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FAST SOLUTION TO FIND QUASI-STATIC PARAMETERS IN ELASTIC CONTACT USING LINEAR DISPLACEMENT AND VARIABLE SFIFFNESS

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ABSTRACT

Hertz theory expresses the dependence between load and contact approach, using a nonlinear function. Usually, an exponent p=1.5 for point contact or p=1.11 for line contact is chosen. If we look at measure units, then a correlation results between load and an exponential function of displacement. $Q = C \cdot ¢^{p}$ *, where Q is the load, C the contact stiffness, δ is the displacement and p an exponent. That observation shows that the contact stiffness has to have Newton/meter^p as measure unit.*

This paper presents a linear dependence between load and displacement, applied either to point contact type or line contact, equivalent with p exponent equal to 1. Results calculated with the new relationship are successfully compared to results calculated with other published relationships and to some available experimental results, presented in literature.

Keywords: *Hertz contact, contact area, contact pressure, exponent p=1, variable rigidity, minimum iterations.*

INTRODUCTION

Assuming a profiled roller element, loaded with a small external load. In this case, the initial contact will be a point contact type and the dependence between load and diplacement is

Q= *contact_stifness_point* *depl^1.5 (1)

When the external load increases, then the contact tends to be a theoretical line contact and, in this case,

Q= *contact_stifnes_line* *depl^1.11. (2)

To determine how the bearing load is distributed along rollers, it is first necessary to develop load-deflection relationships for rolling elements with contacting raceways using an exponent p=1. In this case, the load-displacement relation is

Q(Newton)= *contact_stifness(N/m)* *depl(m)*number(ellipticity). (3)

MATHEMATICAL FORMULATION

Assuming *p=1* and using a slicing technique, a fast approach of the problem is developed to find both the shape and the size of contact domain, as well as the pressure distribution on it. The contact load causes a displacement (ξ) for the mass centre of a particular rolling element. Figures 1 and 2 show the elements used in analysis.

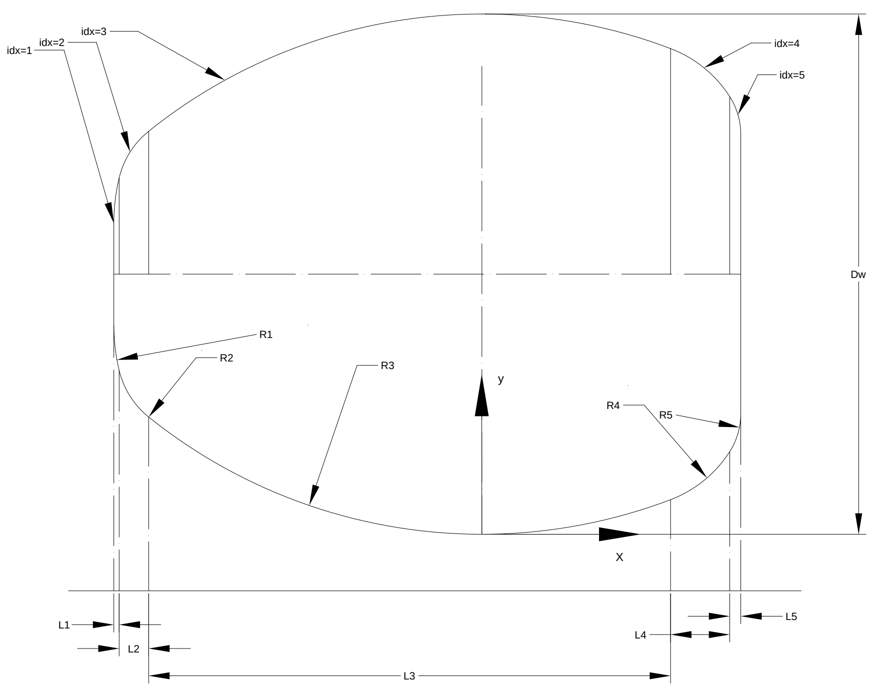

Fig. 1. Elements to describe the rolling elements geometry

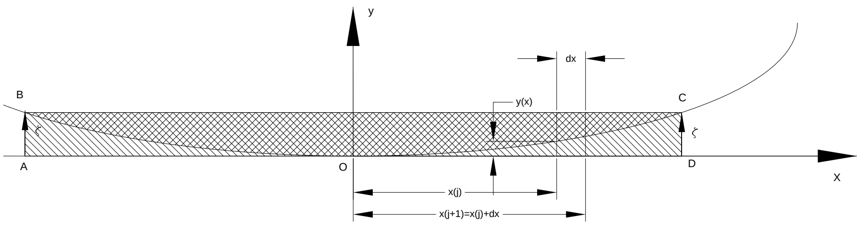

Fig. 2. Elements to describe the contact interference

If a rolling element has *idx=1...N* (usually *idx=1...5*), particular radius (see Fig. 1) according to [1] is possible to express the local load in a slice, according to equation 4:

$$Q_{idx,j} = E0 \cdot k_{idx}^{-0.11} fQ\left(k_{idx}\right) dx_{idx} \cdot \delta_{idx,j} \tag{4}$$

where $E0$ is the equivalent modulus of elasticity for the materials in contact, k_{idx} is the conformity in the *idx* region, dx_{idx} is the slice width, $\delta_{idx,j}$ is the relative approach in contact, corresponding to the *idx* region and for the *j* slice and

$$fQ(k_{idx}) = \frac{0.94896 - 0.09445 \cdot ln(k_{idx})}{1 + 0.45412 \cdot ln(k_{idx})} \tag{5}$$

The external load Q is given as sum of the individual loads in slices:

$$Q = \sum_{idx} \sum_{j} Q_{idx} = \sum_{idx}\left[E0 \cdot k_{idx}^{-0.11}\, fQ\left(k_{idx}\right) \cdot \sum_{j} ¢_{idx} \cdot dx_{idx} \right] \tag{6}$$

Analysing Eq (6) and Figure 2, it results:

$$Q = \sum_{idx} \sum_{j} Q_{idx} = \sum_{idx}\left[CT_{idx} \cdot Area_{idx} \right] \tag{7}$$

where

$$CT_{idx} = E0 \cdot k_{idx}^{-0.11} \cdot fQ\left(k_{idx}\right) \tag{8}$$

$$¢_{idx,j} = s - y\left(x\right) = -\frac{x_{idx,j}^{2}}{2R_{idx}} + s \tag{9}$$

where s is the relative approach in contact, $y(x)$ is used to describe the roller geometry according to Fig. 2 and R_{idx} is the local roller radius corresponding to the *idx* region.

From Figure 2, it results:

$Area_{idx}=Area(ABCD) -Area(ABOCD)$ 10)

According to Eq. 10 and Eq. 9, Eq. 11 may be written:

$$Area_{idx} = \sum_{j} ¢_{idx,j} \cdot dx_{idx} = dx_{idx} \cdot \sum_{j} ¢_{idx,j} = -\sum_{j}\left[\frac{X_{idx,j}^{2}}{2R_{idx}} \cdot dx_{idx} \right] + s \sum_{j} dx_{idx} \tag{11}$$

Because $\sum_{j} dx_{idx} = L_{idx}$ and according to Eq. 7, if we note

$$AA_{idx} = \sum_{j}\left[\frac{X_{idx,j}^{2}}{2R} dx_{idx} \right], \tag{12}$$

it results:

$$Q = \sum_{idx}\left[CT_{idx} \cdot \left(-AA_{idx} + s \cdot L_{idx} \right)\right] = -\sum_{idx} CT_{idx} \cdot AA_{idx} + s \sum_{idx} CT_{idx} \cdot L_{idx} \tag{13}$$

From Eq. 13, we can express the dependence between the external load and the roller displacement according to Eq. 14:

$$s = \frac{Q + \sum_{idx} CT_{idx} \cdot AA_{idx}}{\sum_{idx} CT_{idx} \cdot L_{idx}} \tag{14}$$

For a region *idx*, if we express the roller element geometry as $x_j = x_0 + f \cdot dx_{idx}$ and $dx_{idx}=L_{idx}/N$, where N is the number of slices in the *idx* region, it results

$$AA_{idx} = \sum_{j}\left[\frac{X_{idx,j}^{2}}{2R} dx_{idx} \right] = T\left(L, x0, y0, R\right)_{idx} \tag{15}$$

where

$$T\left(L, x0, y0, R\right) = y0 \cdot L + \frac{x0^{2} \cdot L + x0 \cdot L^{2} + 0.88888 \cdot L^{3}}{2R} \tag{16}$$

The structure of Eq. 16 is due to the simple algebra summation of integer numbers, following the algorithm:

$$\sum_{j}\left[\frac{X_{idx,j}^{2}}{2R} dx_{idx} \right] = \sum_{j=1}^{n} \frac{\left(x0 + j\frac{L}{n} \right)^{2}}{2R} \frac{L}{n} \tag{17}$$

because

$$\sum_{j=0}^{n} j = \frac{n(n+1)}{2} \tag{18}$$

and

$$\sum_{j=0}^{n} j^2 = \frac{n(n+1)(2n+1)}{6} \tag{19}$$

Due to Eqs.16, 18 and 19, when *n* became large, a simply expression results, as Eq. 17.

Usually, a large value for *n* is chooses but, at limits, Eq.16 results. Using Eq.16 and the roller element geometry, for an imposed value of $y0_{idx}$ coordinate, for a particular *idx* region, we can appreciate the load as follow:

$$Q_{idx} = y0_{idx} \cdot \left(\sum_{0}^{idx} CT_{idx} \cdot L_{idx} \right) - \sum_{0}^{idx} CT_{idx} \cdot AA_{idx} \tag{20}$$

Eq. 20 shows that retrieved load does not depend on the number of slices and is a function only of the number of the major regions.

When the number of regions is idx=5 and assuming in every section 100 slices, for example, results that are necessary minimum 500 calculations multiply with number of the iterations. As consequence of Eq. 20, the complexity of algorithm decreases and maximum 5 simply computing expressions are necessary.

If you search a solution which does not correspond to the limited idx regions, then you have to evaluate if $Q_{ext}>(Q_{idx-1})$ and $Q_{ext}<(Q_{idx})$. To solve that problem in only maximum 10 steps, we can include the following algorithm.

Assuming $x=0.1$, then we have to evaluate the minimum value of the Eq. 21.

$$\mathrm{s} = F(idx) = \frac{Q + \sum_{S=0}^{idx-1} CT_S \cdot T_S + CT_{idx} \cdot T(x \cdot L_{idx})}{\sum_{S=0}^{idx-1} CT_S \cdot L_S + CT_{idx} \cdot x \cdot L_{idx}} \tag{21}$$

EXEMPLIFICATION FOR IDX=4 [2]

A rolling element geometry with $R1=2500$ mm, $R2=54$ mm, $R3=764.2$ mm, $R4=0.7$ mm, $x1=5.2914$ mm, $x3=7.3$ mm, $dm=58$ mm and $dw=15$ mm, similarly with [2], and an external load $Q=20$ kN are considered.

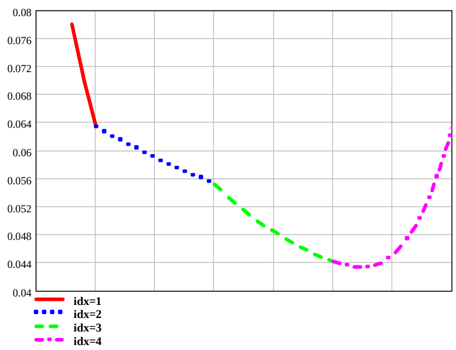

Fig.4a. Minimum value of Eq. 21

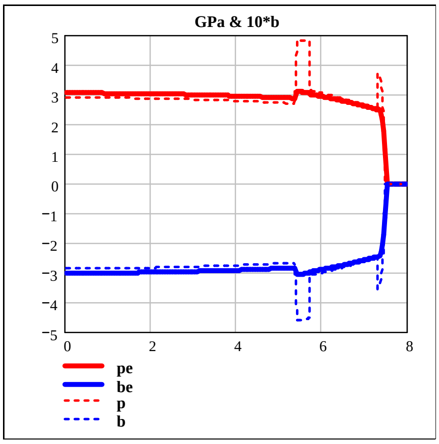

Fig. 4b. Shape of contact pressure and minor semi axes

Applying eq. 21, it results the roller–raceway displacement. With that solution, the quasi-static parameters for the roller-raceway contact are presented in Figures 4a and 4b.

Figure 4a shows that the load covers a part of the region *idx=4*, because a minimum exists in this region. Appling the solution of eq 21, (displacement ***z****=0.04355*), it results the pressure shape of the load and the contact form, according to Figure 4b. Figure 4b presents the following information: *pe* is the contact pressure in a slice when the contact rigidity is a mean of the partial contact rigidity in all *idx* regions; *be* represents the minor semi-axis when the contact rigidity is a mean of the partial contact rigidity in all *idx* regions, *p* represents the contact pressure in a slice, *b* represents the minor semi axis in a slice. In a similar manner, for different external loads as in [2], results are presented in Figures 5a and 5b. These results shows a similarity with [2], when *idx=4*.

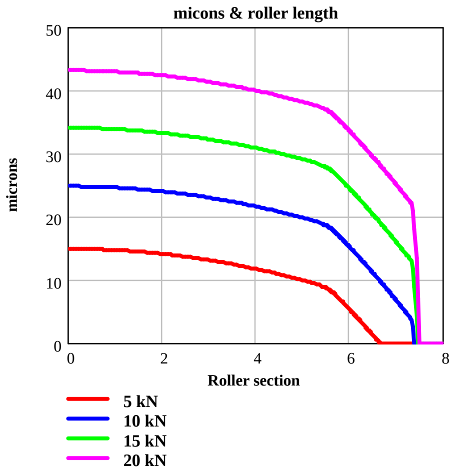

Fig. 5a. Relative displacement for different external loads, along the roller length

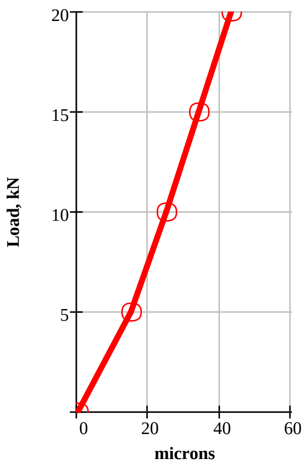

Fig. 5b. Relative displacement in centre of the rolling element

EXEMPLIFICATIONS FOR IDX=2 [3]

A rolling element geometry with *R1=1114 mm*, R2=1.006 mm, x1=6.994 mm, according to [3], and an external load of *Q=33800 N* is considered. Appling eq. 21, the results are presented in Figure 6a and 6b.

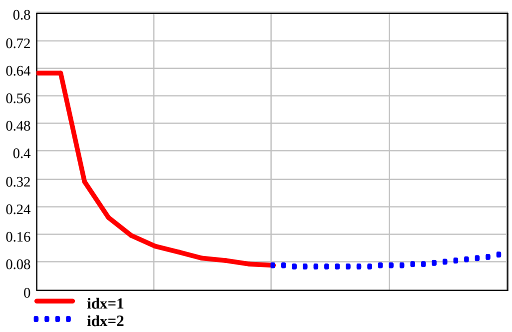

Fig. 6a. Minimum value of Eq. 21

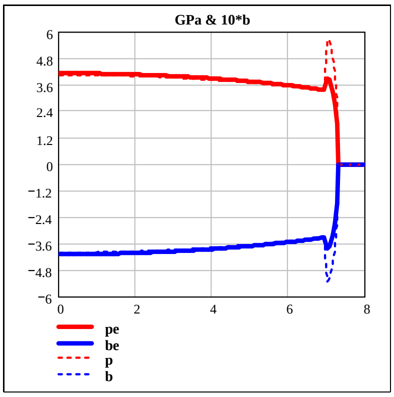

Fig. 6b. Shape of c pressure and minor semi axes

Figure 6a shows that the load covers a part of the region *idx=2*, because a minimum exists in this region. Appling the solution of eq. 21 (displacement=0.065), it results the shape of the load according to Figure 6.b. Similar with Figure 4b, Figure 6.b presents the following information: pe is the contact pressure in a slice when the contact rigidity is a mean of the partial contact rigidity in all idx regions; be represents the minor semi axis when the contact rigidity is a mean of the partial contact rigidity in all idx regions ; p represents the contact pressure in a slice; b represents the minor semi axis in a slice.
These results show a good similarity with [3], when idx=2.

EXEMPLIFICATION FOR IDX=3 [4]

The mathematical model is applied to different roller element geometries, according to [4]. The following geometries were considered: the flat profile, the end tapered profile, the aerospace one and the full crowned profile. A direct application of eq. 21 shows the contact parameters according to Figure 7. These results show a good similarity with [4], when *idx=3*.

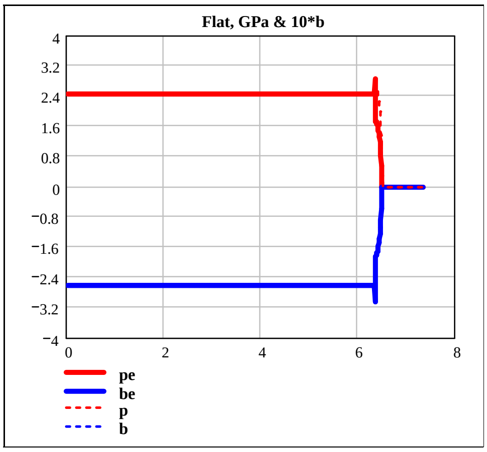

Fig. 7a. Contact parameters for the flat profile

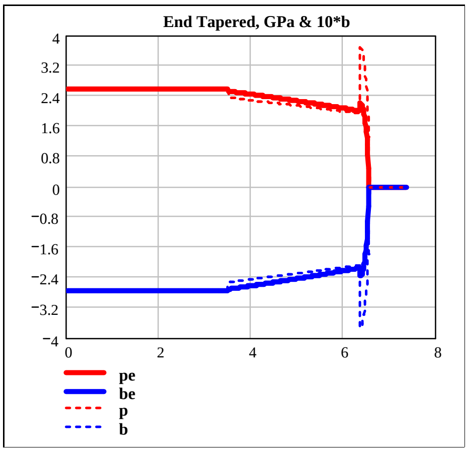

Fig. 7b. Contact parameters for the end tapered profile

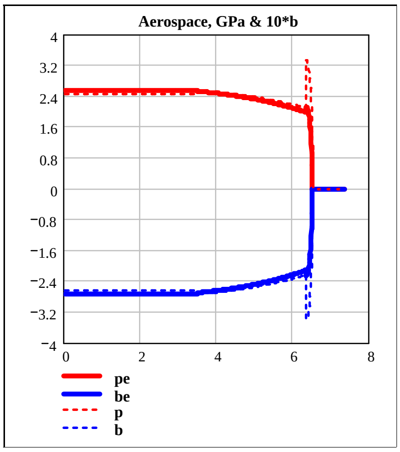

Fig. 7c. Contact parameters for the aerospace profile

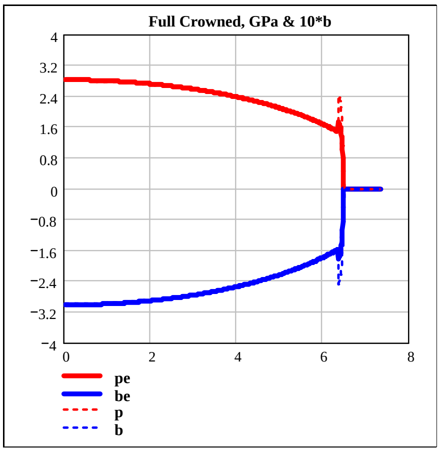

Fig. 7d. Contact parameters for the full crowned profile

In Figure 7, the symbols have the meaning: *pe* is the contact pressure in a slice when the contact rigidity is a mean of the partial contact rigidity in all *idx* regions; *be* represents the minor semi-axis when the contact rigidity is a mean of the partial contact rigidity in all *idx* regions; *p* represents the contact pressure in a slice; *b* represents the minor semi-axis in a slice.

CONCLUSIONS

The mathematic model and Eq.21 minimize the computing time for contact parameters. A single vector with maximum 10 elements replaces large matrices used to evaluate the contact parameters and reduce number of iterations. The method gives approximate solutions with different contact analysis type as half space model and FEM.

The linear dependence between load and displacement verifies the measure units and represents the direct solution of the complex contact problem mathematics. As a conclusion, *Q=rigidity*displacement^1*. The nonlinear effect of displacement exponent 1.5 or 1.11 is replaced by having variable contact stiffness, similarly with multiple parallel springs. The solution can evaluate the volume of stressed material. If different layers of material exist along the rolling direction, then Eq. (21) can be also applied by modifying the equivalent modulus of elasticity and increasing only the idx number of regions.

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