0 = \frac{\partial y_0}{\partial a}; \quad (z_2)_0 = \frac{\partial y_0}{\partial b}; \qquad (9)$$

(8) relation will be noted:

$$dy_o = (z_1)_0 da + (z_2)_0 db \qquad (10)$$

(8) relation expresses y_0 variation when a and b vary infinitesimal. From (6) and (7) relations we can easily find, by differentiation, the following relations:

$$da = 2dx_1 - dx_2 \qquad (11)$$

$$db = dx_1 + 2dx_2 \qquad (12)$$

y_0 value which is, in our case:

$$y_0 = 9,2(x_1)_0 - 0,6(x_2)_0 \qquad (13)$$

where $\{(x_1)_0, (x_2)_0\}$ is the solution of the system 6+7, that means:

$$(x_1)_0 = \frac{2}{5}a + \frac{b}{5} \tag{14}$$

$$(x_2)_0 = -\frac{a}{5} + \frac{2}{5}b \tag{15}$$

As $x_i \geq 0$ (the unnegativity condition) (i=1,2) from (15) the following condition results:

$$-\frac{a}{5} + \frac{2}{5}b \geq 0 \tag{16}$$

or:

$$b \geq \frac{a}{2} \tag{17}$$

Taking into account the relations (14) and (15), the relation (13) becomes:

$$y_0(a,b) = 3{,}8a + 1{,}6b \tag{18}$$

$$dy_0(a,b) = 3{,}8da + 1{,}6db \tag{19}$$

Comparing it to (10) we find:

$$(z_1)_0 = 3{,}8; \quad (z_2)_0 = 1{,}6 \tag{20}$$

In our case:

$$dy\left(x_1, x_2\right) = 9{,}2dx_1 - 0{,}6dx_2 \tag{21}$$

$$dy\left(x_1, x_2\right) = Z_1da + Z_2db \tag{22}$$

Using the relations (11) and (12), the relation (22) becomes:

$$dy\left(x_1, x_2\right) = (2z_1 + z_2)dx_1 + (-z_1 + 2z_2)dx_2 \tag{23}$$

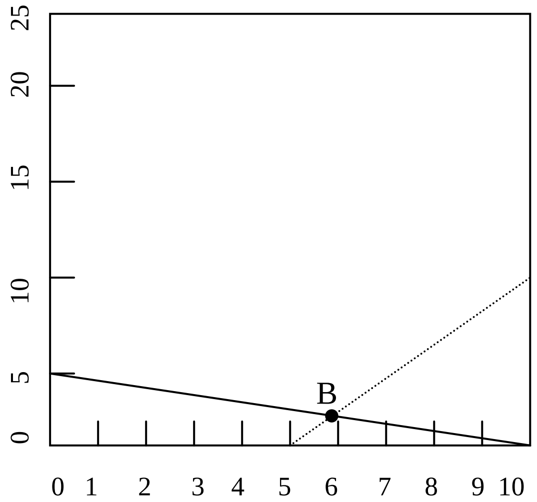
Comparing (21) and (23) we can note the following relations:

$$2z_1 + z_2 = 9{,}2 \tag{24}$$

$$-z_1 + 2z_2 = -0{,}6 \tag{25}$$

Solving the above system we find the values of the new variables in the optimal solution point.

$z_1 = 3{,}8; \; z_2 = 1{,}6$



$x_1 + 2x_2 = 10$

$2x_1 - x_2 = 10$

The optimal admissible solution is for point B(6,2) and:

$y_0 = y_{optim}(6{,}2) = 54$ *currency units*

Conclusion

The main objective in an optimization problem is to find the maximum or minimum of an y objective function. In most of the cases, this function depends on a series of variables x_1, x_2,....x_n, called control variables, whose value we can chose in order to achieve a certain goal.

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