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MATHEMATICAL MODELING OF THERMAL PROCESSING FOR AN AL-SI ALLOY

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ABSTRACT

This paper presents the realization of a mathematical model of a heat treatment process of an Al-Si alloy, with the help of which the values of some mechanical properties are preached. The method of elaborating the mathematical model is a statistical method, namely the method of the active experiment.

With the help of the MATLAB software and based on the elaborated mathematical model, we created a graphic interface that allows the prediction of the values of three mechanical properties depending on the thermal processing conditions of the studied alloy.

KEYWORDS: aluminum alloy, thermal treatment, mathematical modeling, prediction of property values, graphic interface

1. Introduction

Non-ferrous alloys are a category of metal materials with special applications in the machine building industry and beyond, and aluminum and its alloys occupy an extremely important position in this category due to the wide range of potential applications [1, 2].

Aluminum alloys are used in the aerospace, chemical, electrical and electronic industries, consumer goods, automation and computer industry [3].

Although aluminum is the most abundant metal in the earth's crust, it has only recently become widely used in industry, as it is difficult to develop. The wide scope of aluminum and its alloys in the construction of landmarks by plastic deformation, casting or cutting is due to its excellent physical, chemical and mechanical properties, low density, excellent corrosion resistance but also excellent electrical and thermal conductivity [4].

The mechanical properties of these alloys can reach high values by applying appropriate heat treatments. Alloys change the properties of aluminum in a very wide range. Even small amounts of alloying elements have a great effect. Among the most important alloying elements we mention: silicon, copper, magnesium, zinc, nickel, titanium, etc. These alloying elements may dissolve in solid solution (although in small quantities) or may form soluble or insoluble compounds. The soluble material is particularly important because its presence in the structure indicates the possibility of applying a final heat treatment to increase hardness, mechanical strength, corrosion resistance, etc. [5, 6].

Today at European and global level it is possible to produce cheap and high quality in regulated environmental conditions, which means to consume as little energy and fuel as possible, ensure optimal working and living conditions for employees and ensure safety. We must not forget that it is necessary to offer safe products.

Mathematical modeling of processes is a useful basic tool both in the design phase and in the analysis of the operation of installations. By using specialized programs, we can determine an optimal metallurgical process of combining process modeling with computer use. The development of the specific mathematical apparatus and, in particular, of statistical methods has allowed the optimal decision issue to be addressed as a high technical and economic efficiency issue [7].

The mathematical models that can be used are an important source of information necessary for the optimal management of metallurgical processes.

The preliminary experiment clarifies the variation of process factors by performing a series of determinations based on programs (dispersion analysis, correlation analysis, etc.) which allows the selection of factors that significantly influence and highlights the links between factors as well as their contribution to the process [8].

2. Experimental conditions

In the paper, the mathematical model of the heat treatment process applied to an aluminum alloy was performed, by statistical methods, namely the regression analysis by active experiment [8].

Experimental data were obtained from the artificial aging heat treatment applied to an aluminum alloy whose composition is shown in Table 1.

Table 1. *Materials for experimentation*

Al 7Si 0.3Mg	Si	Mg	Al
%	7	0.3	rest

The modeling methods by programming the experiment are different because the metallurgical processes are varied and complex. The most effective methods of programming the experiment is currently those of solving extreme problems, which involve determining the levels of independent sizes (input), u_2, ...u_k, for which the objective function [9]:

$$y = f(u_1, u_2, ..., u_k) \qquad (1)$$

has extreme values (maximum or minimum), as well as the calculation of these values.

The method of the experiment programmed as opposed to the classical method of experimentation, provides for the realization of a number of four experiences near an arbitrarily chosen S1 point, aiming at determining the response surface, on a small area around that point, finding a linear equation of the form [10]:

$$y = b_0 + b_1x_1 + b_2x_2 + b_{12}x_1x_2 \qquad (2)$$

which is a first approximation of the real equation of the process. To find out the extreme value of the Y parameter are necessary in case of its dependence on two factors, of $2^2 = 4$ experiences, so in total 4 experiences.

Experiment programming involves:

- establishing the necessary and sufficient number of experiences and the conditions for their realization;
- determination by statistical methods of the regression equation, which represents with a certain degree of approximation, calculable, the process model;
- determining the conditions for achieving the optimal value of the process performance (the parameter to be optimized).

The use of the active experiment, as a method of mathematical modeling, uses statistical methods in all stages of experimental research:

- before the experiment, by establishing the number of experiences and the conditions for their realization;
- during the experiences by processing the obtained results;
- after concluding the experiment with conclusions regarding the realization of future experiences [9].

We considered as main influencing factors (independent variables) the following technological parameters of heat treatment:

1 - artificial aging temperature – t, [°C];

2 - maintenance time at artificial aging temperature - τ, [hours].

The set of physico-mechanical properties is considered as parameters: Rm, $Rp_{0,2}$, A_5.

To establish the basic level and range of variation of influencing factors we used data from the literature on the nature of solid transformations and behavior in heating and cooling for treatment thermal alloys with aluminum base.

Given the studies and experimental results obtained by some authors on alloys with high-strength aluminum base, we established the conditions of experimentation as follows:

• for aging temperature:
  - basic level: $u_{01} = 204$ °C;
  - variation range: $\Delta u_1 = 56$;

• for aging:
  - basic level: $u_{02} = 15$ hours;
  - variation range: $\Delta u_2 = 10$ hours.

The following notations and symbols were used for the coded representation of the experiment:

Independent variables:

• x_1 - temperature;

• x_2 - time.

Dependent variables (optimization parameters):

Y1 - breaking strength, Rm, [MPa];

Y2 - flow limit, Rp_{02}, [MPa];

Y3 - elongation at break, A_5, [%].

There are the following differences between natural and coded values of xi factors [18]:

$$x_1 = \frac{t - t_0}{\Delta_t}; \qquad x_2 = \frac{\tau - \tau_0}{\Delta_\tau} \qquad (3)$$

Yi values are expressed in natural units.

As the influence of the two factors on the performance of the process (Y) is studied, the 4 experiments according to the experimental matrix in Table 2 were performed.

Next, based on the matrix of the complete factorial experiment, the coefficients of the regression equation (the mathematical model) are calculated. Considering the Yi function as the analytical expression of the first order model, it is in the form [9]:

$$Y_i = c_0 + \sum_{i=1}^{2} c_i \cdot x_i + \sum_{\substack{i=1 \\ j=1 \\ i \neq j}}^{2} c_{ij}\, x_i x_j \quad (4)$$

***Table 2.** Experimental matrix*

Nr. exp.	x_0	x_1	x_2	x_{12}	y_1	y_2	y_3
1	1	1	1	1	140	83	23
2	1	-1	1	-1	323	245	9.3
3	1	1	-1	-1	175	100	8
4	1	-1	-1	1	300	215	10

Following the ephaguation of the specific calculations, the values of the coefficients for the 3 equations that are presented in Table 3 resulted.

***Table 3.** The values of the coefficients of the first order models*

b_i \ y_i	y_1	y_2	y_3
b_0	234.5	160.75	12.5
b_1	-77	-69.25	2.9
b_2	-3	3.25	3.5
b_{12}	-14.5	-11.75	3.9

Therefore, the equation of the mathematical model of order I (4), expressed in coded quantities, for each property, has the form:

$$y_1 = 234.5 - 77x_1 - 3x_2 - 14.5x_1x_2 \quad (5)$$

$$y_2 = 160.75 - 69.25x_1 + 3.25x_2 - 11.75x_1x_2 \quad (6)$$

$$y_3 = 12.5 + 2.9x_1 + 3.5x_2 + 3.9x_1x_2 \quad (7)$$

By replacing the x_i variables with the (3) relationships in the equations (5-7) (the final form of the mathematical model is obtained, expressed in natural quantities (treatment temperature and heat treatment time) for the three properties studied:

$$y_1 = 439.94 - 0.985 \cdot T + 5.004 \cdot \tau - 0.026 \cdot T \cdot \tau \quad (8)$$

$$y_2 = 350.42 - 0.93 \cdot T + 4.4 \cdot \tau - 0.02 \cdot T \cdot \tau \quad (9)$$

$$y_3 = 20.14 - 0.04 \cdot T - 1.18 \cdot \tau + 0.006 \cdot T \cdot \tau \quad (10)$$

With the help of the MATLAB program package and using the results of experimental research we have created a graphical interface that allows the simulation of the values of some mechanical properties for the alloy subjected to the thermal treatment of artificial aging.

For this simulation using MATLAB, the equations of the mathematical model were also used, the equations (8-10).

Through these simulations you can find the value of each property and the conditions (the value of the heat treatment parameters) thermal processing that lead to obtaining those values for each property.

Figure 1 shows under what conditions of temperature and time the value of 236 MPa is obtained for Rm.

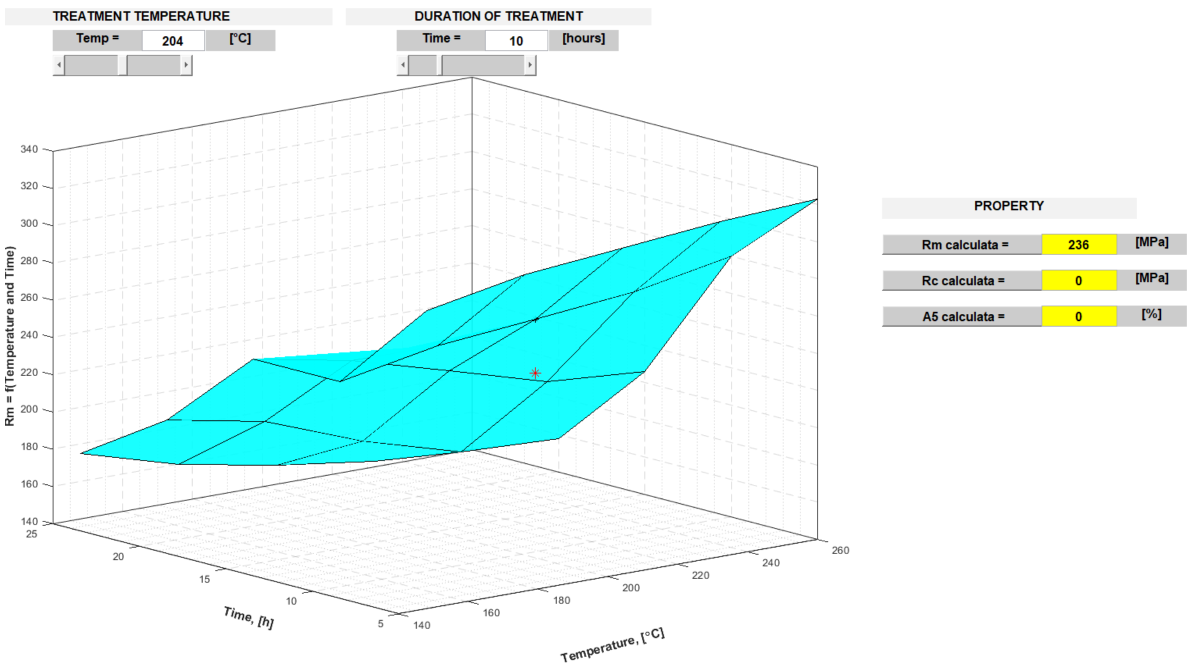

***Fig. 1.** Simulation of Rm [MPa] when the artificial aging temperature is 204 °C and the thermal processing time is 10 hours*

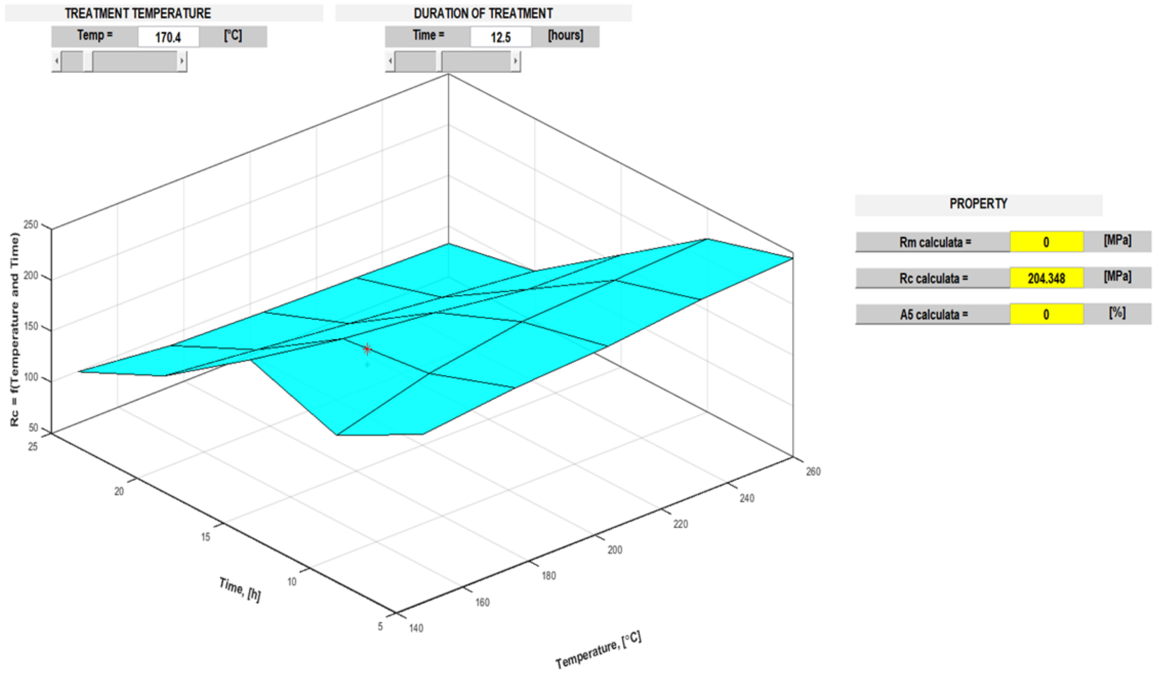

Fig. 2. *$Rp_{0.2}$ finding simulation [MPa] when the temperature is 170.4 °C and the time is 12.5 hours*

Figure 2 shows the simulation to obtain the value of 204.3 MPa for the flow limit as well as under what treatment conditions this value is obtained.

Figure 3 illustrates under what conditions of temperature and artificial aging time is obtained for elongation at break of 10.96 %.

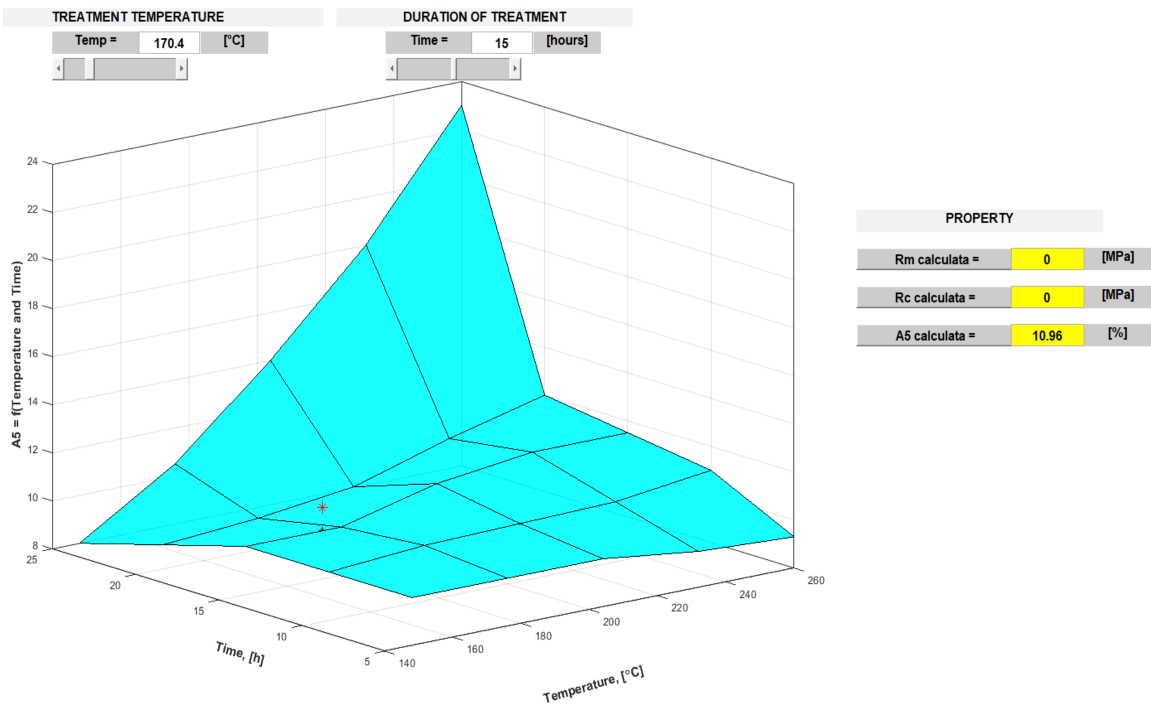

Fig. 3. *Simulation of A5 [%] when the artificial aging temperature is 170.4 °C and the artificial aging time is 15 hours*

3. Conclusions

Following the elaboration of the mathematical model and the realization of the graphical interface with the help of the MATLAB in this paper, the following conclusions can be drawn:

• mathematical modeling of processes is a basic tool useful both in the design phase and in the analysis of the operation of installations.

• the values calculated for the optimized parameter Yi (i = 1... 3) are very close to the measured values, therefore the mathematical model performed allows the simulation of the heat treatment process, by varying the values of the technological parameters, within the experienced limits;

• the elaborated mathematical model allows the calculation simulation to find out the values of the studied mechanical properties;

• the graphical interface made with the help of MATLAB is based on the mathematical model made, for the prediction of the property values according to the two technological parameters of heat treatment, temperature and artificial aging time;

• the mathematical model presented, allows the calculation of optimizing the parameters of the heat treatment process, so as to obtain the optimal complex of the resistance and plasticity properties, with minimal costs.

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