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DETERMINATION OF STRESS CONCENTRATION FACTOR FOR A STEPPED BAR UNDER BENDING LOADING: AN ARTIFICIAL NEURAL NETWORKS APPROACH

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ABSTRACT

Even today, charts and formulas derived from experimental determinations are used to obtain the stress concentration factor.

Stress concentration factors from charts can be converted into numerical values using computational techniques. Stress concentration factor values were collected in a database and an Artificial Neural Network (ANN) model can be developed for improving the database. ANN model provides accuracy in obtaining the stress concentration factors. Using the static stress concentration factor for a stepped bar under a bending load we can quantify the impact of notches in fatigue.

KEYWORDS: Stress Concentration Factor (SCF), Artificial Neural Network (ANN), bending loading

1. Introduction

The main objective of this paper is to develop an ANN model to properly predict the stress concentration factor (K_t) for a stepped bar under bending loading.

Stress concentration factor K_t can be obtained experimentally (especially using photo-elasticity), analytically and published in charts taking into account geometric property. This stress concentration factor is very important in brittle materials. For ductile materials the stress concentration factor is important in fatigue calculation when the safety is critical in the localized yielding hardens material.

In recent years, numerical models have been developed based on experimentally validated data using analytical formulas or by simulation using FEM; these models are obtained using simple regression, ANN approach or FEM simulation with optimization module.

The numerical finite element simulation is often time-consuming but an ANN model can predict with good accuracy the stress concentration factor while reducing the required time for the analysis [16].

Toktas [16], developed an ANN model with high accuracy for the prediction of stress concentration factor (K_t), for a Crankshaft under Bending Loading.

Stress concentration factor K_t is used in determining the fatigue concentration factor K_f.

2. Experimental background

Most structures that are designed have features such as holes, fillets and other notch shapes that cause stress concentration. Fatigue cracks commonly occur at these notches because at this location stresses are highest. An obstruction in a flow field of stresses increases the stresses around the obstruction. Notch is a general term meaning any or all of the fillets, holes, etc. The amount of stress around a notch is:

$$\sigma_{max} = K_t \cdot \sigma_{nom} \tag{1}$$

The stress concentration factor K_t provides the amplification of the stress at a notch relative to a nominal stress σ_{nom}. K_t is usually determined using charts or formulas [10].

The value of σ_{max} computed above is used when evaluating static failure due to yielding by the theory (von Mises) or the maximum shear stress criterion (Tresca) or when evaluating brittle static failure.

When evaluating the effect of notches on the fatigue resistance of a machine part, we use the fatigue stress concentration factor K_f instead of the stress concentration factor K_t. The fatigue stress concentration factor is usually smaller than the stress concentration factor.

The stress concentration factor in fatigue is [19]:

$$K_f = 1 + q(K_t - 1) \tag{2}$$

We use the notch sensitivity factor q to quantify the influence of notches in fatigue [20]:

$$= \frac{1}{1+\frac{\sqrt{a}}{\sqrt{r}}} \tag{3}$$

where $\sqrt{a}$ is Neuber's constant and r is the notch size; Neuber's constant depends on the value of the ultimate tensile strength S_{ut} of the material used [9].

For bending and axial stress, the Neuber's constant can be calculated using the relation (for steel) [20]:

$$\sqrt{a} = 0.246 - 3.08(10^{-3})S_{ut} + 1.51(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3$$

In practice, it is always safe to use K_t instead of K_f.

The elements that influence the stress concentration factor for a stepped bar in bending are represented in Fig. 1. The bar has an applied bending moment M. The parameters involved in the stress concentration factor K_t are (see Fig. 1):

- D = width of the larger section;
- d = width of the smaller section;
- r = radius of fillet;
- h= bar thickness;
- M = applied bending moment.

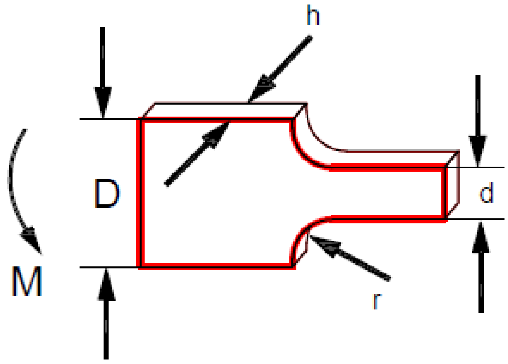

Fig. 1. *Parameters involved in the calculation of the stress concentration factor for a stepped bar in bending* [10, 20]

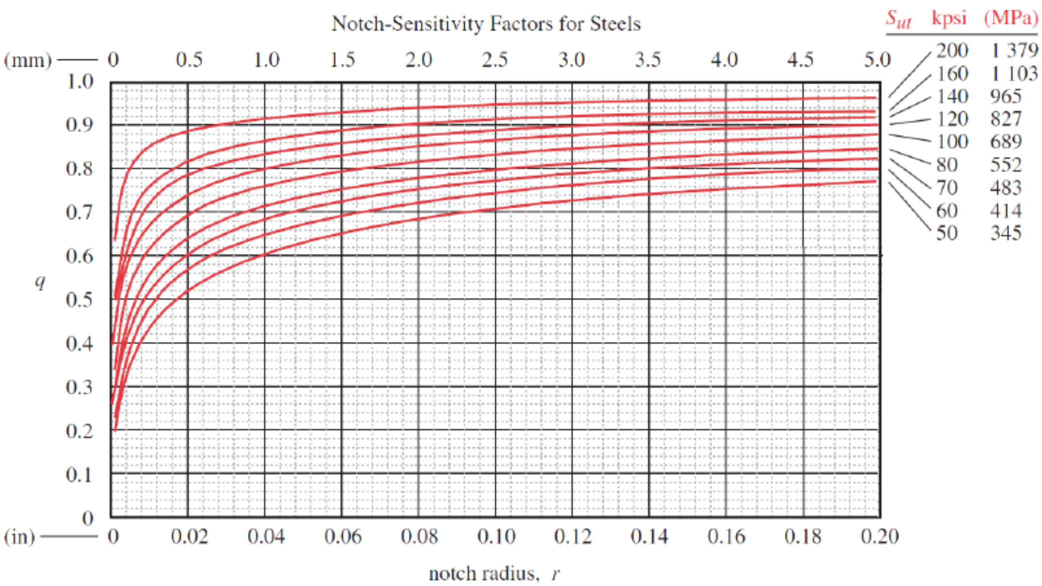

Fig. 2. *Notch Sensitivity Curves for steel* [13]

The stress concentration factor is calculated using formula 6 with coefficients from Table 1 [10].

$$\sigma_{nom} = 6\frac{M}{h \cdot d^2} \tag{4}$$

$$\sigma_{max} = K_t \cdot \sigma_{nom} \tag{5}$$

$$K_t = A\left(\frac{r}{d}\right)^b \tag{6}$$

Table 1. *Coefficients A and b as a function of D/d from relation (6)*

D/d	A	b
3	0.90720	-0.0333
1.2	0.9959	-0.23829
1.01	0.96689	-0.15417

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The stress concentration factors in fatigue is calculated using formula 2 with the notch sensitivity factor q from formula 3 and K_t from the formula 6. The notch sensitivity factor q can be obtained from Fig. 2 as a function of material (Steel) S_{ut} and the notch radius.

The training and the testing data (the experimental data) for stress concentration factor K_t in the case of a stepped bar in bending are given in the tables from Annex A.

3. Neural network modeling

Neural Network is a vast domain of technology where one can implement human brain modelling decisions into computer programs based on error and approximation.

The hierarchical network structure has an Input layer, an Output layer and between them a Hidden layer (Fig. 3).

In fitting problems, we want a neural network to map between a data set of numeric inputs and a set of numeric outputs (targets).

Training a neural network is used to perform a particular function by adjusting the values of the connections (weights + biases) between elements.

Neural networks are adjusted (by training) so that a particular input leads to a specific target.

The Input layer is a layer which communicates with the external environment and represents a pattern to the neural network. The Input layer should represent the condition for which we are training the neural network (Fig. 3).

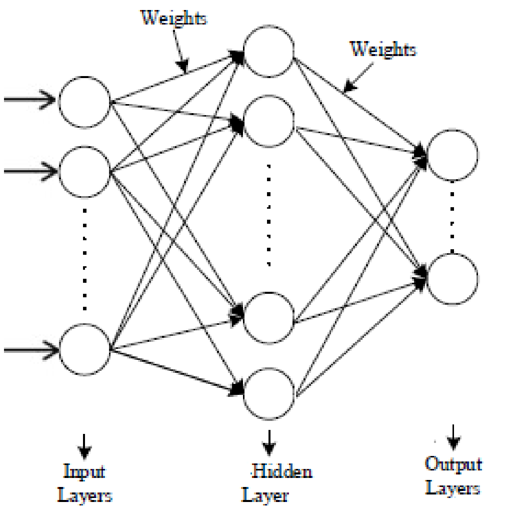

Fig. 3. *ANN structure with one hidden layer* [6]

The Output layer of the neural network represents a pattern to the external environment (Fig. 3).

The number of output neurons should be directly related to the type of work that the neural network is intended to achieve.

The Hidden layer is the collection of neurons which has an activation function applied to it. The Hidden layer provides an intermediate layer between the Input layer and the Output layer (Fig. 3).

The output response (s) using non-linear function is calculated as follows (eq. 7), [7]:

$$s = f(\sum w.e + b) \tag{7}$$

During training, weights (w) and biases (b) are initialized to small random values to avoid sharp saturation in activation functions (f).

The main objective of this study is the development of an ANN model to properly predict the major element influence on the stress concentration factor for stepped barsin bending loading.

The input and output variables are dimensionless quantities and there is no need for their standardization to obtain a fast optimal Regression coefficient R.

Some major parameters used in the final ANN model have been optimized:

- the type of activation functions in hidden and output layers;
- the number of hidden layers;
- the number of neurons in the hidden layer.

Two activation functions need first to be chosen: one applied in hidden layers and the second used in the output layer to determine the appropriate number of hidden neurons and output values.

It is a good practice to use the Sigmoid function the $(f(x) = 1/(1+e^{-x}))$ in hidden layer and the Linear function ($f(x) = x$) in the output layer (S.L.) [7].

It is chosen a 2-4-1 ANN structure and the corresponding numerical model has been expressed on equation (8):

$$Output = f_k(b_k + \sum w_{kj} . f_j(b_j + \sum w_{ji} . x_i)) \tag{8}$$

Where w_{ji} and w_{kj} are respectively the weights that connect input i to hidden layer j and hidden layer j to output layer k, b are biases and f are activation functions.

It is recommended that the number of hidden layers is 1-5 [5, 16], but many times good results can be obtained with a single hidden layer and in this sense many tests are to be made [1]. In practice, one or two hidden layers are sufficient to solve any nonlinear complex problem [1].

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All experiments were carried out using the MATLAB R2014b neural network toolbox [18].

The Levenberg-Marquardt (LM) learning algorithm version was used at the training and testing stages of the Networks.

The input and output variables are dimensionless quantities and without any standardization, we can obtain a fast optimal Regression coefficient R.

The regression coefficients for training different ANN structures and performance from Table 1 are highlighted in Fig. 4, Fig. 5, Fig. 6a; performance in training for the ANN structure 2-4-1 is given in the Fig. 6b.

The data from Table 2 highlights that the ANN-selected model is one with a single hidden layer with 4 neurons.

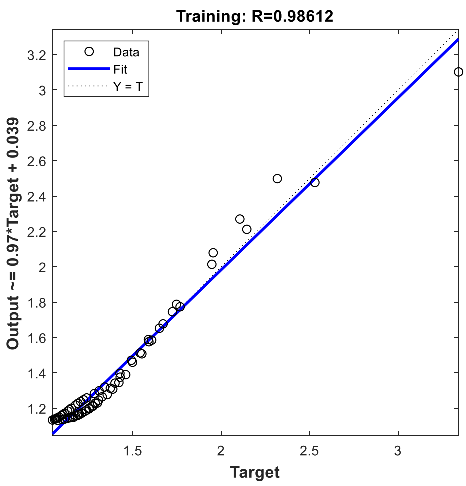


Fig. 4. *ANN structure 2-2-1 R training*

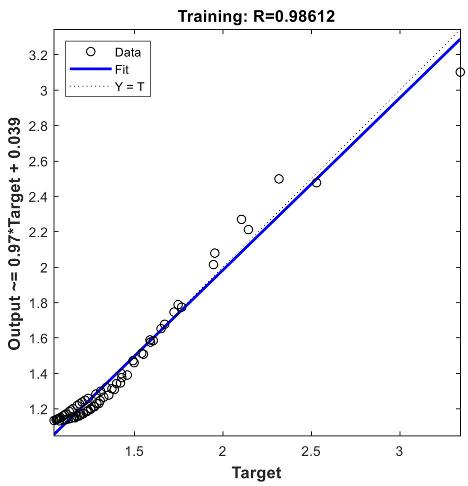
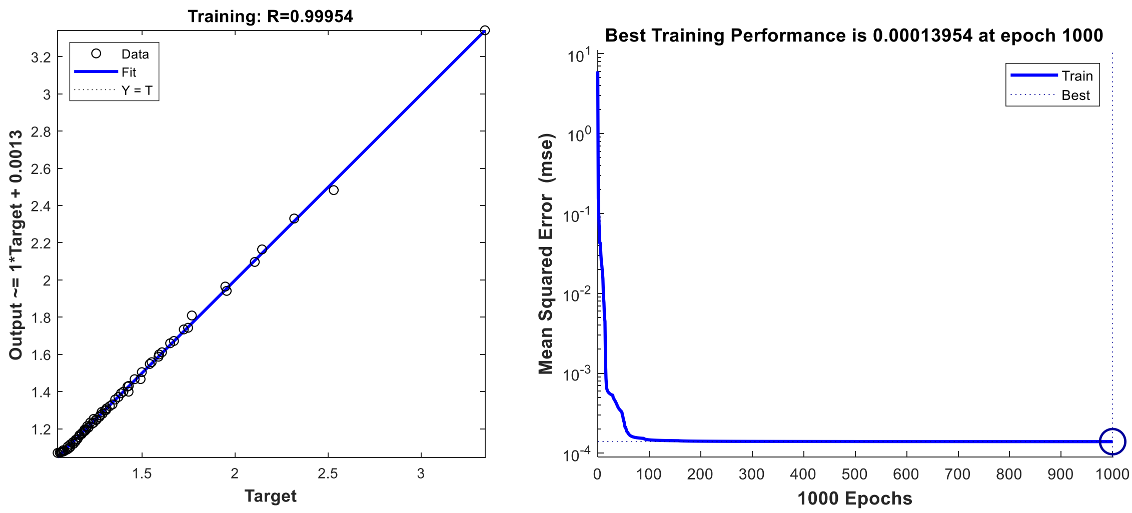


Fig. 5. *ANN structure 2-3-1 R training*



a b

Fig. 6. *a - ANN structure 2-4-1 R training; b - ANN structure 2-4-1 Performance in training*

Table 2. *R-training with performance in training and in testing*

ANN structure	R- training	Performance in training	Performance in testing
2-2-1	0.98612	0.0041	0.0037
2-3-1	0.99391	0.0018	0.0029
2-4-1	0.99954	1.3954e-4	2.7565e-4

Three coefficients are calculated to evaluate statistical network performance: linear regression coefficient R, RMSE and MAPE [18] (Table 2).

The root mean square error (RMSE):

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} \| y_i - \hat{y}_i \|^2}{n}}$$

The mean absolute percentage error (MAPE):

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{\hat{y}_i - y_i}{y_i}\right| \times 100\ \%$$

For linear regression coefficient R we used formula:

$$R = \frac{\sum_{i=1}^{n}(y_i - \bar{y})(\hat{y}_i - \bar{\hat{y}})}{\sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2 \cdot (\hat{y}_i - \bar{\hat{y}})^2}}$$

Where n is the number of experiments, y_i is the actual value, $\hat{y}_i$ is the predicted value, $\bar{y}$ and $\bar{\hat{y}}$ are mean target and predicted output values.

The coefficients calculated to evaluate the statistical network performance of the ANN model for a stepped bar in bending (Table 2) show the numerical model accuracy.

Table 2. *Statistical Performance of training ANN model*

Linear regression coefficient	Root Mean Square Error (RMSE)	Mean absolute percentage error (MAPE) %
0.99954	0.0118	0.58

4. Prediction of stress concentration factor

The Levenberg-Marquardt (LM) optimization algorithm has been used all along this study, in order to find out weights and biases.

The proposed ANN model has a single hidden layer with 4 neurons (Fig. 7).

Stress concentration factors K_t from experimental and predicted for training and testing data are given in the Table 3 and respectively in Table 4.

The graphic representation of the comparison between the experimental results and the predicted ones highlights the accuracy of the ANN model (Fig. 8 and Fig. 9).

The weights and biases values for the ANN model proposed are given in Table 5; with weights and biases thus determined and taking into account the formula (8) values of the stress concentration factor K_t for a stepped bar in bending can be obtained for input data in the field in which the training was carried out.

Weights and biases values are given in Table 5.

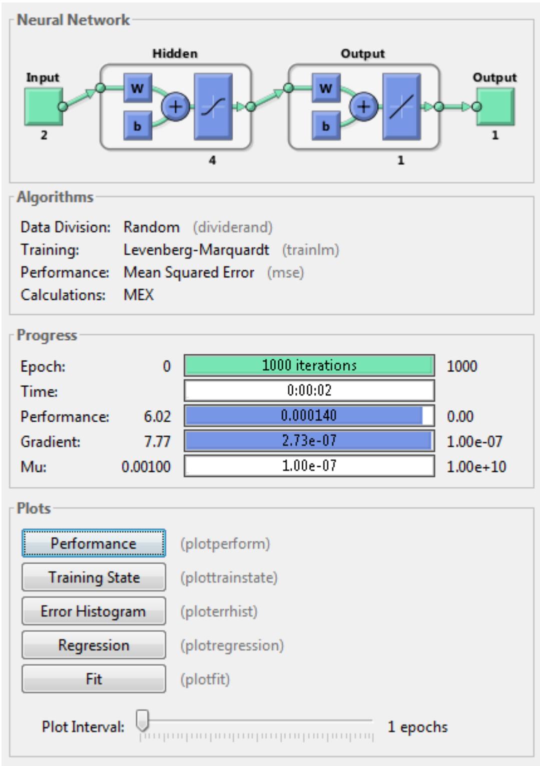


Fig. 7. *ANN model using MATLAB*

Table 3. *Stress concentration factor* K_t *experimental and predicted for training data*

Training data			
r/d	D/d	K_t experimental	K_t predicted
0.02	1.01	1.767288	1.8089
0.04	1.01	1.588172	1.5873
0.06	1.01	1.491934	1.4669
0.08	1.01	1.427209	1.3992
0.12	1.01	1.340725	1.3309
0.14	1.01	1.309238	1.3095
0.16	1.01	1.282561	1.2904
0.2	1.01	1.239188	1.2528
0.22	1.01	1.221113	1.2335
0.24	1.01	1.204841	1.2141
0.26	1.01	1.190065	1.195
0.28	1.01	1.176545	1.1768
0.32	1.01	1.152572	1.1444

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Training data			
r/d	**D/d**	K_t **experimental**	K_t **predicted**
0.34	1.01	1.14185	1.1308
0.36	1.01	1.131832	1.1191
0.4	1.01	1.113596	1.1008
0.42	1.01	1.105251	1.094
0.44	1.01	1.097352	1.0885
0.48	1.01	1.08273	1.0805
0.5	1.01	1.075937	1.0776
0.52	1.01	1.069451	1.0754
0.54	1.01	1.063246	1.0736
0.56	1.01	1.057302	1.0723
0.6	1.01	1.046115	1.0703
0.02	1.2	2.529667	2.4828
0.04	1.2	2.144524	2.1642
0.06	1.2	1.947018	1.9644
0.1	1.2	1.723875	1.7339
0.12	1.2	1.650584	1.6597
0.14	1.2	1.591054	1.5994
0.16	1.2	1.541225	1.5485
0.18	1.2	1.498569	1.5043
0.22	1.2	1.428597	1.4305
0.24	1.2	1.399282	1.3992
0.26	1.2	1.372846	1.3709
0.3	1.2	1.326822	1.3218
0.32	1.2	1.306573	1.3005
0.34	1.2	1.287833	1.2811
0.36	1.2	1.270411	1.2633
0.38	1.2	1.254149	1.2471
0.4	1.2	1.238913	1.2322
0.44	1.2	1.211093	1.2062
0.46	1.2	1.198332	1.1948
0.48	1.2	1.186241	1.1843
0.52	1.2	1.163829	1.1659
0.02	3	3.34211	3.3421
0.06	3	2.317298	2.3302
0.08	3	2.105407	2.0967
0.1	3	1.954488	1.9411
0.14	3	1.747126	1.7428
0.16	3	1.671066	1.672
0.18	3	1.606731	1.6113

Training data			
r/d	**D/d**	K_t **experimental**	K_t **predicted**
0.2	3	1.551282	1.5578
0.24	3	1.459813	1.4664
0.26	3	1.421379	1.4268
0.28	3	1.386698	1.3905
0.3	3	1.355171	1.3572
0.34	3	1.299796	1.2984
0.36	3	1.275266	1.2726
0.38	3	1.252489	1.2488
0.4	3	1.231256	1.2269
0.42	3	1.211394	1.2068
0.44	3	1.192754	1.1882
0.46	3	1.175211	1.1712
0.48	3	1.158657	1.1555
0.5	3	1.142998	1.1411
0.52	3	1.128152	1.1278
0.54	3	1.114049	1.1156
0.56	3	1.100625	1.1044
0.6	3	1.075603	1.0846

Table 4. *Stress concentration factor* K_t *experimental and predicted for testing data*

Testing data			
r/d	**D/d**	K_t **experimental**	K_t **predicted**
0.1	1.01	1.378945	1.3584
0.18	1.01	1.259481	1.2717
0.3	1.01	1.164097	1.1598
0.38	1.01	1.122437	1.1091
0.46	1.01	1.089857	1.084
0.58	1.01	1.051597	1.0712
0.08	1.2	1.818019	1.8304
0.2	1.2	1.461414	1.4654
0.28	1.2	1.348815	1.3452
0.42	1.2	1.224593	1.2186
0.5	1.2	1.174757	1.1747
0.04	3	2.65264	2.7057
0.12	3	1.839245	1.8293
0.22	3	1.502773	1.5099
0.32	3	1.326329	1.3266
0.58	3	1.087826	1.0941

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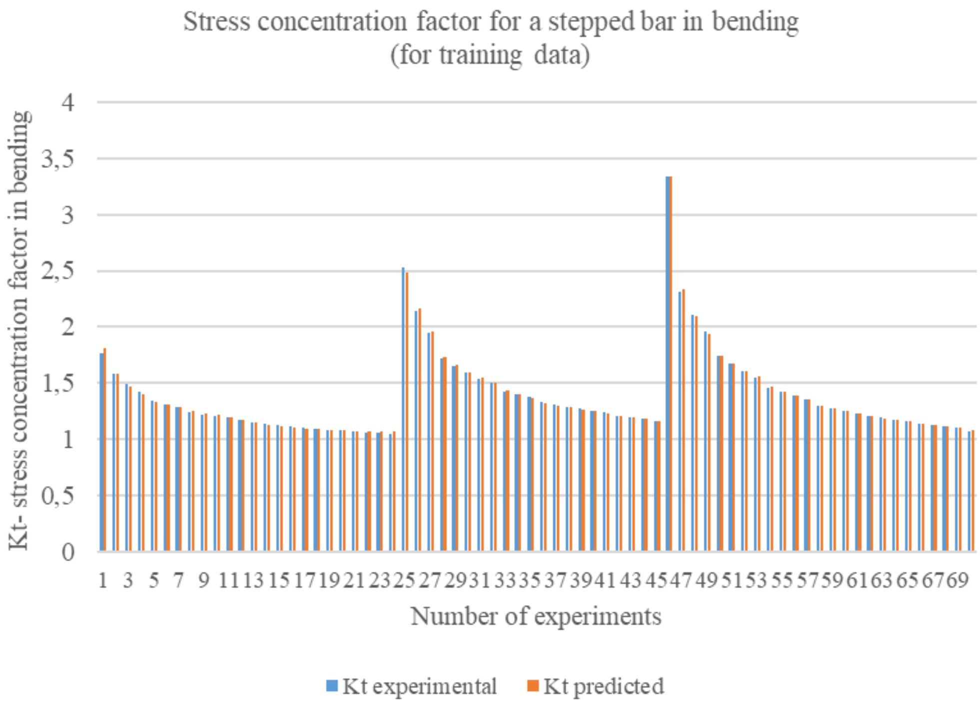

Fig. 8. *Stress concentration factor* K_t *for a stepped flat bar in bending (for training data)*

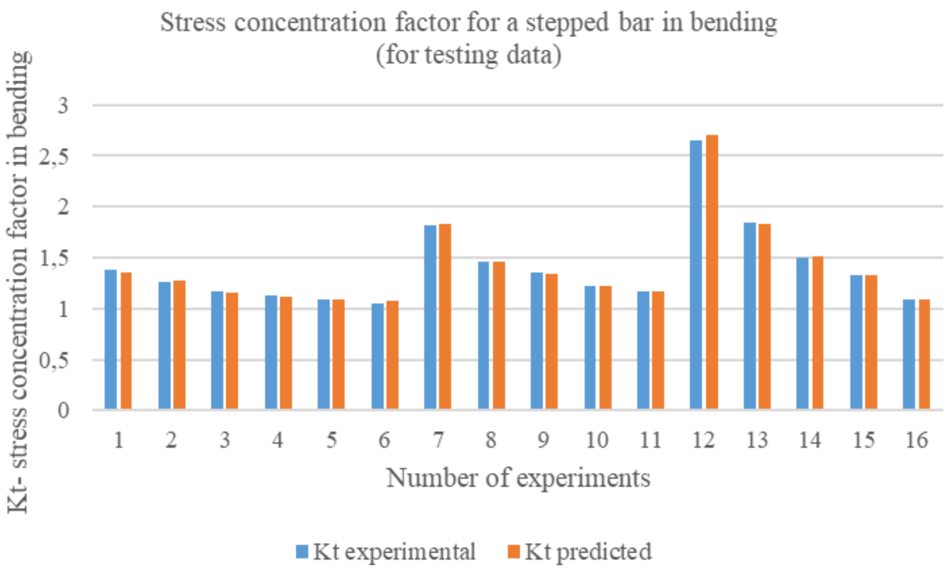

Fig. 9. *Stress concentration factor* K_t *for a stepped flat bar in bending (for testing data)*

Table 5. *The weights and bias values for the proposed ANN model*

the input-to-hidden layer weights	
W1	
0.6064	7.5884
0.6169	-27.0678
-1.8291	-9.137
4.6657	-0.231

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the hidden-to-output layer weights			
W2			
-59.6062	-59.5662	0.1319	-38.2879

the input-to-hidden layer bias
B1
-4.6275
-18.7273
-9.6117
7.0611

the hidden-to-output layer bias
B2
37.3983

5. Conclusions

This study contains the stress concentration factor for the stepped bar in bending position, obtained using formulas from [10] and an ANN approach.

The formula for the stress concentration factor was defined only with the result of experimental studies. It is easier and more practical to determine these values using auxiliary software instead of using formulas.

A new ANN model was developed using Matlab software. Different ANN models were tried and the best model was obtained.

The ANN model provided high accuracy for the prediction of stress concentration factor (K_t) - see Table 2 with Statistical Performance of training ANN model.

Users may misread the values from the charts. By using the ANN model these faults were eliminated.

Using weights and biases of the ANN model, for r/d = 0.045 and D/d = 2 one obtained the stress concentration factor predicted K_t = 3.2801 (see Table 6).

***Table 6.** Test for a pair of input variables that are not found in the training or testing data*

r/d	D/d	K_t predicted
0.045	2	3.2801

References

[1]. Boger Z., Guterman H., *Knowledge extraction from artificial neural network models*, IEEE Systems, Man, and Cybernetics Conference, Orlando, FL, USA, 1997.

[2]. Budynas-Nisbett, *Shigley's Mechanical Engineering Design*, 8th Ed.

[3]. Giannini F., Laveglia V., Rossi A., Zanca D., Zugarini A., *Neural Networks for Beginners A fast implementation in Matlab*, Torch, TensorFlow, March 17, 2017.

[4]. Simon Haykin, *Neural Networks and Learning Machine*, Third Edition, Pearson Prentice Hall, 2009.

[5]. Saurabh Karsoliya, *Approximating Number of Hidden layer neurons in Multiple Hidden Layer BPNN Architecture*, International Journal of Engineering Trends and Technology – vol. 3, issue 6, 2012.

[6]. Abhishek K., Kishore D. J., Prakashkumar Kavalur, Akshay N H., *ANN modeling for prediction of cutting force component during orthogonal turning*, International Research Journal of Engineering and Technology (IRJET), vol. 5, issue 6, June, 2018.

[7]. Souâd Makhfi, Raphaël Velasco, Malek Habak, Kamel Haddouche, Pascal Vantomme, *An Optimized ANN Approach for Cutting Forces Prediction in AISI 52100 Bearing Steel Hard Turning*, p-ISSN: 2163-2669, e-ISSN: 2163-2677, 3(1), p. 24-32, doi: 10.5923/j.scit.20130301.03, 2013.

[8]. Krzysztof L. Molski, *Stress concentration at load-carrying fillet welded cruciform joints subjected to tensile and bending loads*, Acta mechanica et automatica, vol. 13, no. 4, DOI 10.2478/ama-2019-0033, 2019.

[9]. Neuber H., *Theory of Stress Concentration for Shear-Strained Prismatical Bodies with Arbitrary Nonlinear Stress-Strain Law*, ASME Journal of Applied Mechanics, 28, 1961.

[10]. Robert Norton, *Machine Design – An integrated Approach*, 6th Edition, Pearson Publisher, 2014.

[11]. Foram S. Panchal, Mahesh Panchal, *Review on Methods of Selecting Number of Hidden Nodes in Artificial Neural Network*, International Journal of Computer Science and Mobile Computing, vol. 3, issue 11, p. 455-464, November 2014.

[12]. Peterson R. E., *Stress concentration factors*, John Willey and Sons, 1975.

[13]. George Sines, Waisman J. L., *Metal Fatigue*, McGraw-Hill Book Company, 1959.

[14]. Alan J. Thomas, Miltos Petridis, Simon D. Walters, Saeed Malekshahi Gheytassi, Robert E. Morgan, *On Predicting the Optimal Number of Hidden Nodes*, International Conference on Computational Science and Computational Intelligence, 2015.

[15]. Alan J. Thomas, Simon D. Walters, Saeed Malekshahi Gheytassi, Robert E. Morgan, Miltos Petridis, *On the Optimal Node Ratio between Hidden Layers: A Probabilistic Study*, International Journal of Machine Learning and Computing, vol. 6, no. 5, October 2016.

[16]. Ihsan Toktas, Murat Tolga Ozkan, Fulya Erdemir, Nurullah Yuksel, *Determination of Stress Concentration Factor (Kt) for a Crankshaft under Bending Loading: An Artificial Neural Networks Approach*, Politeknik Dergisi, Journal of Polytechnic, ISSN: 1302-0900 (PRINT), ISSN: 2147-9429 (ONLINE).

[17]. Pilkey Walter D., Pilkey Deborah F., *Peterson's Stress Concentration Factors*, 3rd Edition.

[18]. ***, Matlab, The MathWorks, Inc., Natick, Massachusetts, United States, 2018a.
[19]. ***, https://materion.com/-/media/files/alloy/newsletters/technical-tidbits/issue-no-60---effect-of-stress-concentration-on-fatigue-life.pdf.
[20]. ***, https://users.wpi.edu/~cfurlong/me3320/lect14/Lect14.pdf.
[21]. ***, https://research.iaun.ac.ir/pd/jjfesharaki/pdfs/UploadFile_9038.pdf.

ANNEX A

Training data		
r/d	**D/d**	**K_t experimental**
1	**2**	**3**
0.02	1.01	1.767288
0.04	1.01	1.588172
0.06	1.01	1.491934
0.08	1.01	1.427209
0.12	1.01	1.340725
0.14	1.01	1.309238
0.16	1.01	1.282561
0.2	1.01	1.239188
0.22	1.01	1.221113
0.24	1.01	1.204841
0.26	1.01	1.190065
0.28	1.01	1.176545
0.32	1.01	1.152572
0.34	1.01	1.14185
0.36	1.01	1.131832
0.4	1.01	1.113596
0.42	1.01	1.105251
0.44	1.01	1.097352
0.48	1.01	1.08273
0.5	1.01	1.075937
0.52	1.01	1.069451
0.54	1.01	1.063246
0.56	1.01	1.057302
0.6	1.01	1.046115
0.02	1.2	2.529667
0.04	1.2	2.144524
0.06	1.2	1.947018
0.1	1.2	1.723875
0.12	1.2	1.650584
0.14	1.2	1.591054
0.16	1.2	1.541225
0.18	1.2	1.498569
0.22	1.2	1.428597
0.24	1.2	1.399282
0.26	1.2	1.372846
0.3	1.2	1.326822
0.32	1.2	1.306573
0.34	1.2	1.287833
0.36	1.2	1.270411
0.38	1.2	1.254149
0.4	1.2	1.238913
0.44	1.2	1.211093
0.46	1.2	1.198332
0.48	1.2	1.186241
0.52	1.2	1.163829
0.02	3	3.34211
0.06	3	2.317298
0.08	3	2.105407
0.1	3	1.954488
0.14	3	1.747126
0.16	3	1.671066
0.18	3	1.606731
0.2	3	1.551282
0.24	3	1.459813
0.26	3	1.421379
0.28	3	1.386698
0.3	3	1.355171
0.34	3	1.299796
0.36	3	1.275266
0.38	3	1.252489
0.4	3	1.231256
0.42	3	1.211394
0.44	3	1.192754
0.46	3	1.175211
0.48	3	1.158657
0.5	3	1.142998
0.52	3	1.128152
0.54	3	1.114049
0.56	3	1.100625
0.6	3	1.075603

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Testing data		
r/d	**D/d**	**K_t experimental**
1	**2**	**3**
0.1	1.01	1.378945
0.18	1.01	1.259481
0.3	1.01	1.164097
0.38	1.01	1.122437
0.46	1.01	1.089857
0.58	1.01	1.051597
0.08	1.2	1.818019
0.2	1.2	1.461414
0.28	1.2	1.348815
0.42	1.2	1.224593
0.5	1.2	1.174757
0.04	3	2.65264
0.12	3	1.839245
0.22	3	1.502773
0.32	3	1.326329
0.58	3	1.087826