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RACK AND PINION STEERING SYSTEM DESIGN FOR A PASSENGER CAR

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ABSTRACT

The steering system plays a crucial role in the stability of the automobile, especially in the safety of the passengers and pedestrians. The aim of this work is to design a rack and pinion steering system that could equip a passenger car. In this process, many parameters are considered for the correct and effective directional response behavior of the vehicle. 2D models were sketched to validate the kinematic algorithm calculus used to optimize and refine the dimensions of the components of the steering system. After a satisfactory Ackermann percentage was achieved, steering system is designed and analysed in one of the most used CAD and CAE software in automotive, CATIA.

KEYWORDS: rack and pinion, steering kinematics, Ackerman geometry, optimization, CAD, CAE, turning angle, turn centre

1. Introduction

Devised by the English physician Erasmus Darwin in 1758, designed by the German carriage builder George Lankensperger in 1817, and patented by the Anglo-German bookseller Rudolph Ackermann in 1818, Ackermann geometry proposed a new method of steering of horse-drawn carriages, whose stability in turning was very low at that time. This new method aims that the two front wheels should turn about a centre that lies on the extended line of the back axle of the carriage. Respecting this principle means that all four wheels will have concentric paths. These concentric paths of the wheels come with some important advantages: the front wheels both run tangential to the track, so there is no scuffing anymore of the wheels on the road, but more important, the carriage turns more smoothly and the probability of overturning decreases insignificantly. It is essential to know these facts because the automotive steering system is also designed considering this principle discovered by Darwin and Lankensperger [1].

Throughout this paper many parameters will be encountered, so it is absolute necessary to clarify the terminology.

Table 1. *Nomenclature*

Parameter	Notation	Measure unit
Wheelbase	A	[mm]
Wheel track	E	[mm]
Inner tier turning angle	v_i	[°]
Outer tier turning angle	v_e	[°]
Turning radius	R	[mm]

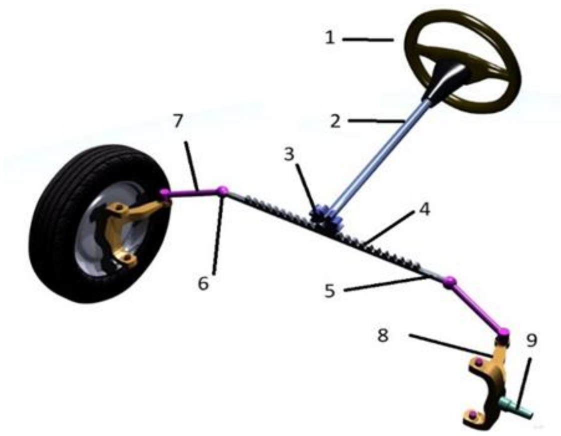

Fig. 1. *Steering system components: 1-steering wheel, 2-steering column, 3-pinion, 4-rack, 5-transverse bar, 6-rod end, 7-tie rod, 8-steering lever, 9-spindle* [2]

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This paper aims to describe the logical steps that must be considered in designing an effective steering system for a passenger car. A steering system is effective if the paths of the wheels are concentric, in other words, the turn centre is placed on the extended line of the back axle. For achieving such concentric trajectories, the inner wheel must be steered more than the outer wheel. In Figure 2 is presented the ideal turn or Ackermann principle.

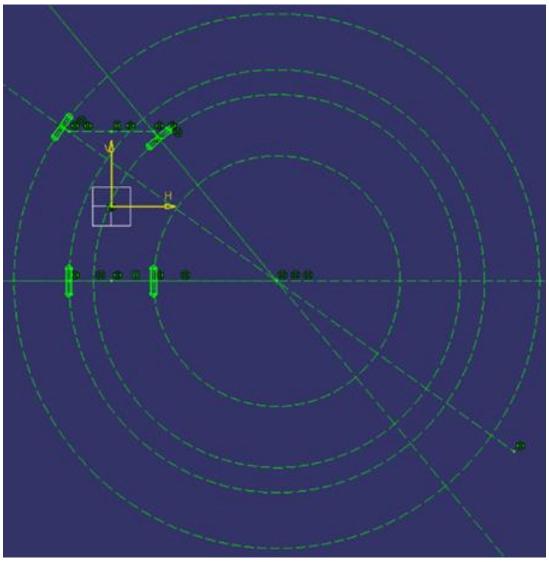


Fig. 2. *Ideal 360 turn/ Ackermann turn*

It is very important to know that such an ideal turn is a consequence of the steering geometry and steering kinematics [3]. Based on Figure 2, the next equation can be deduced, an equation which defines the ideal dependency between the two turning angles of the front wheels.

Theoretically, the ideal turn is based on a very simple equation. In real life, the trajectories of the wheels depend on such a complex system called steering system, and not only. Plotting this first equation, it will result the next chart.

$$v_e = \operatorname{acot}\left(\frac{E}{A} + \cot(v_i)\right) \qquad (1)$$

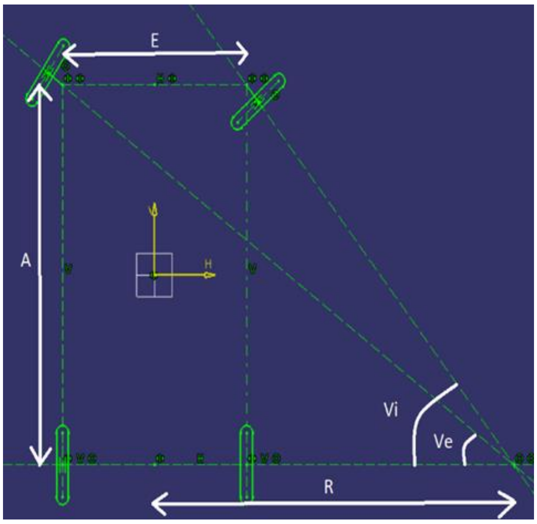
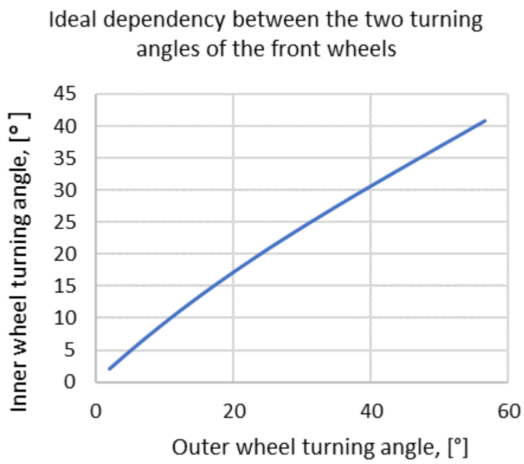


Fig. 3. *Ideal turn angles*



The blue line becomes our objective at this stage and, for achieving such an ideal curve, a kinematic algorithm is needed.

2. Kinematic algorithm

Every calculus algorithm has an input and an output.

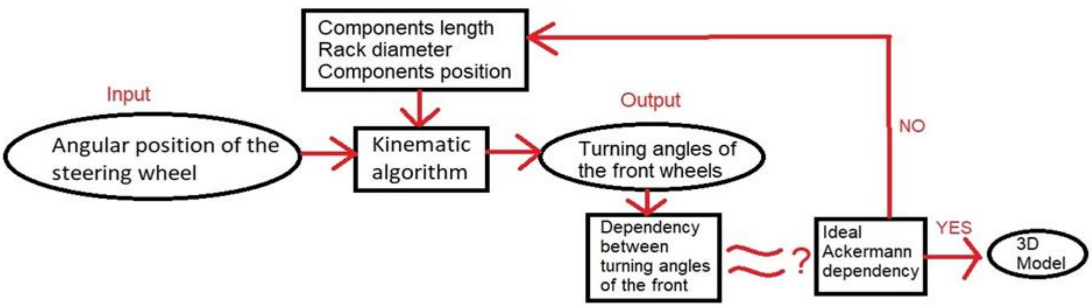


Fig. 4. *Kinematic algorithm overview*

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2.1. Ackermann's angles

The theory proposes that the possibility of achieving Ackermann geometry is greater if the intersection point of the extension of steering arms is placed in the middle of the rear axle [5].

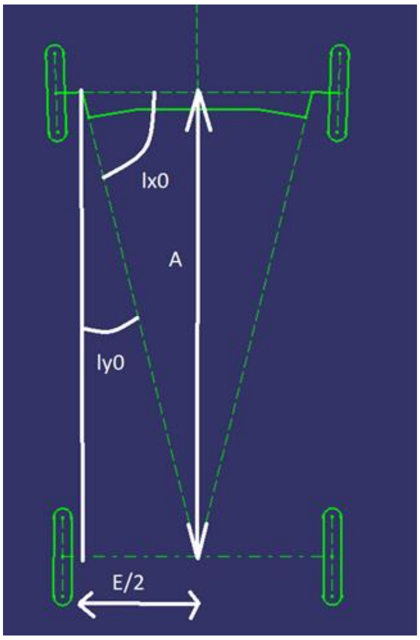

Fig. 5. *Ackermann angles*

Verry important for the following calculations are the two angles from Figure 5.

$$l_{y0} = \arctan\left(\frac{\frac{E}{2}}{A}\right) \quad ; l_{x0} = 90° - l_{y0} \qquad (2)$$

2.2. Length of the tie rod

As in the figure 1, in figure 6 we see the following components: l-steering arm, b-tie rod, t-transverse bar, B and C- rod ends and A- steering axis.

$$x_A = \frac{E}{2};\ y_A = 0;\ x_C = \frac{t}{2} \qquad (3)$$

$$x_B = x_A - l\sin(l_{y0}) \qquad (4)$$

$$b = \sqrt{(x_C - x_B)^2 + (y_C - y_B)^2} \qquad (5)$$

For finding out the length of the tie rod, the first two unknown parameters, t and y_C, have been introduced. Till now, we analysed the steering linkage only in neutral position, in other words, the car is not turning. The car is steering only if the rack is moving in the rack and pinion box. Now, the third unknown parameter is introduced.

$$x = \pi \cdot d_p \cdot \frac{\varphi_v}{360^0} \qquad (6)$$

In equation 6: x - transverse bar displacement in x direction, the only direction the rack is moved by the driver via steering wheel; d_p - pinion diameter, the third unknown parameter, and φ_v - our input parameter, the angular position of the steering wheel. For passenger cars, the steering wheel makes 1.5 turns. This means that our input vector is [-540°, +540°].

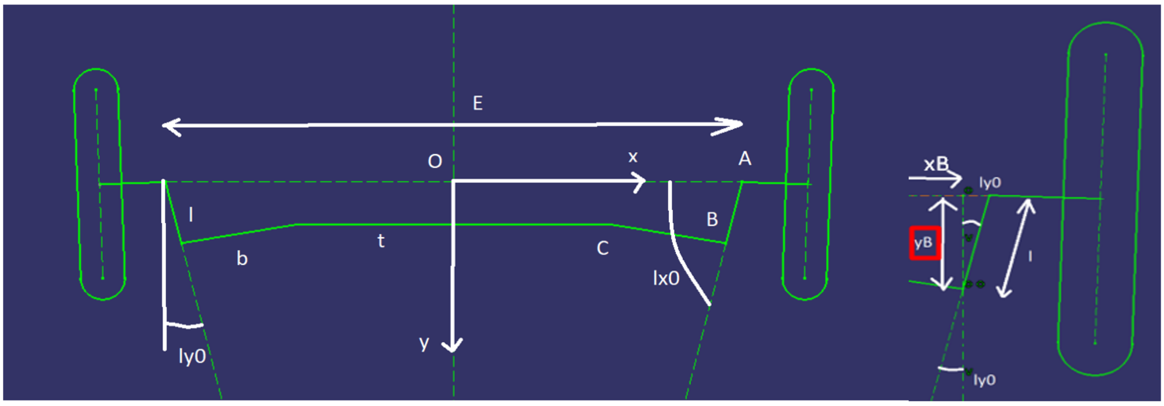

Fig. 6. *Length of the tie rod*

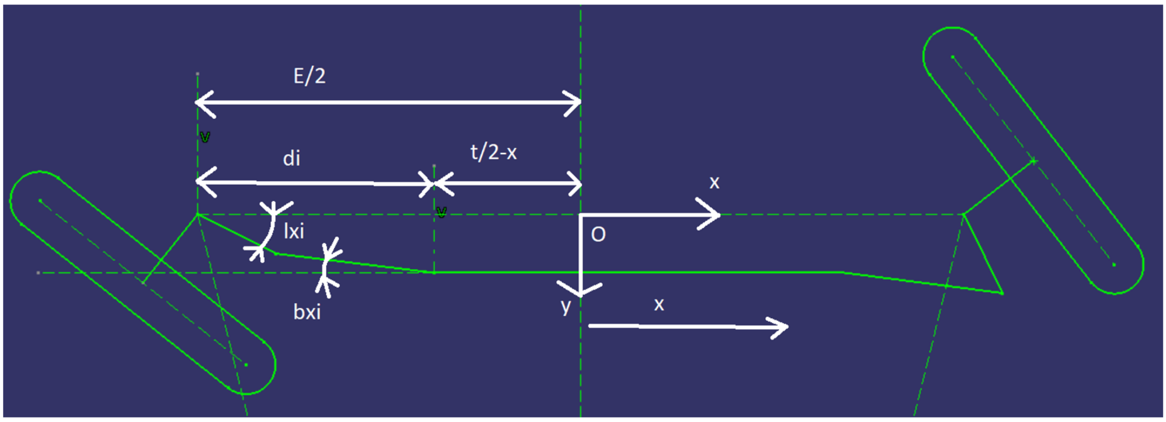

Fig. 7. *Top view of the steering linkage for finding the turn angle of the inner wheel*

2.3. Turn angle of the inner wheel

$$d_i = \frac{E}{2} - \frac{t}{2} + x \tag{7}$$

$$b \cdot \cos(b_{xi}) = d_i - l \cdot \cos(l_{xi}) \tag{8}$$

$$b \cdot \sin(b_{xi}) = y_C - l \cdot \sin(l_{xi}) \tag{9}$$

$$\begin{gathered} b^2 = d_i^2 + {y_C}^2 + l^2 \\ -2 \cdot d_i \cdot l \cdot cos(l_{xi}) \end{gathered} \tag{10}$$

$$\begin{gathered} -2 \cdot y_C \cdot l \cdot sin(l_{xi}) \\ 2 \cdot d_i \cdot l \cdot cos(l_{xi}) \\ +2 \cdot y_C \cdot l \cdot sin(l_{xi}) \end{gathered} \tag{11}$$

$$\begin{gathered} +b^2 - d_i^2 - {y_C}^2 - l^2 = 0 \\ P \cdot cos(l_{xi}) + Q \cdot sin(l_{xi}) + R = 0 \end{gathered} \tag{12}$$

$$\begin{gathered} P = 2 \cdot d_i \cdot l \ ; \ Q = 2 \cdot y_C \cdot l \ ; \\ R = b^2 - d_i^2 - {y_C}^2 - l^2 \end{gathered} \tag{13}$$

$$sin(l_{xi}) = \frac{2 \cdot tan\,(\frac{l_{xi}}{2})}{1 + tan\,(\frac{l_{xi}}{2})^2} \tag{14}$$

$$cos(l_{xi}) = \frac{1 - tan\,(\frac{l_{xi}}{2})}{1 + tan\,(\frac{l_{xi}}{2})^2} \tag{15}$$

$$sin(l_{xi}) = \frac{2 \cdot z}{1 + z^2} \tag{16}$$

$$cos(l_{xi}) = \frac{1 - z}{1 + z^2} \tag{17}$$

$$P \cdot \frac{1 - z}{1 + z^2} + Q \cdot \frac{2 \cdot z}{1 + z^2} + R = 0 \tag{18}$$

$$(R - P)z^2 + 2Qz + P + R = 0 \tag{19}$$

$$a = (R - P)\ ;\ b = 2Q;\ c = P + R \tag{20}$$

$$\Delta = b^2 - 4ac \tag{21}$$

$$z = \frac{-b - \sqrt{\Delta}}{2a} \tag{22}$$

$$z = tan\left(\frac{l_{xi}}{2}\right) \tag{23}$$

$$l_{xi} = 2 \cdot arctan\,(z) \tag{24}$$

$$\beta_i = l_{x0} - l_{xi} \tag{25}$$

In these equations we used for the first time “l”, so we introduced our last unknown parameter. "β_i" is the angular variation of the inner wheel. There still is a toe angle, the angle that each tire makes with the longitudinal plane of the vehicle. For most passenger cars, the front tires are not only for steering, but also for traction. Because of this, even if the angular position of the steering wheel is “0”, the front tires will be very little steered, that angle being called toe-out angle. Therefore, also this small angle must be considered for the final value [4].

$$v_i = \beta_i - \gamma_0 \tag{26}$$

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2.4. Turn angle of the outer wheel

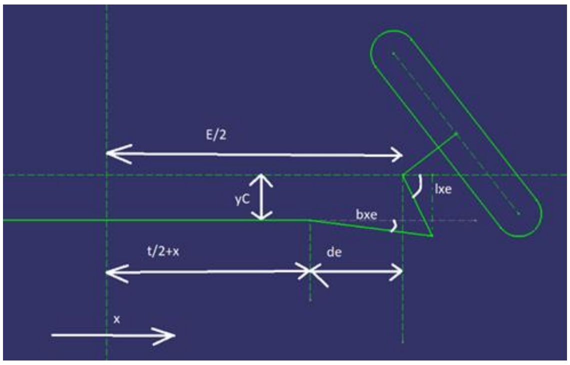

Fig. 8. *Top view of write half steering linkage*

The calculation method is almost identical with only a few differences.

$$d_e = \frac{E}{2} - \frac{t}{2} - x \tag{27}$$

$$b \cdot cos(b_{xe}) = d_e + l \cdot cos(l_{xe}) \tag{28}$$

$$b \cdot sin(b_{xe}) = l \cdot sin(l_{xe}) - y_C \tag{29}$$

$$\begin{gathered} b^2 = d_e^2 + {y_C}^2 + l^2 \\ 2 \cdot d_e \cdot l \cdot \cos(l_{xe}) \end{gathered} \tag{30}$$

$$\begin{gathered} -2 \cdot y_C \cdot l \cdot \sin(l_{xe}) \\ 2 \cdot \mathrm{d}_e \cdot l \cdot \cos(l_{xe}) \\ -2 \cdot y_C \cdot l \cdot \sin(l_{xe}) \end{gathered} \tag{31}$$

$$\begin{gathered} -b^2 + d_e^2 + {y_C}^2 + l^2 = 0 \\ P \cdot \cos(l_{xe}) + Q \cdot \sin(l_{xe}) + \mathrm{R} = 0 \end{gathered} \tag{32}$$

$$\begin{gathered} P = 2 \cdot d_e \cdot l \ ; \ Q = -2 \cdot y_C \cdot l \ ; \\ \mathrm{R} = -b^2 + d_e^2 + {y_C}^2 + l^2 \end{gathered} \tag{33}$$

From this point, equations (14)-(24) are the same, but this time we are trying to find the outer wheel turning angle, “l_{xe}", not “l_{xi}". As we found out "l_{xe}", based on Figure 9 we can deduce the angular variation of the outer wheel.

$$\beta_e = (180 - l_{xe}) - l_{x0} \tag{34}$$

$$v_e = \beta_e + \gamma_0 \tag{35}$$

In equation (52), as in (26), the γ_0 is a toe out angle.

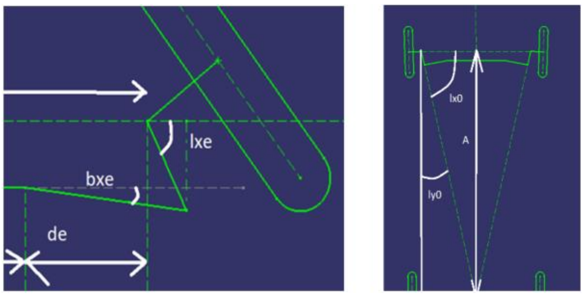

Fig. 9. *Turning angle of the write wheel*

3. Results

We see that there is no chance for an ideal rack and pinion steering system. Mathematically, it is impossible to obtain a 'blue curve' for every angular position of the steering wheel for a rack-and-pinion steering system. If we cannot obtain a “blue curve” for all steering wheel positions, then we can try to obtain it for large turning angles at least, where the speed of the vehicle is very low, and the car cannot take advantage of the slip angle.

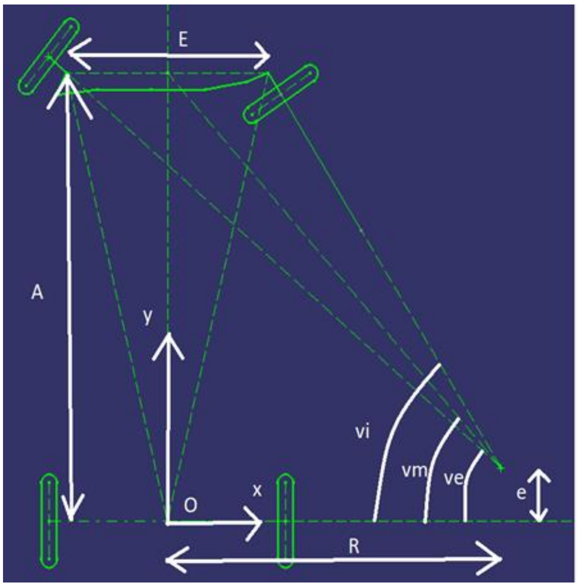

Fig. 10. *Ackermann error*

$$R = \frac{E}{2} \cdot \frac{tan(v_e) + \tan(v_i)}{\tan(v_i) - \tan(v_e)} \tag{36}$$

$$e = A - \left(R - \frac{E}{2}\right) tan(v_i) \tag{37}$$

As we can see, the relative error between our results and Ackermann principle is less than 5% for very tight curves. Moreover, the fact that in the less tight curve domain, the error is very high is normal.

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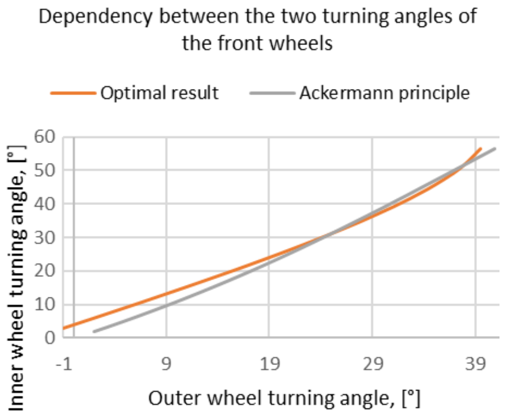
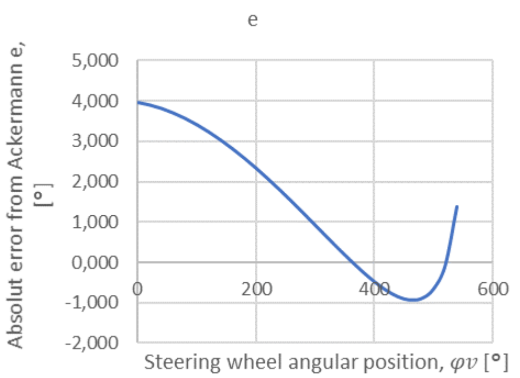
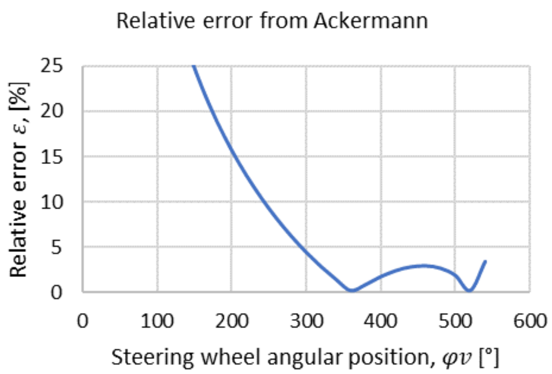
In this domain, the car is running almost straight with a higher speed, which increases the slip angle. Slip angle will reduce this high error. The values of slip angle are very unpredictable because they are based on many complex aspects regarding vehicle dynamics. In this paper an Ackermann curve in low speeds domain (extreme angular steering wheel positions) is desired to be obtained.

$$\varepsilon = \frac{|v_{ideal} - v_{real}|}{v_{ideal}} \cdot 100$$







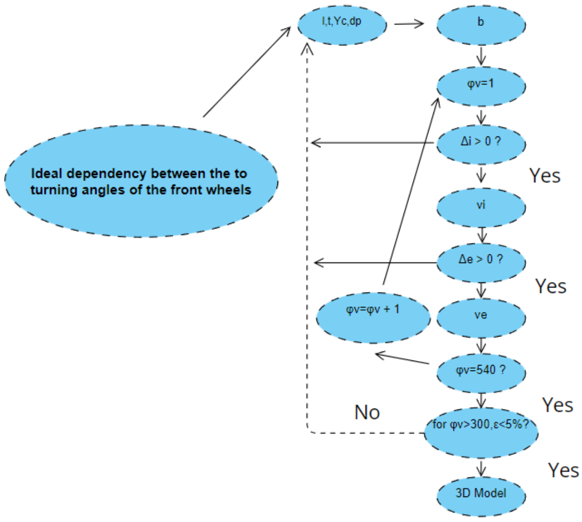


Fig. 11. *Kinematic algorithm*

After a lot of iterations these are the winning values which define the steering linkage geometry:

Table 2. *Winning values*

Notation	Measure unit	Value
l	[mm]	150
t	[mm]	700
y_C	[mm]	100
d_p	[mm]	22

The only known from the beginning parameters were:

Table 3. *Known dimensions*

Parameter	Notation	Measure unit	Value
Wheelbase	A	[mm]	2650
Wheel track	E	[mm]	1530

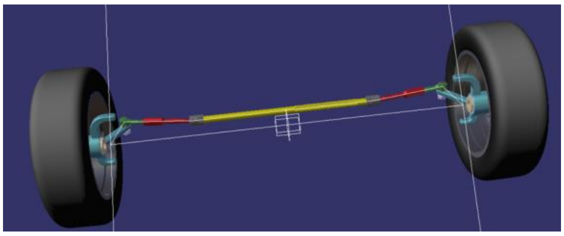


Fig. 12. *Steering linkage CAD*

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4. CAD and CAE

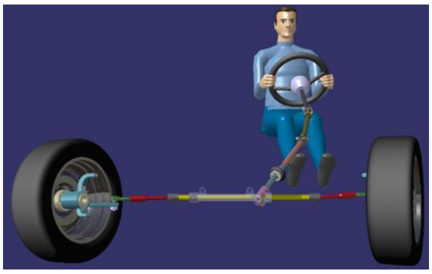

***Fig. 13.** Steering system CAD*

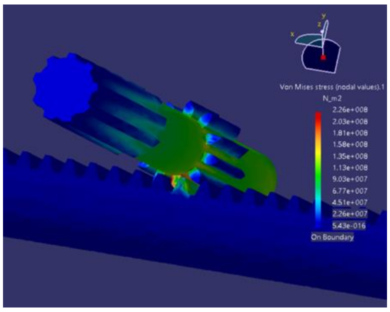

***Fig. 14.** Finite Element Analyse for rack and pinion*

***Table 4.** Materials*

Component	Material
Rack and Pinion	42CrMo4
Tie rod	High strength steel
Rod end	PH13-8Mo
Steering knuckle	Al 7057-T6
Steering column	Carbon fiber

5. Conclusions

This paper presents a possible approach to the design and optimization of a such a complex system as a steering system. The project started with a very solid objective, an algorithm calculus has been created based on 2D sketches, the dimensions of the steering linkage were continuously changed based on the algorithm. If 2D model helped the kinematic improvement of the system, later, a CAD model has been created for running kinematic simulations and finite element analyses. It is also well to know that even if Ackermann is the most used name in steering system design, there were another 2 men, Darwin and Lnakensperger, who invented a steering linkage geometry used in horse-drawn carriages for improving the stability, a geometry which is also used today in our cars steering system. We also saw that it is very hard to keep all four wheels on concentric paths with a mechanical steering system. Also, to be noticed that a toe out angle is often used for a better steerability.

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