THE ANNALS OF "DUNAREA DE JOS" UNIVERSITY OF GALATI
FASCICLE IX. METALLURGY AND MATERIALS SCIENCE
Nº. 4 - 2022, ISSN 2668-4748; e-ISSN 2668-4756
Article DOI: https://doi.org/10.35219/mms.2022.4.01

CONSOLIDATION OF CENTRAL COLUMNS OF CIVIL MULTISTORY STRUCTURES TO INCREASE THE CRITICAL BUCKLING FORCE

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ABSTRACT

Civil multi-story structures are used as office buildings or residential buildings. The central pillars on the ground floor of the civil structures will have the largest gravitational load and implicitly will be the pillars most stressed in compression. In this paper, the authors propose to increase the critical buckling load of the central pillars on the ground floor in the event that it is necessary to strengthen these pillars, due to the change in the purpose of the building and the modification of the loads on the floors. To increase the buckling load in the case of these columns, it is proposed to increase the moment of inertia of the columns on the base region. To highlight how the critical loss of stability load changes, a theoretical model is made for the column with constant section and another for the column with variable section in steps. Finally, the results obtained for the critical load will be compared and it will be highlighted what is the increase in buckling load for the pole with variable section in relation to the pole with constant section.

KEYWORDS: buckling force, multistory structures, moment of inertia

1. Introduction

Gravitational loads in the case of these structures are permanent loads (self-weight of the floor), quasi-permanent loads (self-levelling piles) and useful loads. The gravity loads collected from the floors through the beams drain through the pillars to the foundations and finally to the good foundation ground. For regular structures in plan and elevation we encounter 3 types of pillars in terms of gravity loads flowing through them. Corner columns that take the load from ¼ of the area of a slab mesh, marginal columns that take the load from ½ of the area of a slab mesh and central columns that take the load from the surface of a slab mesh. From fig.1 it can be seen that the pillars on the ground floor have the largest gravitational effect. The most stressed columns in compression will be the central columns that collect the gravitational load from all the floors above it. Central columns on the ground floor are loaded with the highest compression forces, therefore in their case the increase of the critical force of loss of stability is justified. If the loads with compressive forces become high and at marginal posts, the cross-section at the base can also be increased in their case as well. In general, central columns have high compressive loads due to gravity loads, and marginal columns are loaded with high axial forces due to seismic actions that load and unload marginal columns with compressive forces. The solution proposed in the paper aims to increase the cross-section of the pillars only at the base. In this case, the foundation that will take over the capable moment of the pole must also be checked. In the case of the consolidation of a multi-story civil structure where the incarnation on the floors has been modified, the degree of compression stress on the pillars on the ground floor is different. That's why the area on which the section is died is done according to the type of pillar, the degree of loading with axial forces and the action to which the structure is subjected. If the section enlargement were to be done along the entire length of the pillars, then the space on the ground floor would be considerably reduced. Hence the need to increase the section of the pillars depending on their position within the ground floor and the degree of loading with the axial compression force. The article presents the case of a pillar whose cross-section is increased by half the length of the pillar. Obviously, the civil structures to which the consolidation is done can be metal or concrete. In the case of metal structures, the consolidation is done by welding additional elements and in the case of concrete structures by adding perimeter concrete.

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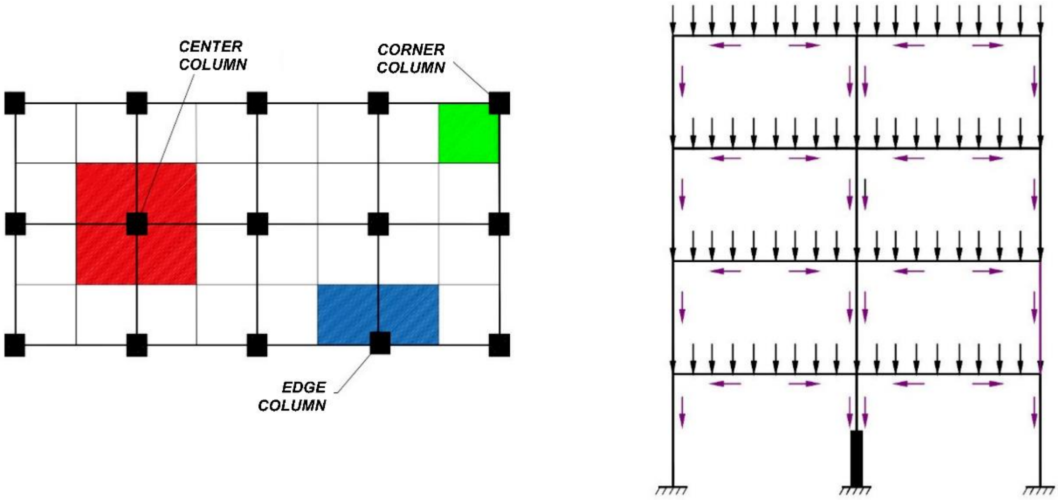

Fig. 1. *Gravitational afferents on the columns*

2. Theoretical calculation model for sliding embedded-recessed bar with constant section and with variable section in steps

We first consider the case of the bar with constant section at which the critical loss of stability force is determined.

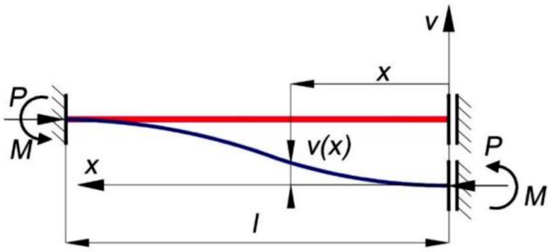

Fig. 2. *The deformed shape of the bar with constant section*

For the bar in Fig. 2, write the equation of the average deformed fiber on the deformed shape of the bar to determine the critical load [1].

For a current section the bending moment variation expression is:

$$M(x) = P \cdot v - M \tag{1}$$

From the approximate deformed mean fiber differential equation, it follows:

$$\frac{d^2v}{dx^2} = -\frac{M(x)}{EI}$$
$$\frac{d^2v}{dx^2} + \frac{P \cdot v}{EI} = \frac{M}{EI} \tag{2}$$

$$k^2 = \frac{P}{EI}$$

The solution of the inhomogeneous second-order differential equation with constant coefficients is:

$$\begin{gathered} v(x) = v_o(x) + v_p(x); \\ v(x) = C_1 \sin(kx) + C_2 \cos(kx); \\ v_p(x) = C; \\ \frac{P \cdot v_p}{EI} = \frac{M}{EI}; \\ v_p = \frac{M}{P} \end{gathered} \tag{3}$$

The general solution of the differential equation and its derivative are:

$$\begin{gathered} v(x) = C_1 \sin(kx) + C_2 \cos(kx) + \frac{M}{P}; \\ v'(x) = C_1 k \cos(kx) - C_2 k \sin(kx). \end{gathered} \tag{4}$$

The integration constants are determined from the existence conditions of the deformed shape as an equilibrium configuration:

$$\begin{gathered} x = 0; \\ v(0) = 0; \\ v'(0) = 0; \\ x = l; \\ v'(l) = 0. \end{gathered} \tag{5}$$

By applying the boundary conditions, the homogeneous system of equations results:

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$$\begin{cases} C_2 + \frac{M}{P} = 0 \rightarrow C_2 = -\frac{M}{P}; \\ kC_1 = 0; \\ C_2 k \sin(kl) = 0. \end{cases} \quad (6)$$

The critical force of loss of stability results from the condition that the determinant of the homogeneous system of equations is zero.

$$C_2 k \sin(kl) = 0; \quad (7)$$
$$(kl) = n\pi; n = 1,2,3 \ldots$$

The first 3 values for critical instability forces are:

for n=1

$$k_1 l = \pi; \quad k_1^2 = \frac{P_{cr1}}{EI}; \quad P_{cr1} = \frac{\pi^2 EI}{l^2}. \quad (8)$$

for n=2

$$k_2 l = 2\pi; \quad k_2^2 = \frac{P_{cr2}}{EI}; \quad P_{cr1} = \frac{4\pi^2 EI}{l^2}. \quad (9)$$

for n=3

$$k_3 l = 3\pi; \quad k_3^2 = \frac{P_{cr3}}{EI}; \quad P_{cr3} = \frac{9\pi^2 EI}{l^2}. \quad (10)$$

For the bar with variable section in steps in Fig. 3, the critical load for loss of stability is determined using the equation of the average deformed fiber, and the balance is written on the deformed shape of the bar [1].

The sectional bending stresses on the section with stiffness module EI and on the section with stiffness module 4EI are:

$$M_{EI}(x) = P \cdot v_1 - M; \quad (11)$$
$$M_{4EI}(x) = P \cdot v_2 - M.$$

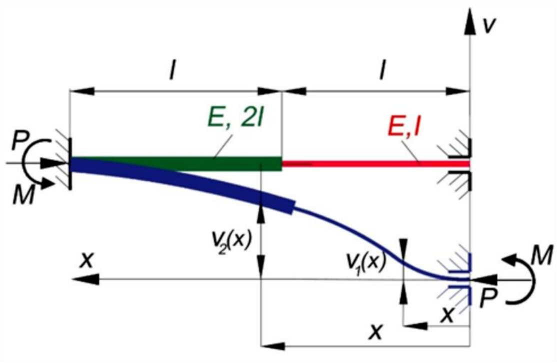

Fig. 3. *The deformed shape of the bar with variable section*

Using the stress variation expressions, the differential equations of the average fiber deformed on the 2 sections result:

$$\frac{d^2 v_1}{dx^2} = -\frac{M_{12}(x)}{EI}; \quad \frac{d^2 v_1}{dx^2} + \frac{P \cdot v_1}{EI} = \frac{M}{EI}; \quad \frac{d^2 v_2}{dx^2} = -\frac{M_{23}(x)}{4EI}; \quad \frac{d^2 v_2}{dx^2} + \frac{P \cdot v_2}{4EI} = \frac{M}{4EI}. \quad (12)$$

If the notations are made,

$$\frac{P}{4EI} = \alpha^2; \quad \frac{P}{EI} = (2\alpha)^2. \quad (13)$$

then the differential equations of the two sections become:

$$\frac{d^2 v_1}{dx^2} + (2\alpha)^2 v_1 = \frac{M}{EI}; \quad \frac{d^2 v_2}{dx^2} + \alpha^2 v_2 = \frac{M}{EI}. \quad (14)$$

The solutions of the differential equations and their derivatives are:

$$v_1(x) = C_1 \sin(2\alpha x) + C_2 \cos(2\alpha x) + \frac{M}{P};$$
$$v_2(x) = C_3 \sin(\alpha x) + C_4 \cos(\alpha x) + \frac{M}{P}; \quad (15)$$
$$v_1'(x) = 2\alpha C_1 \cos(2\alpha x) - 2\alpha C_2 \sin(2\alpha x);$$
$$v_2'(x) = \alpha C_3 \cos(\alpha x) - \alpha C_4 \sin(\alpha x).$$

Applying the boundary conditions yields:

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$$\begin{gathered}
x=0 \rightarrow v_1(0)=0; \\
C_2+\frac{M}{P}=0 \rightarrow C_2=-\frac{M}{P}; \\
x=0 \rightarrow v_1{}'(0)=0; \\
2\alpha C_1=0 \rightarrow C_1=0; \\
x=l; v_1(l)=v_2(l); \\
C_2\cos(2\alpha x)-C_3\sin(\alpha x)-C_4\cos(\alpha x)=0; \\
x=l; v_1{}'(l)=v_2{}'(l); \\
2\alpha C_2\sin(2\alpha l)+\alpha C_3\cos(\alpha l)-\alpha C_4\sin(\alpha l)=0; \\
x=2l; v_2{}'(2l)=0; \\
\alpha C_3\cos(2\alpha l)-\alpha C_4\sin(2\alpha l)=0.
\end{gathered} \tag{16}$$

If αl=x is written, the system of equations (16) becomes:

$$\begin{cases}
-C_3 sinx-C_4 cosx-\frac{M}{P}cos2x=0; \\
-C_3 cosx+C_4 sinx+2sin2x=0; \\
C_3 cos2x-C_4 sin2x=0.
\end{cases} \tag{17}$$

The system of equations (17) admits nontrivial solutions if the determinant of the coefficients is zero. Applying this condition results:

$$\begin{vmatrix}
-sinx & -cosx & -cos2x \\
-cosx & sinx & 2sin2x \\
cos2x & -sin2x & 0
\end{vmatrix}=0. \tag{18}$$

After solving the transcendental equation above in the Matlab program [2], the solutions are obtained:

x1= 0
x2=-acos((2^(1/2)*3^(1/2))/6)=-1.1503
x3=acos((2^(1/2)*3^(1/2))/6) =1.1503

The lowest critical force is obtained for x=1.1503, and is determined with the relation:

$$\begin{gathered}
x=\alpha l; \alpha=\frac{x}{l}; \\
\frac{P_{cr}}{4EI}=\alpha^2=\left(\frac{x}{l}\right)^2
\end{gathered} \tag{19}$$

$$P_{cr}=\frac{4EI\cdot 1{,}32}{l^2}=\frac{5{,}292EI}{l^2}=2{,}14\frac{\pi^2 EI}{(2l)^2};$$

3. Results and conclusions

From the analysis of the results obtained for the critical force of loss of stability in the case of the bar with constant section and the bar with variable section in steps, it follows:

- the critical force for a bar of length 2l, embedded at one end and sliding embedded at the other end is:

$$P_{cr\_ct}=\frac{\pi^2 EI}{(2l)^2}$$

- the critical force for a step-variable bar of length 2l embedded at one end and sliding embedded at the other end is:

$$P_{cr\_var}=2{,}14\frac{\pi^2 EI}{(2l)^2}$$

- the ratio between the critical forces in the case of the bar with constant section and the bar with variable section in steps is:

$$\frac{P_{cr\_var}}{P_{cr\_ct}}=2{,}14$$

In conclusion, an increase in the critical loss of stability force for the central columns on the ground floor can be obtained by increasing the section of the columns at the base.

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[1]. Bănuț V., Teodorescu M. E., *Calculul geometric neliniar al structurilor de rezistență*, București, Conspress, 2010.
[2]. *****, MathWorks Inc. MATLAB, Math. Graphics. Programming., [Interactiv]. Available: https://www.mathworks.com/products/matlab.html, accessed in 23.07.2022.