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RESEARCH OF REGULARITIES OF SUSPENDED MINE MONORAIL MOTION

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ABSTRACT

There have been studied the regularities of rolling stock motion along the suspended mine monorail. Goal of the article is to find out the interdependence between the amplitude of suspended units side-sway and carriage displacement during the motion along the monorail. The given model represents the two-mass mechanical system, the free oscillations, which are described with two generalized coordinates. There has been determined the connection between amplitude of suspended units side-sway and carriage displacement during motion along the monorail. Received dependences determine interconnection between amplitude of suspended units side-sway and carriage displacement during the motion along the monorail that allows for determining well-grounded parameters of existing and new-projecting monorails.

Keywords: monorail, system, rolling stock, displacement, side-sway, mathematical model

1. INTRODUCTION

Suspended monorails are getting wider application at modern coal mines [1, 2]. Their main advantages are the possibility of subsidiary cargo transporting along the crooked mine workings with alternating sign. However in the real-life environment at the motion of suspended mine monorail, inevitably it appears the rolling stock side-sway, which influences onto motion safety.

A number of works are devoted to researches of suspended monorails. In the works [3] there have been given the results of research of monorail profile as the rolling stock oscillations source, there have been determined analytic dependences of disturbances and their parameters. There have been fixed the factors influencing onto monorail stock drifts values. Researches [4] are devoted to peculiarities of forming extra charge onto arched support of district workings with suspended monorails.

The present work is the continuation of the indicated researches. The goal of the article is to find out the interdependence between the amplitude of suspended units side-sway and the carriage displacement, during the motion along the monorail.

2. RESEARCH AND RESULTS

Let's examine the model of suspended rolling stock and carriage during the motion along monorail with constant speed (Fig. 1). The given model represents the two-mass mechanical system, the free oscillations of which are described with two generalized coordinates x – carriage transverse displacement relative to monorail on horizontal and φ - displacement angle of longitudinal axis of rolling stock body from vertical. Let's mark reduced mass of carriage m_t, reduced mass of body m_k and distance from suspension points to centre of body mass l.

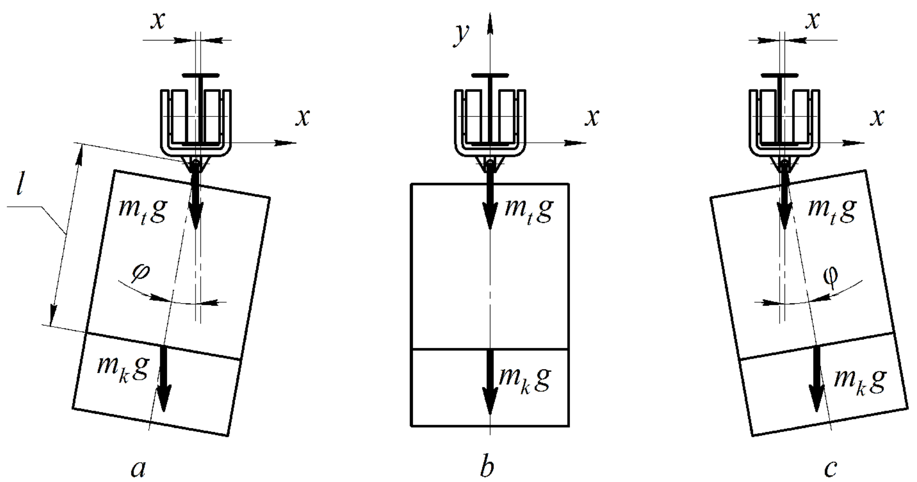

Fig.1. Design model of placement of suspended stock and carriage on the monorail: *a, c* – during motion; *b* – at rest.

Kinetic energy of system is $T = T_t + T_k$, where T_t – carriage kinetic energy, T_k – body kinetic energy.

As carriage during the motion can drift along axis X, so for it we have

$$T_t = \frac{1}{2} m_t \dot{x}^2 . \tag{1}$$

Body executes a linear motion along axis X and turns according to carriage onto angle φ. That's why its speed is determined as $\vec{v}_k = \vec{v}_t + \vec{v}_{kw}$, where $\vec{v}_t$ – speed of carriage displacement, modulus of which is $\dot{x}$; $\vec{v}_{kw}$– speed of body angular motion according to carriage, correspondingly modulus of which is $v_{kw} = l \cdot \dot{\varphi}^2$.

Using cosine theorem we have

$$v_k^2 = v_t^2 + v_{kw}^2 - 2v_k \cdot v_{kw} \cos(180 - \varphi) == \dot{x}^2 + l^2 \dot{\varphi}^2 + 2\dot{x} \cdot \dot{\varphi} \cdot l \cos\varphi .$$

Then, it is

$$T_k = \frac{1}{2} m_k v_k^2 . \tag{2}$$

Taking into account (1) and (2) kinetic energy of all the system it will be

$$T = \frac{m_t \dot{x}^2}{2} + \frac{m_k}{2}\left(\dot{x}^2 + l^2 \cdot \dot{\varphi}^2 + 2\dot{x} \cdot \dot{\varphi} \cdot l \cos\varphi\right). \tag{3}$$

Work of gravity on the virtual displacement $\delta \vec{x}$, leaving constant the generalized coordinate φ , determines the work of gravity on this displacement

$$\delta A_x = m_t \vec{g} \cdot \delta \vec{x} + m_k \vec{g} \cdot \delta \vec{x} .$$

As the angle between $\vec{g}$ and $\delta \vec{x}$ is right, so $\delta A_x = 0$. Therefore generalized force Q_x , doing this work, equals to zero.

Thereafter work of gravities on the virtual displacement φ , leaving constant the generalized coordinate $\delta \vec{x}$, will be

$$\delta A_\varphi = m_k \vec{g} \cdot \delta \vec{r}_k = m_k g \cdot \delta r_k \cos\left(90^0 + \varphi\right),$$

where

$$\delta r_k = l \cdot \delta\varphi .$$

Then,

$$\delta A_\varphi = m_k g l \cdot \sin(\varphi) \cdot \delta\varphi .$$

Generalized force, corresponding to this coordinate, equals to

$$Q_\varphi = -m_k g l \sin\varphi . \qquad (6)$$

Let's draw up the Lagrange equation as

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) - \frac{\partial T}{\partial x} = Q_x ;$$

$$\frac{d}{dt}\left[(m_t + m_k)\dot{x} + m_k l \dot{\varphi} \cos\varphi\right] = 0 ; \qquad (7)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\varphi}}\right) - \frac{\partial T}{\partial \varphi} = Q_\varphi ;$$

$$\frac{d}{dt}\left(m_k l^2 \dot{\varphi} + m_k l \dot{x} \cos\varphi\right) - \left(-m_k l \dot{x} \dot{\varphi} \sin\varphi\right) = -m_k g l \sin\varphi .$$

Received equations (7) let's equate to the system

$$\begin{cases} (m_t + m_k)\ddot{x} + m_k l \cos\varphi \cdot \ddot{\varphi} = m_k l \dot{\varphi}^2 \sin\varphi; \\ m_k \cos\varphi \ddot{x} + m_k l \ddot{\varphi} = -m_k g \sin\varphi. \end{cases} \qquad (8)$$

Let's have transformation and put a mark $m_k \to \mu_k m_k$, where μ_k - a parametric coefficient. As the result we'll get

$$\begin{cases} (m_t + m_k)\ddot{x} + \mu_k m_k l \cos\varphi \cdot \ddot{\varphi} = m_k l \dot{\varphi}^2 \sin\varphi; \\ \cos\varphi \ddot{x} + l \ddot{\varphi} = -g \sin\varphi. \end{cases} \qquad (9)$$

If Taylor series are used and functions are trigonometrically represented as sums of power series eliminating summands of the third and higher power, then received earlier system is possible to be represented as

$$\begin{cases} (m_t + m_k)\ddot{x} + \mu_k m_k l \ddot{\varphi}\left(1 - \dfrac{\varphi^2}{2}\right) = m_k l \dot{\varphi}^2\left(\varphi - \dfrac{\varphi^3}{6}\right); \\ \ddot{x}\left(1 - \dfrac{\varphi^2}{2}\right) + l\ddot{\varphi} = -g\left(\varphi - \dfrac{\varphi^3}{6}\right). \end{cases} \qquad (10)$$

According to parameter μ_k we can write

$$x = x_0 + \mu_k x_1 + \mu_k^2 x_2 + \mu_k^3 x_3 + \dots, \qquad (11)$$

$$\varphi = \varphi_0 + \mu_k \varphi_1 + \mu_k^2 \varphi_2 + \mu_k^3 \varphi_3 + \dots \qquad (12)$$

Using method of numerous scales [7, 8] and taking into account time scales

$$x(\mu_k, t) = x(\mu_k; T_0, T_1, T_2, T_3 \dots),$$
$$\varphi(\mu_k, t) = \varphi(\mu_k; T_0, T_1, T_2, T_3 \dots),$$

where $T_0 = t$, $T_1 = \mu_k t$, $T_2 = \mu_k^2 t$; $T_3 = \mu_k^3 t$ …, we have

$$\begin{cases} m_t \dfrac{\partial^2 x_0}{\partial T_0^2} + \mu_k \left[m_t \left(\dfrac{\partial^2 x_1}{\partial T_0^2} + 2\dfrac{\partial^2 x_0}{\partial T_0 T_1} \right) \right] + m_k \dfrac{\partial^2 x_0}{\partial T_0^2} + \\ + \mu_k m_k l \left[\dfrac{\partial^2 \varphi_0}{\partial T_0^2} \left(1 - \dfrac{\varphi_0^2}{2} \right) - \left(\dfrac{\partial \varphi_0}{\partial T} \right)^2 \left(\varphi_0 - \dfrac{\varphi_0^3}{6} \right) \right] + \dots = 0; \\ \dfrac{\partial^2 x_0}{\partial T_0^2} \left(1 - \dfrac{\varphi_0^2}{2} \right) + l\, \dfrac{\partial^2 \varphi_0}{\partial T_0^2} + g \left(\varphi_0 - \dfrac{\varphi_0^3}{6} \right) + \mu_k l \left(\dfrac{\partial^2 \varphi_1}{\partial T_0^2} + 2\dfrac{\partial^2 \varphi_0}{\partial T_0 \partial T_1} \right) + \\ + \mu_k \left[\left(1 - \dfrac{\varphi_0^2}{2} \right) \left(g\varphi_1 + \left(\dfrac{\partial^2 x_1}{\partial T_0^2} + 2\dfrac{\partial^2 x_0}{\partial T_0 \partial T_1} \right) \right) - \varphi_0 \varphi_1 \dfrac{\partial^2 x_0}{\partial T_0^2} \right] + \dots = 0. \end{cases} \qquad (13)$$

Received equations describe the first form of oscillations of motion of examining mechanical system.

Taking into account the guidelines [5, 6] let's have the analysis of oscillations form stability. Let's study the turn of the main coordinate system onto angle *α*. With this, the new generalized coordinates will be $\tilde{x}$ and $\tilde{\varphi}$. For these coordinates the equations of system motion have the form

$$\begin{cases} (m_t + \mu_k m_k)(\ddot{\tilde{x}} \cos\alpha - \ddot{\tilde{\varphi}} \sin\alpha) - \\ - \mu_k m_k l \left(\dot{\tilde{x}} \sin\alpha + \dot{\tilde{\varphi}} \cos\alpha \right)^2 \left[\tilde{x} \sin\alpha + \tilde{\varphi} \cos\alpha - \dfrac{1}{6} \left(\tilde{x} \sin\alpha + \tilde{\varphi} \cos\alpha \right)^3 \right] + \\ + \mu_k m_k l \left(\ddot{\tilde{x}} \sin\alpha + \ddot{\tilde{\varphi}} \right) \left[1 - \dfrac{1}{2} \left(\tilde{x} \sin\alpha + \tilde{\varphi} \right)^2 \right] = 0; \\ (\ddot{\tilde{x}} \cos\alpha - \ddot{\tilde{\varphi}} \sin\alpha) \left[1 - \dfrac{1}{2} \left(\tilde{x} \sin\alpha + \tilde{\varphi} \cos\alpha \right)^2 \right] + l(\ddot{\tilde{x}} \sin\alpha + \ddot{\tilde{\varphi}} \cos\alpha) + \\ + g \left[\tilde{x} \sin\alpha + \cos\alpha - \dfrac{1}{6} \left(\tilde{x} \sin\alpha + \tilde{\varphi} \right)^3 \right] = 0. \end{cases}$$

For the first form of oscillations, the linearized equation in variations will be

$$\begin{aligned} & \left(1 - \frac{1}{2} x^2 \sin\alpha \right) (-\ddot{v} \sin\alpha + vg \cos\alpha) + \\ & + \cos\alpha \left(\ddot{v} l - v x \ddot{x} \sin\alpha \cos\alpha \right) = 0, \end{aligned} \qquad (14)$$

where x - variation of variable $\tilde{x}$; v - variation of variable $\tilde{\varphi}$.

Using Hill method and taking into account that $x = A\cos\omega t$, the linearized equation (14) can be represented as

$$\ddot{v}\left[l - k_1 + \frac{A^2 k_1^2}{4\left(1 + k_1^2\right)}\left(1 + \cos 2\omega t\right)\right] = \\ = -v\left[g + \frac{A^2}{2\left(1 + k_1^2\right)}\left(\omega^2 - \frac{g k_1}{2}\right)\left(1 + \cos 2\omega t\right)\right], \qquad (15)$$

where k_1 - angular coefficient of straight-line approximation of oscillation form, equals to $k_1 = tg\alpha$.

The equation solutions (15) determine the stability thresholds and correspond to periods T and $2T$, where T – period of coefficients in linearized equation. They can be found as

$$v = a_1\cos\omega t + a_2\cos 2\omega t + a_3\cos 3\omega t + a_4\cos 4\omega t + a_5\cos 5\omega t + \ldots,$$

$$v = a_1\cos\frac{\omega}{2}t + a_2\cos 2\omega t + a_3\cos\frac{3\omega}{2}t + a_4\cos 2\omega t + a_5\cos\frac{5\omega}{2}t + \ldots .$$

Using this type of expansion in equations in variations for different harmonics, we get the systems of algebraic equations of expansion coefficient. Solutions of these systems determine stability thresholds of related forms of mechanical system oscillations.

In Fig. 2, the solution of equations system (10) has been represented for the following values of the parameters: m_t = 5000 kg, m_k =1342 kg, l =0,4 m at starting conditions φ_0 =0.026; x_0 =0; $\dot{\varphi}_0$ =0 and $\dot{x}_0$ =0.

From the graph given in Fig. 2, one can see that body displacement onto angle 0.05 rad leads to a maximum displacement of 25 mm, for m_k =1342 kg, and for m_k =342 kg – 14 mm. The calculations show that with increase of body suspension length l , from 0.4 till 1.0 m, the carriage displacements are 75 mm and 40 mm, respectively. With this approximately into 1.5 times their increases the period of carriage and body oscillations.

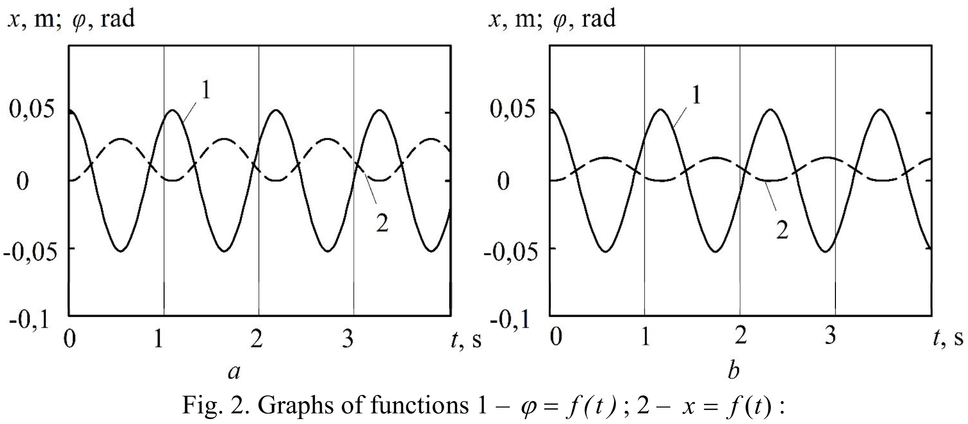

Fig. 2. Graphs of functions 1 – $\varphi = f(t)$; 2 – $x = f(t)$:
a – at $m_k = 1340\,kg$; b – m_k =340 kg

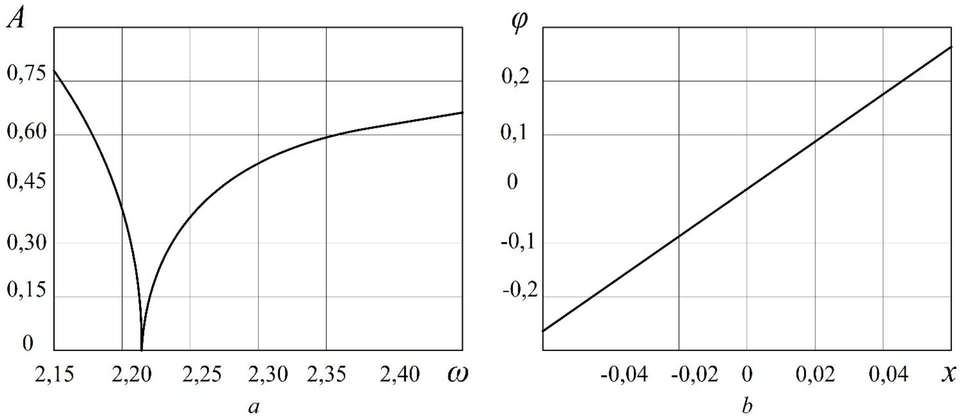

Fig. 3. Dependences $A = f(\omega)$ and $\varphi = f(x)$: *a* - stability range threshold;
b – path of motion in area of instability

Fig. 3*a* has shown the stability range of related oscillations form, determined with the help of Hill method. For internals, there are typical unstable oscillations of mechanical system, which has loss of stability of the first form that leads to the appearing of a couple of new oscillations forms, nonlocal ones. The dependences of φ on x, for unstable motion, are given in Fig. 3*b*.

3. CONCLUSION

Received dependences determine interconnection between the amplitude of suspended units side-sway and the carriage displacement during the motion along the monorail allows for determining well-grounded parameters of existing and new-projecting monorails.

With the goal to accurate the received dependences in future, it is planned to carry out theoretical researches taking into account forced oscillations, caused by effect of disturbances from horizontal and vertical irregularities of the monorail.

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