Mechanical Testing and Diagnosis
ISSN 2247 – 9635, 2012 (II), Volume 2, 21-26

PARTIAL JOURNAL BEARINGS WITH COUPLE STRESS FLUIDS: AN APPROXIMATE CLOSED-FORM SOLUTION

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ABSTRACT

Under certain external unforeseen disturbances, the rotor-bearings' system exhibits self excited oscillating behaviours. The instability occurs when the speed exceeds a certain onset, appearing as self excited orbital motions induced by the fluid film dynamic forces. In this paper, an approximate closed form solution for the Reynolds' equation for the particular case of the partial arc journal bearings in unsteady operating conditions, is proposed. The analysis considers the case of the incompressible coupled stress fluid under the classical hypothesis.

Keywords: Journal bearings, couple stress fluid, oil wedge pressure

1. INTRODUCTION

Journal bearings are usually used in a wide variety of machines, where satisfactory performances are necessary for proper functioning, such as pumps, turbines, compressors, etc. Under certain external unforeseen disturbances the rotor-bearings system exhibits self excited oscillating behaviours. The oil film instability is of primary importance for high speed rotating machines. In fact, it is well known that the dynamic behaviour of a rotor on lubricated bearings is strongly affected by the fluid film characteristics. The instability occurs when the speed exceeds a certain onset and appears as self excited orbital motions induced by the fluid film dynamic forces. The fluid film forces directly rise up by the gap oil film pressure field, which is essentially related to the lubricant viscosity.

It is well known that the additives are typically added to petroleum oils to modify their physical properties, such as the pour point, the foaming or the viscosity–temperature behaviour, the chemical behaviour, such as the detergency, the oxidation or the corrosion and to improve the wear and extreme pressure resistance. With reference to the long-chain organic compounds additives, e.g., the length of the polymer chain may be a million times the characteristic diameter of a water molecule, the experimental studies have shown marked load enhancement and friction reduction [1-3]. The increasing use of the complex fluids as lubricants has received widespread interests owing to the development of modern machine elements. The experimental investigations have also shown that the use of complex fluids can decrease the sensitivity to the shear rate change, improving the stabilization of the lubricating properties. According to the observation in strip squeeze film flow, the polymer thickened oil gives a significant load enhancement as compared to a Newtonian one under similar conditions [3]. In the first work about the short model journal bearing by Oliver [4], the presence of the dissolved polymer in the lubricant produces a load enhancement and a friction reduction.

Since the classical continuum theory neglects the fluid particles size, this approach is not suitable for describing the rheological behaviour of these kinds of non-Newtonian complex fluids. However, the micro-continuum theory takes into account the intrinsic motion of the material constituents; it is developed by the polar theory of complex fluids characterized by the classical Cauchy stresses, as well as by the couple stresses resulting from the spin of microelements in fluids (Ariman and Sylvester [5] and Stokes [6]).

In particular, the Stokes micro-continuum theory [6] is a generalization of the conventional theory, which allows the study of the polar effects, such as the presence of the couple stresses, the body couples and the non-symmetric tensors, involving the rotational velocity field with the dimensional effect of the particles.

Application of this theory is found for the peristalsis mechanisms by Srivastava [7] and Shehawey and Mekheimer [8], the line contacts by Das [9], the rolling elements by Sinha and Singh [10] and Bujurke and Naduvinami [11], the externally pressurized bearings by Lin [12], the squeezing films by Bujurke and Jayaraman [13] and Lin [14], the slider bearings by Ramaniah [15], the finite bearings by Chiang et al. [16], the short journal bearings by Naduvinamani et al. [17], Chiang et al. [18] and Lin [19].

Since the dynamic characteristics account for priority importance in the design of the bearings, in the papers [18-20], the linear stability of the short journal bearings is illustrated.

In this paper, the effects of the parameter related to the couple stress fluids on the solution of the unsteady Reynolds equation governing the film pressure are explored in the case of a partial infinitely long bearing.

2. THE MODEL FOR THE COUPLE STRESS FLUID FILM PRESSURE

2.1. The Equations from Stokes' Theory

The here analysed system consists of a rigid, symmetric and balanced rotor, supported by equal cylindrical bearings. The symmetry about the rotor middle plane allows for limiting the analysis to one of the two halves into which the system is subdivided by the above mentioned plane.

Based upon the classical conception of hydrodynamics, the Stokes model allows for the inspection of the polar effects, such as the presence of couple stresses, the body couples and the non-symmetric tensor. This couple stress fluid is a peculiar case of a non-Newtonian lubricant and takes account of the particle-size effects of the blending additives with large molecules [6]. The isothermal conditions will be assumed to prevail throughout the present investigation. The couple stresses might be expected to appear in noticeable magnitudes in liquids containing additives with large molecules. These couple stresses may be significant, particularly under lubrication conditions where thin films usually exist. The couple stresses introduce non-linear terms in the relationship between the shear stresses and the velocity gradients. As a result, the lubricant should be considered as non-Newtonian and it is characterized by two constants: the dynamic viscosity μ and the couple stress property η.

The continuity and momentum equations governing the motion of an incompressible coupled stress fluid under the classical hypothesis (Stokes [6]), are:

$$\rho \frac{D\mathbf{V}}{Dt} = -\nabla p + \rho \mathbf{F} + \frac{1}{2} \rho \nabla \times \mathbf{C} + \mu \nabla^2 \mathbf{V} - \eta \nabla^4 \mathbf{V} \tag{1}$$

$$\nabla \cdot \mathbf{V} = 0 \tag{2}$$

where the vectors **V**, **F** and **C** represent the velocity, the body force per unit mass and body couple per unit mass, respectively; ρ is the density, p is the pressure, μ is the shear viscosity, and η is a material constant responsible for the couple stress fluid property.

Assuming that the fluid film is thin, the body forces and the body moments are absent and fluid inertia is small as compared to the viscous shear, the equations governing the motion of the lubricant, given in the Cartesian coordinates, reduce to:

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial y^2} - \eta \frac{\partial^4 u}{\partial y^4} \tag{3}$$

$$\frac{\partial p}{\partial y} = 0 \tag{4}$$

$$\frac{\partial p}{\partial z} = \mu \frac{\partial^2 w}{\partial y^2} - \eta \frac{\partial^4 w}{\partial y^4} \tag{5}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{6}$$

The boundary conditions at the bearing surface:

$$u(x,0,z) = v(x,0,z) = w(x,0,z) = 0 \tag{7.1}$$

$$\left.\frac{\partial^2 u}{\partial y^2}\right|_{y=0} = \left.\frac{\partial^2 w}{\partial y^2}\right|_{y=0} = 0 \tag{7.2}$$

while the boundary conditions at the journal surface are described by:

$$u(x,h,z) = U \tag{8.1}$$

$$v(x,h,z) = V \tag{8.2}$$

$$w(x,h,z) = 0 \tag{8.3}$$

$$\left.\frac{\partial^2 u}{\partial y^2}\right|_{y=h} = \left.\frac{\partial^2 w}{\partial y^2}\right|_{y=h} = 0 \tag{8.4}$$

Integrating the (3) and (5) applying the above boundary conditions, the velocity components can be derived as:

$$u = U\frac{y}{h} + \frac{1}{2\mu}\frac{\partial p}{\partial x}\left\{ y(y-h) + 2\ell^2 \left[1 - \frac{\cosh\left(\frac{2y-h}{2\ell}\right)}{\cosh\frac{h}{2\ell}} \right] \right\} \tag{9}$$

$$w = \frac{1}{2\mu}\frac{\partial p}{\partial z}\left\{ y(y-h) + 2\ell^2 \left[1 - \frac{\cosh\left(\frac{2y-h}{2\ell}\right)}{\cosh\frac{h}{2\ell}} \right] \right\} \tag{10}$$

where:

$$\ell = \sqrt{\frac{\eta}{\mu}} \tag{11}$$

is the characteristic length of the additives. In the (11), η has the dimension of momentum. The measurement methods and the procedures for ℓ have been proposed as in [6]. Integrating the continuity equation (6) with respect to y using the velocity components *u* and *w*, with the above boundary conditions, the modified form of the Reynolds equation can be derived [18]:

$$\frac{\partial}{\partial x}\left(\bar{g}\left(\bar{h};\ell\right)\frac{\partial \bar{p}}{\partial x}\right)+\frac{\partial}{\partial \bar{z}}\left(\bar{g}\left(\bar{h};\ell\right)\frac{\partial \bar{p}}{\partial z}\right)=6\mu\left(U\frac{\partial \bar{h}}{\partial x}+2V\right) \tag{12}$$

with:

$$\bar{g}\left(\bar{h};\ell\right)=\bar{h}^3-12\ell^2\left(\bar{h}-2\ell\tanh\frac{\bar{h}}{2\ell}\right) \tag{13}$$

The journal speeds are given by:

$$U=\omega R \qquad V=\frac{\partial \bar{h}}{\partial \bar{t}} \tag{14}$$

The governing model for the hydrodynamic lubrication pressure in the shaft-bearing wedge is a dimensionless modified form of (12):

$$\frac{\partial}{\partial \theta}\left(g(h;\tau)\frac{\partial p}{\partial \theta}\right)+\frac{\partial}{\partial z}\left(g(h;\tau)\frac{\partial p}{\partial z}\right)=\frac{\partial h}{\partial \theta}+2\frac{\partial h}{\partial t} \tag{15}$$

$$g(h;\tau)=h^3-12\tau^2\left(h-2\tau\tanh\frac{h}{2\tau}\right) \tag{16}$$

In (15) and (16) the following dimensionless variables have been introduced:

$$p_{ref}=6\mu\omega\left(\frac{R}{C}\right)^2 \qquad p=\frac{\bar{p}-p_0}{p_{ref}} \qquad h=\frac{\bar{h}}{C}=1+\varepsilon\cos\theta \qquad \tau=\frac{\ell}{C} \qquad z=\frac{\bar{z}}{R} \tag{17}$$

2.2. Infinitely Long Partial arc Bearing: Couple Stress Fluid and Lubrication Equation

For the journal bearings with couple stress fluids, the analytical solutions of the Reynolds equation are not generally achievable and numerical methods must be involved. This is the case of the infinitely long bearing model: the partial differential equation (15), reduced as follows, for which an exact solution can't be obtained:

$$\frac{\partial}{\partial \theta}\left[g(h;\tau)\frac{\partial p}{\partial \theta}\right]=\frac{\partial h}{\partial \theta}\left(1-2\dot{\varphi}\right)+2\dot{\varepsilon}\cos\theta \tag{18}$$

In eq. (18), the function for accounting the couple stress effect is:

$$g(h;\tau)=h^3-12\tau^2\left(h-2\tau\tanh\frac{h}{2\tau}\right) \tag{19}$$

Such a function replaced by:

$$\tilde{g}(h;\tau)=(h(\theta)-\tau\varepsilon)^3=(1+\varepsilon\cos\theta-\tau\varepsilon)^3 \tag{20}$$

allows a closed-form integration for an approximation of eq. (18). The numerical calculations show that eq. (20) allows results for being close to the exact ones within the normal operating conditions of this tribological pair [20, 21]. In this way, the differential equation for the infinitely long partial arc bearing in the reference system as in [22] (Fig. 1) can be integrated to give the following expressions:

$$p_L\left(\theta,\varepsilon,\dot{\varepsilon},\dot{\phi};\tau\right)=(1-2\dot{\phi})\,I_1-2\,\dot{\varepsilon}\,I_2+c_1\,I_3+c_2 \tag{21}$$

and:

$$I_1=\int\frac{h(\theta)}{\tilde{g}(h;\tau)}d\theta \tag{22}$$

$$I_2 = \int \frac{\sin\theta}{\tilde{g}(h;\tau)} d\theta \tag{23}$$

$$I_3 = \int \frac{1}{\tilde{g}(h;\tau)} d\theta \tag{24}$$

while the constants c_1 and c_2 are calculated by imposing the void pressure at $\theta = \theta_1$ and $\theta = \theta_1+\beta$.

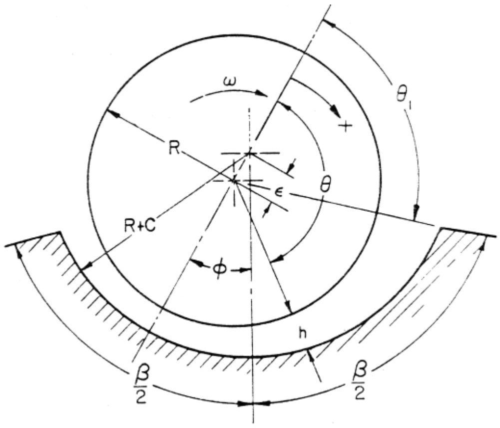

Fig. 1. Partial bearing and reference system [22]

Then, the approximate solution for the unsteady infinitely long bearing with couple stress fluid can now be written as in eq. (21) in a closed form expression, here not reported for the sake of briefness (Fig. 2) for couple stress parameter: τ=0 (Newtonian), τ=0.1, τ=0.2.

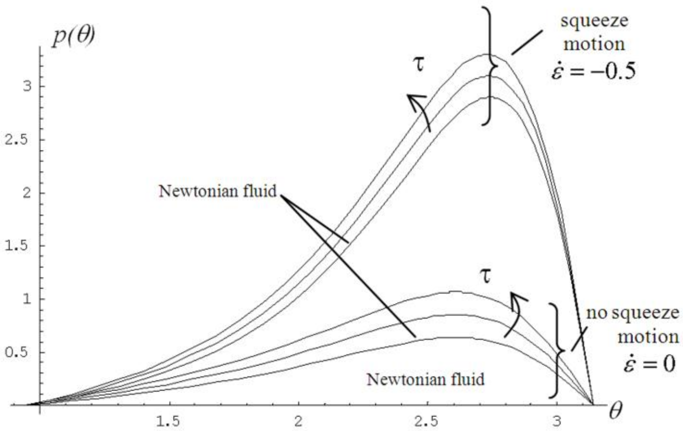

Fig. 2. Dimensionless pressure in the oil wedge with: $\theta_1 = \pi/3$, $\beta = 2\pi/3$, $\dot{\varepsilon} = 0, -0.5$

3. CONCLUSIONS

An approximate closed form solution for the partial arc journal bearings in unsteady operating conditions has been proposed in this paper.

The results compare the Newtonian fluid case with those at different levels of couple stress parameters. As expected, a sensible difference on the bearing load has been calculated. The availability of such a solution for the pressure field in the oil wedge allows an effective development of a numerical algorithm for the rotor behaviour exploration.

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