ELASTOPLASTIC MODEL FOR THE STEEL BARS UNDER THERMAL TREATMENTS

Petrică ALEXANDRU, Octavian POTECAŞU, Florentina POTECAŞU

"Dunarea de Jos" University of Galati
palexandru61@yahoo.com

ABSTRACT

In order to understand material behavior, during the stresses action produced by thermal effects of volume, was started from Hooke relations, whereat was added the expressions that take into account these effects. Have been chose the expressions in cylindrical coordinates for the relations simplification reason, but and because cylindrical pieces are frequent in practical thermal treatments

Keywords : Hooke relations, thermal strain, quenching bar.

1. Introduction

The Hooke relations in cylindrical coordinates in the elastic domain:

$$\varepsilon_r = \frac{1}{E}\left[\sigma_r - \nu(\sigma_t + \sigma_z)\right];$$

$$\varepsilon_t = \frac{1}{E}\left[\sigma_t - \nu(\sigma_r + \sigma_z)\right];$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_t + \sigma_r)\right];$$

$$\gamma_{rz} = \frac{2(1+\nu)}{E}\tau_{rz}; \qquad (1)$$

where:

- $\varepsilon_r, \varepsilon_t, \varepsilon_z$ -the specific elongations on three directions, radial, tangential direction and respective on the height cylinder;
- E_e -the modulus of longitudinal elasticity;
- $\sigma_r, \sigma_t, \sigma_z$ -the normally stresses on three, one radial, tangential direction and respective on the height cylinder;
- ν -the Poisson coefficient;
- γ_{rz} -tangential strain;
- τ_{rz} -tangential stress.

2. Proposed model

The completion with thermal volume effect presupposes considers as much expansion of pure thermal how much nature and these produced by transformations of phase, as well as plastic deformation [1]. Through the addition in relations (1) of terms eh comprise these modifications was obtained:

$$\varepsilon_r = \frac{1}{E}\left[\sigma_r - \nu(\sigma_t + \sigma_z)\right] + \varepsilon_r^{pl} + \varepsilon_{vol};$$

$$\varepsilon_t = \frac{1}{E}\left[\sigma_t - \nu(\sigma_r + \sigma_z)\right] + \varepsilon_t^{pl} + \varepsilon_{vol};$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_t + \sigma_r)\right] + \varepsilon_z^{pl} + \varepsilon_{vol};$$

$$\gamma_{rz} = \frac{2(1+\nu)}{E}\tau_{rz} + \gamma_{rz}^{pl}; \qquad (2)$$

where:

- $\varepsilon_r^{pl}, \varepsilon_\theta^{pl}, \varepsilon_z^{pl}, \gamma_{rz}^{pl}$ -the plastic deformations on radial, tangential direction, on height and respective, the tangential strain in a plan perpendicular to radius and measured in direction of cylinder height;
- ε_{vol} -the thermal volume effect.

Envisaged as much his dilatation the contraction produced of the temperature variation in the existence domain of a certain how much phase and the volume modifications motivated of the phase transformations must as the volume deformation be write:

$$\varepsilon_{vol} = \varepsilon_0 + \beta\Delta T \qquad (3)$$

Through ε_0 is taken into account variation of volume to crossing abs existing structure to surrounding temperature to proper structure temperature whereat is found out the supposed

Through the substitution in the relations (2) expressions (3) was obtained.

$$\varepsilon_r = \frac{1}{E_e}\left[\sigma_r - \nu(\sigma_\theta + \sigma_z)\right] + \varepsilon_r^{pl} + \varepsilon_0 + \beta\Delta T ;$$

$$\varepsilon_\theta = \frac{1}{E_e}\left[\sigma_\theta - \nu(\sigma_r + \sigma_z)\right] + \varepsilon_\theta^{pl} + \varepsilon_0 + \beta\Delta T ;$$

$$\varepsilon_z = \frac{1}{E_e}\left[\sigma_z - \nu(\sigma_\theta + \sigma_r)\right] + \varepsilon_z^{pl} + \varepsilon_0 + \beta\Delta T ;$$

$$\gamma_{rz} = \frac{2(1+\nu)}{E_e}\tau_{rz} + \gamma_{rz}^{pl} ; \tag{4}$$

The plastic components of strains caused through analysis at every turn of time state every volume element from angle of behavior to deformation. Was calculated the intensity compliant stresses power hypotheses of plasticity (Mises $\sigma^k = \sqrt{(\sigma_r - \sigma_\theta)^2 + (\sigma_\theta - \sigma_z)^2 + (\sigma_{zr} - \sigma_r)^2 + 6\tau_{rz}^2}$ And if these exceeds the resistance values to proper deformation temperature and material structure

$$\sigma^k \geq R_d^k ,\ R_d^k = \sqrt{2}R_e ,$$

Re represents the tension for which material pass through stage plastic to the draft unidirectional solicitation), the element into analyses was considered element of volume according as the figure 1 present. in elasto-plastic domain of deformation. Calculated the plastic deformations was considered that elastic cumulated energy after touch critical state is changed to in mechanical energy of plastic deformation. When the mechanical energy of shape variation exceeds the critical value any growth of total energy be changed into mechanical energy of plastic deformation.

If noted the time whereat consisted the touched critical stage with t_f, and with $t_s = t_f - \Delta t$, the time whereat was did previous analysis of behavior to deformation, have been presupposed a linear variation of tensions intensity σ^k, unitary efforts, σ_r, σ_θ, σ_z, τ_{rz} and temperature of which depending the mechanical characteristics of material σ_r, σ_θ, σ_z, τ_{rz},in the very short interval $\Delta t = t_f - t_s$ represents as a matter of fact a step of time. How on this moment t_s, the volume element is still in the elastic domain of deformation with $\sigma^k < R_d^k$, and on the passed moment t_f in one elasto-plastic, characterized through $\sigma^k > R_d^k$, was considered the moment of time whereat is produced the equality $\sigma^k = R_d^k$, is in middle time interval,

$$t_k = t_s + \frac{t_f - t_s}{2} = t_s + \frac{\Delta t}{2} .$$

Accordingly, result

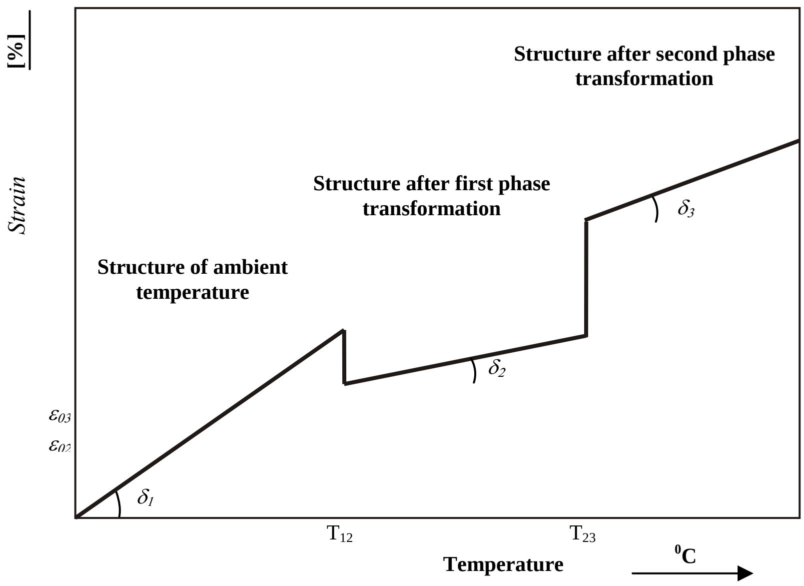

Fig. 1. *Thermal strain variation in a case of material with multiple phase transformations.*
-T12, T23-The critical temperatures of phase transformations;
- ε_{01}, ε_{02} *-The strains variation from critical temperatures T12, T23;*
- $\beta_1 = tg\delta_1$, $\beta_2 = tg\delta_2$, $\beta_3 = tg\delta_3$, *The dilation coefficients of three structures.*

$$\sigma_r^k = \frac{\sigma_r^s + \sigma_r^f}{2};$$

$$\sigma_\theta^k = \frac{\sigma_\theta^s + \sigma_\theta^f}{2};$$

$$\sigma_z^k = \frac{\sigma_z^s + \sigma_z^f}{2};$$

$$\tau_{rz}^k = \frac{\tau_{rz}^s + \tau_{rz}^f}{2};$$

(5)

$$\varepsilon_r^k = \frac{1}{E_e}\left[\sigma_r^k - \nu\left(\sigma_\theta^k + \sigma_z^k\right)\right] + \varepsilon_r^{pl} + \varepsilon_0 + \beta\Delta T;$$

$$\varepsilon_\theta^k = \frac{1}{E_e}\left[\sigma_\theta^k - \nu\left(\sigma_r^k + \sigma_z^k\right)\right] + \varepsilon_\theta^{pl} + \varepsilon_0 + \beta\Delta T;$$

$$\varepsilon_z^k = \frac{1}{E_e}\left[\sigma_z^k - \nu\left(\sigma_r^k + \sigma_\theta^k\right)\right] + \varepsilon_z^{pl} + \varepsilon_0 + \beta\Delta T;$$

$$\gamma_{rz}^k = \frac{2(1+\nu)}{E_e}\tau_{rz}^k + \gamma_{rz}^{pl};$$

(6)

Theses estimates are convenient in the case of bodies division in a big number of elements. In figure 2 is presented the variation of stress σ_r depending on strain over a time step in which is produced the crossing in the elasto-plastic domain of deformation.

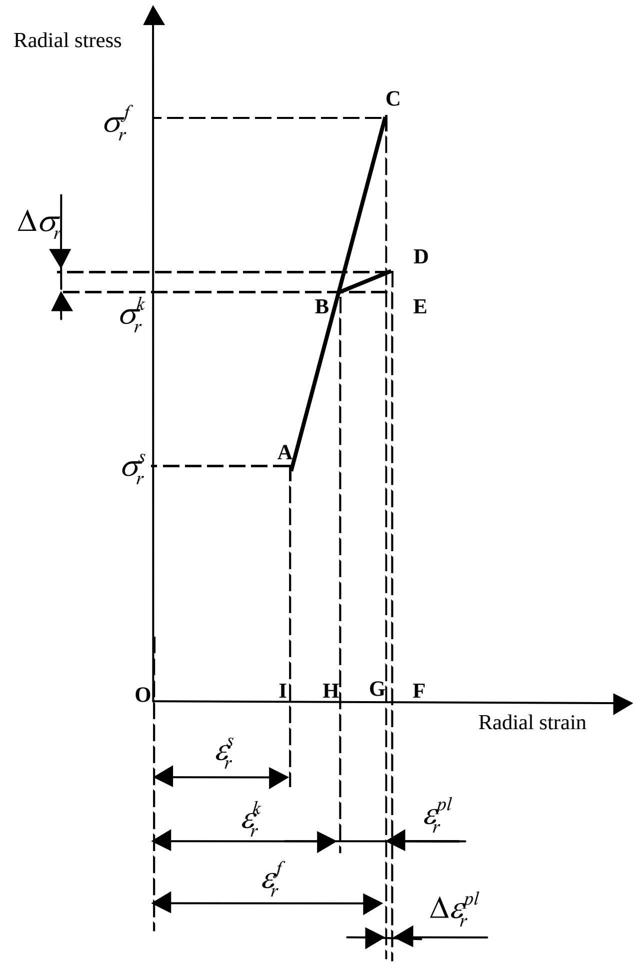

Fig. 2. *Approximation calculation of plastic radial strain into elementary volume.*

σ_r^s, σ_r^k, σ_r^f *-The radial stress in elastic domain from the beginning, middle and end of time step;*

ε_{re}^s, ε_{re}^k, ε_{re}^f *-The radial strains in elastic domain from the beginning, middle and end of time step;*

$\varepsilon_r^{pl\Delta t}$ *- The radial strains in plastic domain from the end of time step;*

$\Delta\sigma_r$, $\Delta\varepsilon_r$ *-The approximation error of calculation of radial strain and stress in plastic domain.*

$$L_r = \frac{1}{2}\left(\sigma_r^f + \sigma_r^k\right)\left(\varepsilon_r^f - \varepsilon_r^k\right),$$

where represents the mechanical energy of elastic deformation stores in material after touch critical state. The trapeziums area HBCG can be divided in the triangle BCE and the rectangle HBEG. If these two areas are shared

$$\frac{S_{BCE}}{S_{HBEG}} = \frac{\frac{1}{2}\left(\sigma_r^f - \sigma_r^k\right)\left(\varepsilon_r^f - \varepsilon_r^k\right)}{\sigma_r^k\left(\varepsilon_r^f - \varepsilon_r^k\right)} = \frac{\sigma_r^f - \sigma_r^k}{\sigma_r^k} \approx 0 .$$

Because, $\left(\sigma_r^f - \sigma_r^k\right) << \sigma_r^k$ (the variation tension is very small beside the own value) what permits the approximation calculus

$$L_r = \sigma_r^k\left(\varepsilon_r^f - \varepsilon_r^k\right),$$

this will be consumed through a plastic deformation production

$$L_r = \sigma_r^k\left(\varepsilon_r^f - \varepsilon_r^k\right) = \sigma_r^k \varepsilon_r^{pl\Delta t} ,$$

where through $\varepsilon_r^{pl\Delta t}$, was noted the plastic deformation produced in the element of volume on the analyzed time Δt, who confer for this, a value of the difference between the radial deformation appropriate of a the exhausted The deformation can be calculated without took count of the cold working (considering on duration of time step is very small in order to do not influence the deformation resistance of material) with a simple relation

$$\varepsilon_r^{pl\Delta t} = \varepsilon_r^f - \varepsilon_r^k$$

step of time and one appropriate the of critical attained domain. The other plastic strains was write likewise:

$$\varepsilon_\theta^{pl\Delta t} = \varepsilon_\theta^f - \varepsilon_\theta^k ,$$

$$\varepsilon_z^{pl\Delta t} = \varepsilon_z^f - \varepsilon_z^k ,$$

$$\gamma_{rz}^{pl\Delta t} = \gamma_{rz}^f - \gamma_{rz}^k .$$

These are added with the sums of plastic deformations already produced in previous interval Δt in the considers element.

For the stresses depending on deformations then next set of equation resulted:

$$\sigma_r = \frac{E_e}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_r + \nu(\varepsilon_\theta + \varepsilon_z) - (1+\nu)\beta\Delta T - (1-2\nu)\varepsilon_r^{pl} - (1+\nu)\varepsilon_0\right]$$

$$\sigma_\theta = \frac{E_e}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_\theta + \nu(\varepsilon_r + \varepsilon_z) - (1+\nu)\beta\Delta T - (1-2\nu)\varepsilon_\theta^{pl} - (1+\nu)\varepsilon_0\right]$$

$$\sigma_z = \frac{E_e}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_z + \nu(\varepsilon_\theta + \varepsilon_r) - (1+\nu)\beta\Delta T - (1-2\nu)\varepsilon_z^{pl} - (1+\nu)\varepsilon_0\right]$$

$$\tau_{rz} = \frac{E_e}{2(1+\nu)}\left(\gamma_{rz} - \gamma_{rz}^{pl}\right).$$

4. Simulation to water quenching of steel bar

As with most heat-treatment processes, the quenching conditions continue to be established empirically by trial-and-error procedures. A good prediction of microstructure and hardness distribution, as well as distortion and residual stresses, after quenching, clearly is desirable from the standpoint of both cost savings and enhanced product quality.

In the present paper a mathematical model, based on the finite-difference method, has been developed to predict the interaction among temperature, stress, water quenching process [4]. Internal stresses are induced by density changes resulting from cooling and phase transformations during quenching; but in same time the evolution of the microstructure depends on the thermal history as well as stress development in the part. The block diagram for the proposed mathematical model is shown in figure 3[1,2].

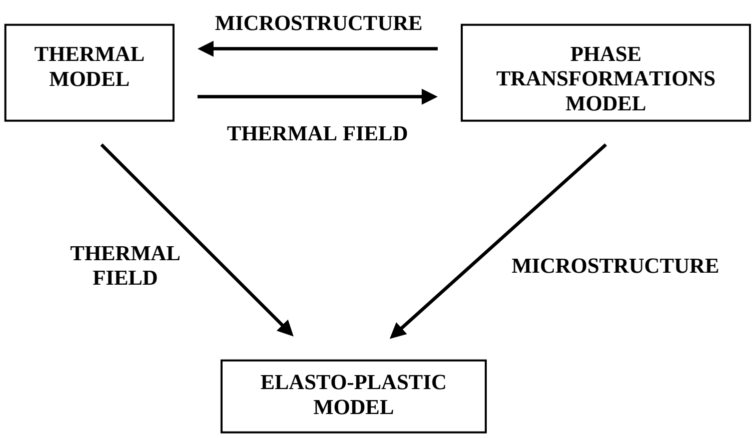

Fig.3 *General configuration of the mathematical model for the quenching steel bars.*

The stress influence at the phase transformations have been neglected in internal stress models of quenching. The applicability of this model for the simulation in the water quenching of a round steel bars *ϕ80x240 mm, 1C45 plain carbon grade was* compared with of the experimental measured and determined results. Figure 4 shows predicted frontal and lateral surface profiles after quenching as well as a comparison to experimental distortions after quenching. Good agreement can be observed.

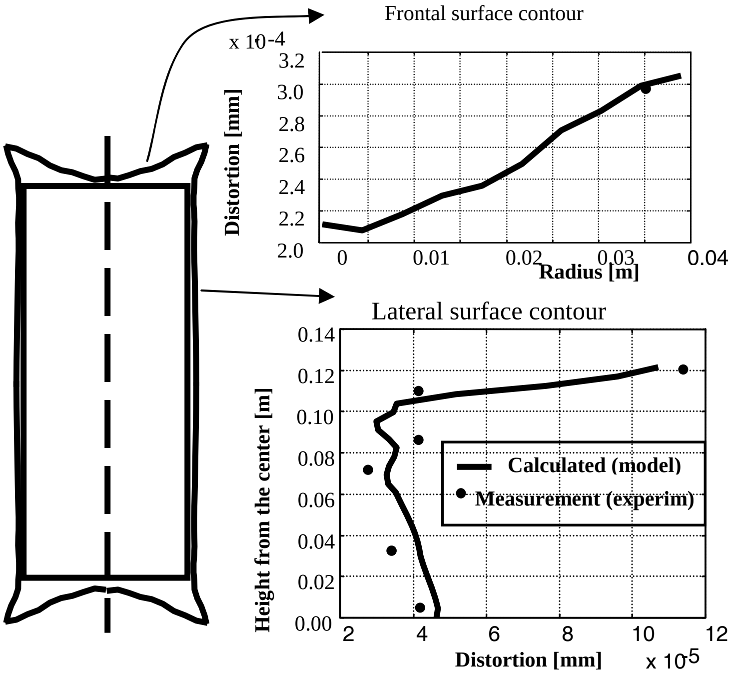

Fig. 4. *Model prediction for the distortion at the middle length surface and the frontal surface contour of the quenched bar (ϕ 80x240 mm, 1C45)*

5. Conclusions

The use of precedent equations (7) is indicated for the thermal treatments modeling by numerical method, such us the finite elements method or finite differences method, but in completion with the other terms depending on the specifies case.

The model can be used to optimize process parameters, at least in a comparative manner. Their use is limited currently by the large material database required as input.

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