THE ANNALS OF “DUNAREA DE JOS” UNIVERSITY OF GALATI
FASCICLE IX. METALLURGY AND MATERIALS SCIENCE
N°. 3 - 2022, ISSN 2668-4748; e-ISSN 2668-4756
Article DOI: https://doi.org/10.35219/mms.2022.3.07

THE PROBABILISTIC METHOD FOR DETERMINATION THE INERTIA CHARACTERISTICS OF 3D BODIES WITH MONTE CARLO ALGORITHMS

Marius BOTIȘ, Costel PLEȘCAN
Transilvania University of Brasov, Department of Civil Engineering, Romania
e-mail: plescan.costel@unitbv.ro

ABSTRACT

This study determines the inertial characteristics of bodies with probabilistic algorithms of the Monte Carlo type. To determine the mechanical moments of inertia in the case of three-dimensional bodies, the bodies are decomposed into simple bodies for which the moments of inertia are calculated, which are added to Steiner's relations. If the bodies have complex shapes to determine the moments of mechanical inertia, numerical or probabilistic algorithms can be used. This paper presents the main stages in the use of probabilistic algorithms for calculating moments of inertia and finally a program in MATLAB that allows determining the moments of inertia for bodies.

KEYWORDS: Monte Carlo method, moments of inertia of three-dimensional bodies, probabilistic algorithms

1. Introduction

To determine the moments of inertia of the three-dimensional bodies with the Monte Carlo type algorithm, we start from the calculation relations of the moments of inertia in the case of a system of concentrated masses distributed in space.

In the case of a system of n-concentrated masses located in space, the tensor of the mechanical moments of inertia (Billingsley, 2018) is determined starting from the definition relations of the mechanical moments of inertia.

Axial mechanical moments of inertia with respect to the x y and z axis:

Axial mechanical moments of inertia with respect to the x y and z axis:

$$\mathbf{J_{xx}} = \int_A (\mathbf{y^2} + \mathbf{z^2})\mathbf{dm} = \sum_{\mathbf{i=1}}^{\mathbf{n}} \mathbf{m_i}(\mathbf{y_i^2} + \mathbf{z_i^2});$$

$$J_{yy} = \int_A (x^2 + z^2)dm = \sum_{i=1}^{n} m_i(x_i^2 + z_i^2);$$

$$J_{zz} = \int_A (x^2 + y^2)dm = \sum_{i=1}^{n} m_i(x_i^2 + y_i^2)$$

Centrifugal moment of inertia:

$$J_{xy} = \int_A xydm = \sum_{i=1}^{n} m_i x_i\, y_i\ .$$

$$J_{xz} = \int_A xzdm = \sum_{i=1}^{n} m_i x_i z_i\ ;$$

$$J_{yz} = \int_A yzdm = \sum_{i=1}^{n} m_i y_i z_i;$$

The inertia tensor of the mechanical moments of inertia in matrix form becomes:

$$[J] = \begin{bmatrix} \sum_{i=1}^{n} m_i(y_i^2 + z_i^2) & -\sum_{i=1}^{n} m_i x_i\, y_i & -\sum_{i=1}^{n} m_i x_i\, z_i \\ -\sum_{i=1}^{n} m_i x_i\, y_i & \sum_{i=1}^{n} m_i(x_i^2 + z_i^2) & -\sum_{i=1}^{n} m_i y_i\, z_i \\ -\sum_{i=1}^{n} m_i x_i\, z_i & -\sum_{i=1}^{n} m_i y_i\, z_i & \sum_{i=1}^{n} m_i(y_i^2 + x_i^2) \end{bmatrix}$$

Similarly, the tensor of the geometric moments of inertia is determined, which in the form of a matrix becomes:

THE ANNALS OF “DUNAREA DE JOS” UNIVERSITY OF GALATI
FASCICLE IX. METALLURGY AND MATERIALS SCIENCE
N°. 3 - 2022, ISSN 2668-4748; e-ISSN 2668-4756
Article DOI: https://doi.org/10.35219/mms.2022.3.07

$$[I]=\begin{bmatrix}\sum_{i=1}^{n}V_i(y_i^2+z_i^2) & -\sum_{i=1}^{n}V_i x_i y_i & -\sum_{i=1}^{n}V_i x_i z_i\\ -\sum_{i=1}^{n}V_i x_i y_i & \sum_{i=1}^{n}V_i(x_i^2+z_i^2) & -\sum_{i=1}^{n}V_i y_i z_i\\ -\sum_{i=1}^{n}V_i x_i z_i & -\sum_{i=1}^{n}V_i y_i z_i & \sum_{i=1}^{n}V_i(y_i^2+x_i^2)\end{bmatrix}$$

From the established relations (3) and (4) it can be observed that the knowledge of the tensor of the geometric moments of inertia also implies the knowledge of the tensor of the geometric moments of inertia.

$$[J]=\rho[I]$$

2. The algorithm for calculating the moments of inertia with the Monte Carlo method

If we know how to calculate the mechanical and geometric moments of inertia for a system of concentrated masses, we can explain how to calculate the geometric and mechanical inertia tensor with the Monte Carlo method.

For the calculation of the moments of inertia for three-dimensional bodies with the Monte Carlo method, infinitesimal masses that are calculated with the following equation.

$$dm=\frac{M}{N}$$

- N is the number of probabilistically generated points in the surface domain for which the moments of inertia are determined;
- M the total mass of the surface.

The higher the number of points generated on the surface domain, the higher the probability of tending to the exact solution. By using the Monte Carlo algorithm, the calculation relations for the mechanical inertia tensor and the geometric inertia tensor become.

Tensor of mechanical moment inertia:

$$[J]=\begin{bmatrix}\sum_{i=1}^{n}dm(y_i^2+z_i^2) & -\sum_{i=1}^{n}dm x_i y_i & -\sum_{i=1}^{n}dm x_i z_i\\ -\sum_{i=1}^{n}dm x_i y_i & \sum_{i=1}^{n}dm(x_i^2+z_i^2) & -\sum_{i=1}^{n}dm y_i z_i\\ -\sum_{i=1}^{n}dm x_i z_i & -\sum_{i=1}^{n}dm y_i z_i & \sum_{i=1}^{n}dm(y_i^2+x_i^2)\end{bmatrix}$$

Tensor of geometric moments of inertia:

$$[I]=\begin{bmatrix}\sum_{i=1}^{n}dV(y_i^2+z_i^2) & -\sum_{i=1}^{n}dV x_i y_i & -\sum_{i=1}^{n}dV x_i z_i\\ -\sum_{i=1}^{n}dV x_i y_i & \sum_{i=1}^{n}dV(x_i^2+z_i^2) & -\sum_{i=1}^{n}dV y_i z_i\\ -\sum_{i=1}^{n}dV x_i z_i & -\sum_{i=1}^{n}dV y_i z_i & \sum_{i=1}^{n}dV(y_i^2+x_i^2)\end{bmatrix}$$

where:

- x_i, y_i, z_i are the coordinates of the concentrated masses dm.
- V is the volume delimited by the three-dimensional body for which the inertia tensor and the geometric tensor are determined.

3. Presentation of MATLAB programs for calculating inertia tensors using the Monte Carlo method

To determine the efficiency of Monte Carlo type algorithms, the moments of inertia will be determined for a cube. In the cases analysed, the moments of inertia will be determined both probabilistically and with exact methods. The following analysis was performed to compare the results obtained analytically with the results obtained with algorithms such as Monte Carlo probabilistic algorithm.

3.1. Implementing the Monte Carlo algorithm in MATLAB for a cube

Matlab code implementation of the Monte Carlo method [2] for determining the moments of inertia for a square of mass M, the axial moments of inertia and the centrifugal moment of inertia for a square have the analytical expressions below:

Axial moments of inertia:

$$I_{yy}=I_{xx}=\frac{2}{3}Ma^2$$

Centrifugal moment of inertia:

$$I_{xy}=-\frac{1}{4}Ma^2$$

Below is a Matlab [3] code for determining the moments of inertia for a cube with side a and mass M. A program for calculating the inertia tensor in Python code is presented in [4].

```
%Calculation of moments of inertia for a cube:
clear;clf;
%Monte Carlo probabilistic algorithm
mass_cube=1;
edge=0.5;
```

Nº. 3 - 2022, ISSN 2668-4748; e-ISSN 2668-4756
Article DOI: https://doi.org/10.35219/mms.2022.3.07

```
n_mass=10000;
dm_point=mass_cube/n_mass;
Ixx_mc=0;Ixy_mc=0;n=0;
while n<n_mass
x_range=edge*rand;y_range=edge*rand;z_range=edge*rand;
scatter3(x_range,y_range,z_range,30,'MarkerEdgeColor',[0 0 0],...
        'MarkerFaceColor',[1 0 0],...
        'LineWidth',2)
hold on;
rtemp=[x_range y_range z_range];
Ixx_mc=Ixx_mc+dm_point*(x_range^2+z_range^2);
Ixy_mc=Ixy_mc-dm_point*(x_range*y_range);
n=n+1;
end
hold off;
%Theoretical values
Ixxt=2/3*mass_cube*edge^2;
Ixyt=-1/4*mass_cube*edge^2;
Ixx_mc
Ixxt
Ixy_mc
Ixyt
```

Following the analysis, the results presented in Fig. 1 and Fig. 2 were obtained, which are compared with the analytical results obtained.

- For 600 randomly generated points Fig. 1, the theoretical and probabilistic results are:

Theoretical value of moment of inertia
Ixxt = 0.1667
Value moment of inertia with Monte Carlo
Ixx= 0.1720
Theoretical value of moment of inertia
Iyyt =0.1667
Value moment of inertia with Monte Carlo
Iyy= 0.1682
Theoretical value of moment of inertia
Ixyt = -0.0625
Value moment of inertia with Monte Carlo
Ixy= -0.0625

- For 10000 randomly generated points Fig. 2, the theoretical and probabilistic results are:

Theoretical value of moment of inertia
Ixxt = 0.1667
Value moment of inertia with Monte Carlo
Ixx= 0.1655
Theoretical value of moment of inertia
Iyyt =0.1667
Value moment of inertia with Monte Carlo
Iyy= 0.1635
Theoretical value of moment of inertia
Ixyt = -0.0625
Value moment of inertia with Monte Carlo
Ixy= -0.0629

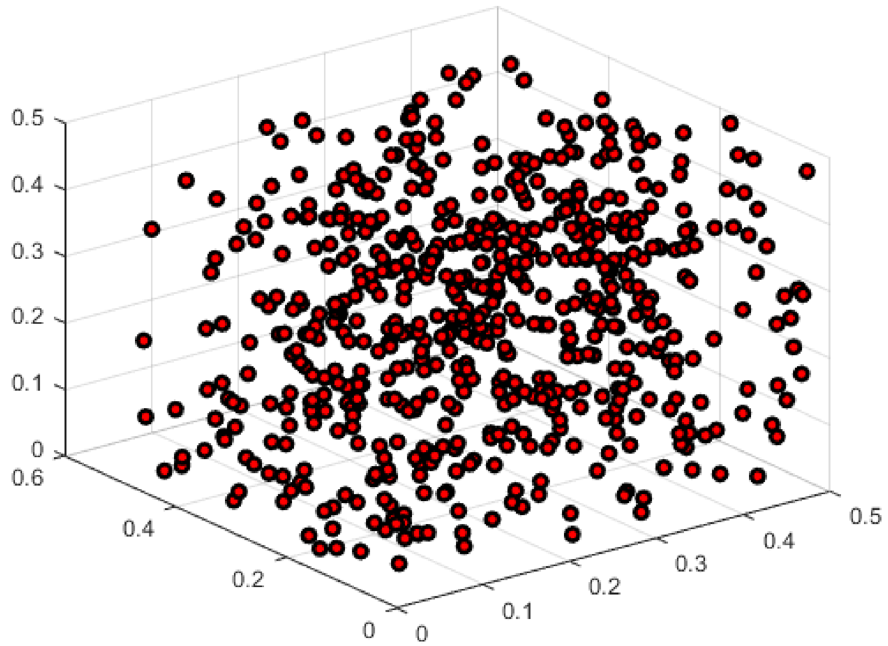

Fig. 1. *Generating 600 points randomly*

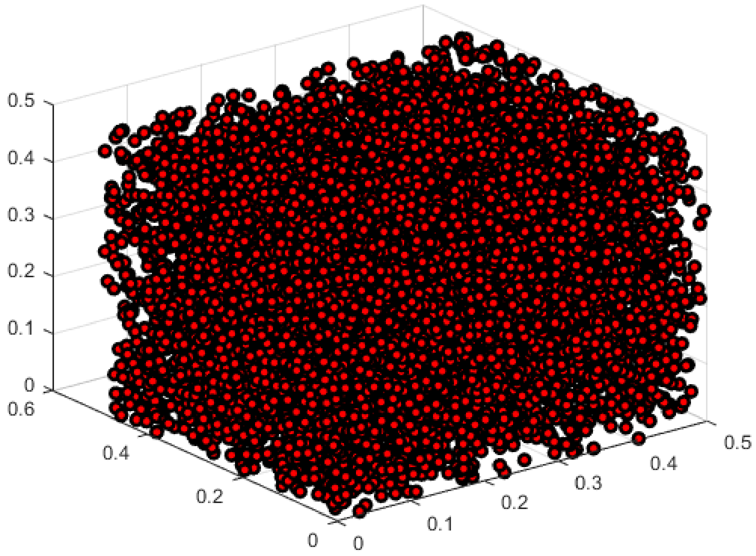

***Fig. 2.** Generating 10000 points randomly*

4. Results and conclusions

The analysis of the obtained results shows:

- the Monte Carlo method generates solutions that have a probabilistic character;

- in the calculation of the inertia tensor, the result depends on the number of masses generated in the field occupied by the analysed body;

- the method can be applied if the geometric shape of the body is complex and the calculation with exact methods becomes difficult.

To determine the inertia tensor in the case of flat surfaces, the calculation algorithm is presented in the paper [5].

References

[1]. Billingsley J., *Essentials of Dynamics and Vibration*, Publishing House Springer, 2018.

[2]. Barbu A., Zhu S., *Monte Carlo Methods*, Publishing House Springer ,2020.

[3]. ***, https://www.mathworks.com/products/matlab.html, accessed: 22.00.2021.

[4]. ***, https://www.youtube.com/watch?v=SbTSATs-DBA.

[5]. Botiș M., Pleșcan C., *Matlab Program for Determining the Inertia Characteristics of Flat Surfaces with Monte Carlo Algorithms*, The Annals of "Dunarea De Jos" University of Galati, p. 22-26, June 2022.