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ANALYTICALLY FORMULA TO APPRECIATE THE CONTACT LOAD FOR TRANSITION PROCESS FROM ELASTIC TO PLASTIC

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ABSTRACT

This paper intents to establish some simply analytical relations used to describe the limit load in the ball – flat surface point contact. The mathematical model takes into account the Brinell surface hardness. All these mathematical formulas assure the transition continuity between elastic and elastic-plastic phenomenon.

Keywords: Hertz contact, Brinell hardness test, indentation contact radius

1. INTRODUCTION

Literature [4, 5, 6] presents complex analysis of transition theory between elastic contact and plastic contact phenomenon. The present model considers a slice technique and some observations as are presented in [1, 2]. To prove the validity of the developed formulas, the load and the ball correspond to E10-standards procedure [3] and [7]. Some different values of Brinell hardness were considered. According to [3], it is necessary to load a 10 mm ball with 29430 N.

2. ANALYTICAL RELATIONS

Assuming D the ball diameter, and a flat surface with a HB (Brinell hardness) Ry≈0.5*D, k=1.03, E=230769 MPa and xHBN=13.5*HB [2], according to [1,2] results Eq. 1:

$$xNHB = \frac{0.282*E*k^{-0.11}*\delta*fp(k)}{Ry*\sqrt{k^{-0.11}*\frac{\delta}{Ry}}*fb(k)*1.15617*\pi} \quad \text{[MPa]} \tag{1}$$

To obtain the flat surface displacement, d, it is necessary to extract d from Eq.1, resulting:

$$\delta = 41.4786*\frac{fb(k)^2}{fp(k)}*\frac{1}{E}^2*Ry*k^{0.11} = 2.9053626*D*\frac{xNHB^2}{E^2} \tag{2}$$

According to [2], the semi-axes formula is

$$b = Ry*fb(k)*1.15617*\sqrt{k^{-0.11}*\frac{\delta}{Ry}} \tag{3}$$

From Eq. 3 and Eq. 2, when Ry≈0.5*D, k=1.03, => the contact diameter, noted bb2, according to Eq. 4, becomes:

$$bb2=2*Ry*fb(k)*1.15617*\sqrt{k^{-0.11}*\frac{\delta}{Ry}}=3.3597*D*\frac{xNHB}{E}=1.9711*\sqrt{\delta D} \quad (4)$$

In these conditions the retrieved external load, noted Fcalc, which corresponds to xHBN is presented in Eq.5.

$$Fcalc = 1.956 * \pi * \frac{D^2}{E^2} * xNHB^3 \text{ [N]} \quad (5)$$

3. MODEL CONTINUITY

To describe the model continuity form, the first point where the contact pressure is xHBN to a full area with xHBN, some elements are presented in Fig. 1. In this case, because a single contact region exists, the idx parameter is constant and idx=1.

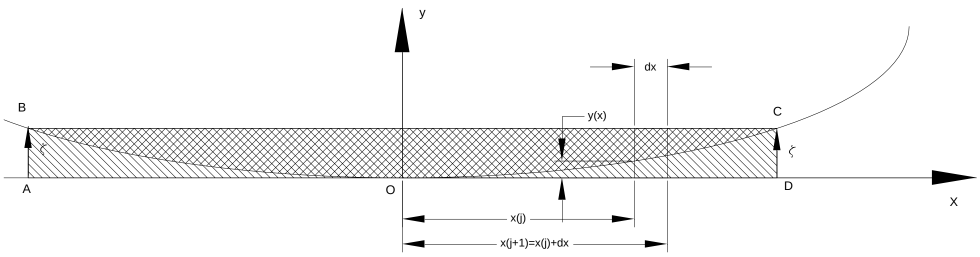

Fig. 1. Elements to describe the contact interference

According to [1], the contact load into a slice is presented in Eq. 6

$$Q_{idx,j} = E0.k_{idx}^{-0.11} fQ(k_{idx}).dx_{idx}.\delta_{idx,j} \quad (6)$$

where E0 is equivalent modulus of elasticity, k_{idx} – is the conformity in the idx region, dx_{idx} is the slice width, $\delta_{idx,j}$ is the relative approach in contact corresponding to idx region and for the j slice and

$$fQ(k_{idx}) = \frac{0.94896 - 0.09445 \cdot \ln(k_{idx})}{1 + 0.45412 \cdot \ln(k_{idx})}$$

The external load Q is given as the sum of the individual loads in slices:

$$Q = \sum_{idx}\sum_j Q_{idx,j} = \sum_{idx}\left[E0.k_{idx}^{-0.11}.fQ(k_{idx}) * \sum_j \delta_{idx,j}.\mathrm{d}x_{idx}\right] \quad (7)$$

Analysing Eq (7) and Figure 1, results:

$$Q = \sum_{idx}\sum_j Q_{idx,j} = \sum_{idx}[CT_{idx} * Area_{idx}] \quad (8)$$

when

$$CT_{idx} = E0.k_{idx}^{-0.11}.fQ(k_{idx}) \quad (9)$$

$$\delta_{idx,j} = \zeta - y(x) = -\frac{X_{idx,j}^2}{2*R_{idx}} + \zeta \quad (10)$$

From Figure 1, it results:

$Area_{idx}$=Area(ABCD) -Area(ABOCD) (11)

According to Eq. 10 and Eq. 11, results Eq.7:

$$Area_{idx} = \sum_j \delta_{idx,j}.\mathrm{d}x_{idx} = \mathrm{d}x_{idx}.\sum_j \delta_{idx,j} = -\sum_j\left[\frac{X_{idx,j}^2}{2*R_{idx}} * \mathrm{d}x_{idx}\right] + \zeta.\sum_j \mathrm{d}x_{idx} \quad (12)$$

Because $\sum_j \mathrm{d}x_{idx} = L_{idx}$, and according to Eq. 12, if note:

$$AA_{idx} = \sum_j\left[\frac{X_{idx,j}^2}{2*R} * \mathrm{d}x_{idx}\right] \quad (13)$$

It results:

$$Q = \sum_{idx}[CT_{idx} * (-AA_{idx} + \zeta.L_{idx})] = -\sum_{idx} CT_{idx}.AA_{idx} + \zeta.\sum_{idx} CT_{idx}.L_{idx} \quad (14)$$

From Eq.14, we can express the dependence between the external load and the roller displacement, according to Eq. 15:

$$\zeta = \frac{Q+\sum_{idx} CT_{idx}.AA_{idx}}{\sum_{idx} CT_{idx}.L_{idx}} \quad (15)$$

For a region idx, if we express the roller element geometry as follows, $x_j = x_0 + j * dx_{idx}$ and $dx_{idx}=L_{idx}/N$, where N is a number of slices in the idx region, it results:

$$AA_{idx} = \sum_j \left[\frac{X^2_{idx,j}}{2*R} * \mathrm{d}x_{idx}\right] = T(L, x0, y0, R)_{idx} \quad (16)$$

where:

$$T(L, x0, y0, R) = y0 * L + \frac{x0^2.L + x0.L^2 + 0.33338*L^3}{2R} \quad (17)$$

The structure of Eq.17 is due to the simple algebra summation of integer numbers, following the algorithm: $\sum_j \left[\frac{X_j^2}{2*R}.dx_{idx}\right] = \sum_{j=1}^{n} \frac{\left(x0+j\frac{L}{n}\right)^2}{2*R}.\frac{L}{n}$ (18)

because

$$\sum_{j=0}^{n} j = \frac{n*(n+1)}{2} \quad (19)$$

and

$$\sum_{j=0}^{n} j^2 = \frac{n*(n+1)(2*n+1)}{6} \quad (20)$$

For idx=1, from Eq 15 results the expression of the relative approach, according to Eq 21:

$$\delta = \frac{0.5*F + C1*T1}{C1*L1}, \quad (21)$$

L1=bb2*0.5

T1=0.33338/(2*D*0.5)*L1^3

C1=E*fQ(k)*k^(-0.11)

We note that FN is the maximum imposed load from E10 Standard (the load which corresponds to Brinell number);

max=FN/Fclac;

ori_N∈[-1,...max-1].

Using Eq. 21 and Eq.4, when F= Fcalc(1+ori_N) and for k=1.03, it results:

$$\delta = \frac{Fcalc*(1+ori\_N)*(0.31985+0.1531)}{D*xNHB} * \left(1 - 0.07381 * \frac{ori\_N}{max-1}\right) \text{ [mm]} \quad (22)$$

where 0.07381 is a small correction factor due to k=1.03.

Using the displacement computed with eq. 22 results the imprint diameter. The imprint diameter is determined using Eq. 23.

$$BB2 = \sqrt{D^2 - (D - 2*\delta)^2} \qquad \text{[mm]} \quad (23)$$

Note. All these relations retrieve the numerical values indicated in E10 standard and are close to Hertz contact theory for central pressure close to xHBN.

Observation:

3^(2/3)=2.08003

According to Hertz theory, the total relative approach is

$$\delta H = \delta^* * \frac{SR}{2}\left[\frac{F}{E*SR}\right]^{2/3} * [3]^{2/3} \quad (24)$$

To find the exact point where elastic tends to plastic, we have to solve a simply equation:

$$\frac{\text{Fcalc*(1+ori\_N)*0.47295}}{\text{D*xNHB}} * \left(1\text{-}0.07381*\frac{\text{ori\_N}}{\text{max-1}}\right) - \frac{1}{2.08} * \left[\frac{\text{Fcalc*(1+ori\_N)}}{\text{k}}\right]^{1/1.5} = 0 \quad (25)$$

=>ori_N, usually <-0

4. NUMERICAL EXAMPLES WITH 10 MM BALL

When a 10 mm ball is pressed with a load of 3000 kgf or with 1500 kgf, the imprint diameter is presented in Fig. 2, as a function of hardness HB or the equivalent HRC hardness.

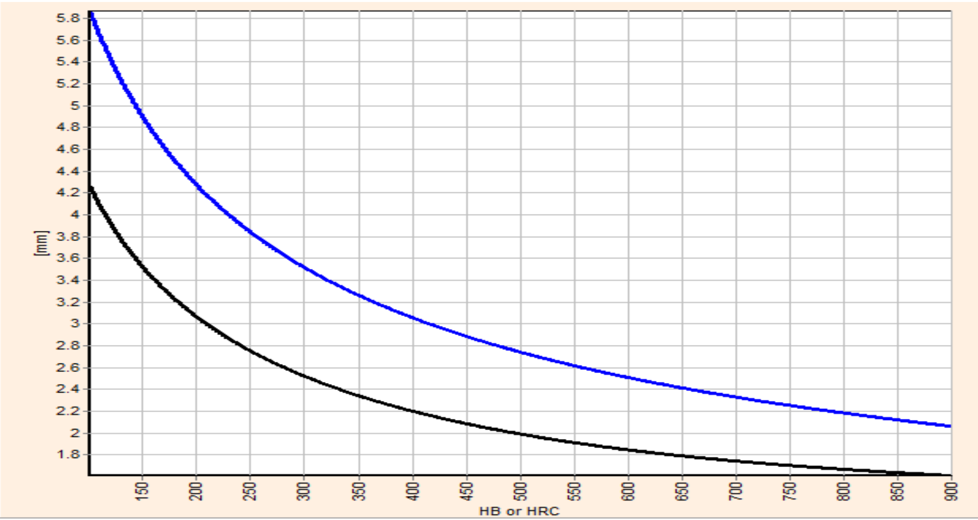

Fig. 2. Imprint diameter for a 10 mm ball computed as a function of material hardness HB. Upper line corresponds to 3000 kgf, while the lower line is for 1500 kgf.

5. NUMERICAL EXAMPLES WITH 5 MM BALL

When a 5 mm ball is pressed with a load of 3000 kgf, or with 1500 kgf, the imprint diameter is presented in Fig. 3, as function of hardness HB or the equivalent HRC hardness.

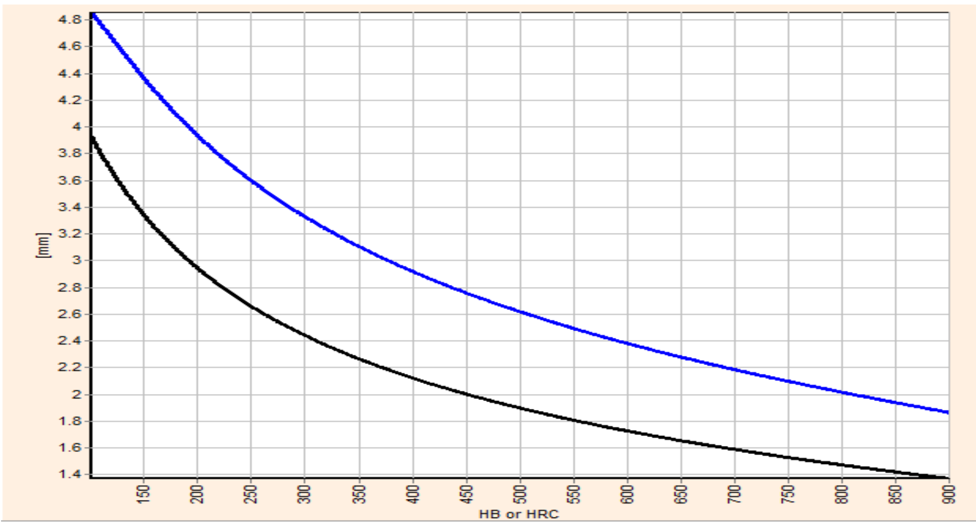

Fig. 3. Imprint diameter for a 5 mm ball computed as a function of material hardness HB. Upper line corresponds to 3000 kgf, while lower line corresponds to 1500 kgf

6. CONCLUSIONS

The mathematic model presents some analytically formula to transition from elastic point contact (see equations 1, 2, 4 and 5) to full Brinell phenomenon (Eq. 22). Also, Eq. 22 and Eq. 23 retrieve the experimental data form [3] and [7].

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