Mechanical Testing and Diagnosis
ISSN 2247 – 9635, 2012 (II), Volume 3, 60-65

ASPECTS REGARDING SHAFT COUPLING USING ELASTIC ELEMENTS

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ABSTRACT

The paper presents several aspects concerning the shaft coupling using elastic elements, more precisely, with corrugated bellows. The degree of freedom necessary for an accurate coupling is found. The advantages and the inconveniences of the presented coupling are analyzed. As main benefits, the simplicity and relatively reduced costs are highlighted. The mentioned major disadvantages are the infinite degree of freedom of the coupling elements and the occurrence of a vibration phenomenon that affects the quality of the motion transmission.

Keywords: Shaft coupling, corrugated bellows, finite element analysis

1. INTRODUCTION

A frequent challenge in machinery is coupling shafts with imposed parallel axis. Two methods are used in order to avoid the missalignments of the shafts: the elastic elements coupling and the coupling by the kinematic chains consisting in rigid elements linked by lower pairs [1]. The benefits and the inconveniences of these two methods are equally analyzed in the present paper.

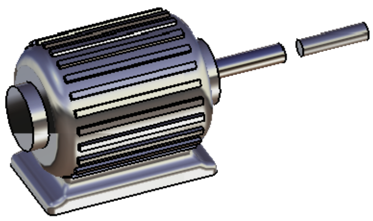

Fig. 1. Scheme of the shaft alignment

Many of the mechanical systems are activated by rotation motors, a large fraction of which being the electrical ones. Concretely, the motor's shaft is coupled to a shaft of the mechanical system, like a generator, a pump or any mechanical system. A problem to be solved for an accurate system functioning, emphasized by Piotrowski [2], in a vast work, is the correct alignment between the motor shaft and the shaft of the mechanical system (Fig. 1).

2. THEORETICAL CONSIDERATIONS

An incorrect positioning of the two shafts can lead, for a diminished phenomenon, to large vibrations in the mechanical system when the shafts are strongly loaded and for a pronounced missalignment, to system blocking. The jamming occurs because, by the rigid shaft coupling of the shafts linked to the ground by rotation pairs, the mobility degree of the obtained system is, according to Gruebler-Kutzbach relation: $DOF = 1 \cdot 6 - 2 \cdot 5 = -2$, showing an overconstrained system.

The relative position of the two shafts is précised by two parameters, presented in Fig. 2, as the length of the common normal a and the crossing angle α .
The relative position of the two shafts can be conveniently expressed using the dual angle:

$$\hat{\alpha} = \alpha + \varepsilon a \qquad (1)$$

where $\varepsilon^2 = 0$ is the dual unity introduced by Clifford, cited by Dimentberg [3]. The dual numbers found an immediate applicability in robotics by dual trigonometric functions. Therefore, it can be written:

$$\begin{aligned} \sin\hat{\alpha} &= \sin\alpha + \varepsilon a \cos\alpha, \\ \cos\hat{\alpha} &= \cos\alpha - \varepsilon a \sin\alpha. \end{aligned} \qquad (2)$$

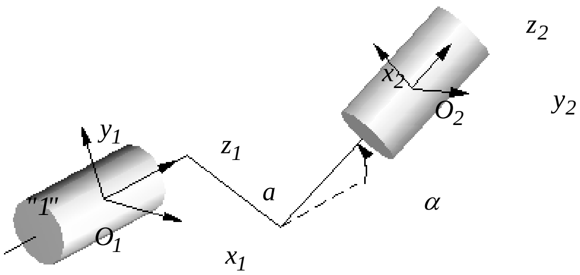

Fig. 2. The parameters and the reference systems used for characterizing the relative position of the two shafts

Attaching to the two shafts the reference systems $O_1x_1y_1z_1$ and $O_2x_2y_2z_2$ respectively, the position of the last system is fully précised when the position of the origin O_2 is known with respect to "1" system and the orientation of the system "2" axes are known referred to system "1"; hence, six independent scalar parameters must be précised.

In the case when for characterizing the orientation of system "2" axes there are used Euler parameters or Rodrigues parameters [4], there are necessary four parameters linked by a relation. Using the dual angle, the correct alignment condition is merely expressed as:

$$\hat{\alpha} = 0 \rightarrow \begin{cases} \alpha = 0 \\ a = 0 \end{cases} \qquad (3)$$

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3. PROPOSED MODEL AND ANALYSIS

A coupling between two shafts using compliant elements, namely corrugated bellows that can support different types of loadings [5-7], is modelled using CAD and presented in Figure 3.

A finite element analysis was made in order to emphasize the influence of the natural vibrations. Figure 5 presents the refined meshing of the model and Figure 6 presents the list of ten natural vibration frequencies found by the numerical structural analysis.

For each vibration mode, the deformation of the elastic element is different [8]. For the case that the whole motor angular velocity is close to or coincides to one of these values, the amplitudes of the deformations increase and, also, the stresses from the material increase, as long as they are within the elastic range; but with increased stresses, the material reaches the plastic domain and the coupling element gets permanent deformations and the irreversible break down can happen.

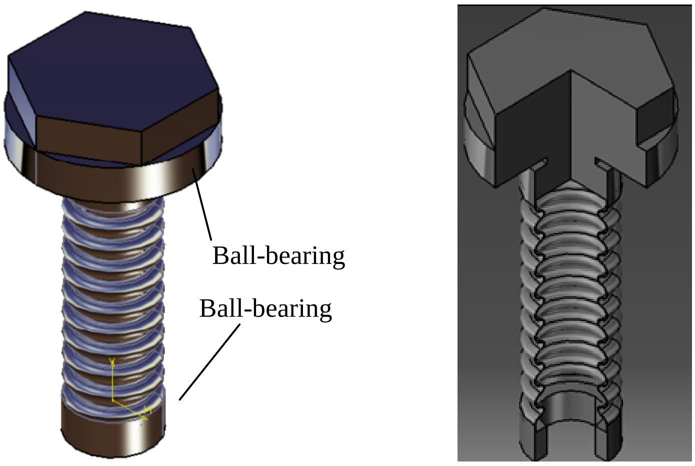

Fig. 3. CAD modelling of the corrugated bellow coupling

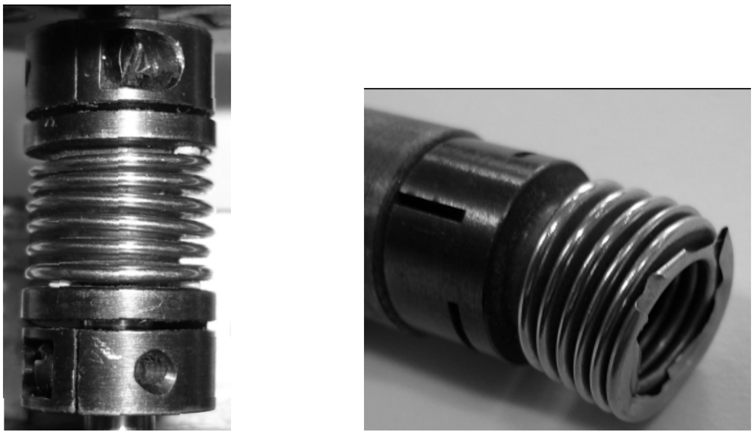

Fig. 4. Actual corrugated bellow and the broken one

Fig. 5. Finite element modelling of the corrugated bellow coupling

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Number of modes	Frequency (Hz)
1	170,691
2	472,044
3	2964,79
4	3145,08
5	3813,86
6	6753,74
7	7065,7
8	7647,43
9	8618,77
10	8640,36

Fig. 6. The list of first ten natural vibration frequencies

For the first vibration mode, the elastic element always deforms axially (Fig. 7) and the traction or compression stresses are supported by the bearings coupling the shaft to the casing.

For the second vibration mode, the deformation of the elastic element is of twisting (Fig. 8) [9, 10]. The relative rotation of the two shafts cannot be supported by the bearings because it is over imposed to the theoretical rotation motion required to be transmitted between the two shafts.

The cases for values *t=0.3 mm*, *t=0.1 mm* and *t=0.5 mm* were analyzed and the results are presented in Table 1.

Table 1. Natural vibration frequencies for three thicknesses

Mod number	**t= 0.1 mm**	**t=0.3 mm**	**t=0.5 mm**
1.	170.69	315.06	478.56
2.	472.04	730.51	895.69
3.	2964.79	3540.86	3838.39
4.	3145.08	3669.42	4009.27
5.	3813.86	4651.23	5357.28
6.	6753.74	7773.61	7952.44
7.	7065.70	7928.65	8104.29
8.	7647.43	8866.25	9339.61
9.	8618.77	8970.75	9650.63
10.	8640.36	9282.79	10755.90

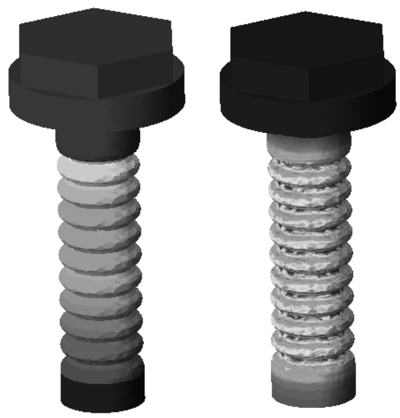

Fig. 7. The deformations and von Mises stresses for the first vibration mode

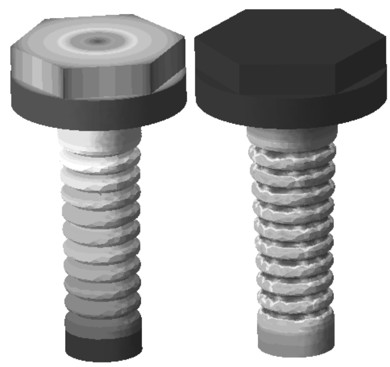

Fig. 8. Deformations and von Mises stresses for the second vibration mode, f = 730.51Hz

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For the third vibration mode, the deformations (flexural deformations) are given by bending (Fig. 9). Similar to the axial deformations, these are supported by the elements of the bearings coupling the shafts to the shells.

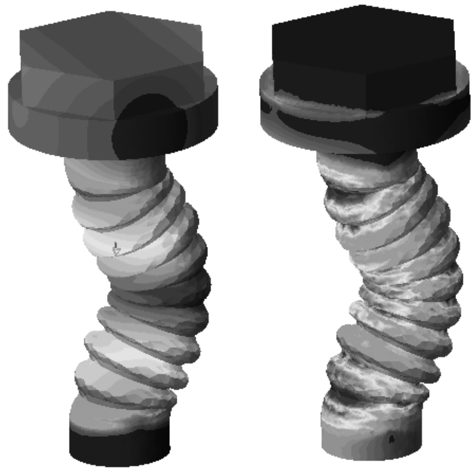

Fig. 9. The deformations and von Mises stresses for the third vibration mode, $f = 3540.86 Hz$

For the above examples, the elastic element dimensions were randomly chosen. The influence of the variation of the geometrical parameters upon the fundamental vibration modes is evidenced next. As a variable parameter, the thickness of the metal sheet from which the corrugated bellow is made was considered (Fig. 10).

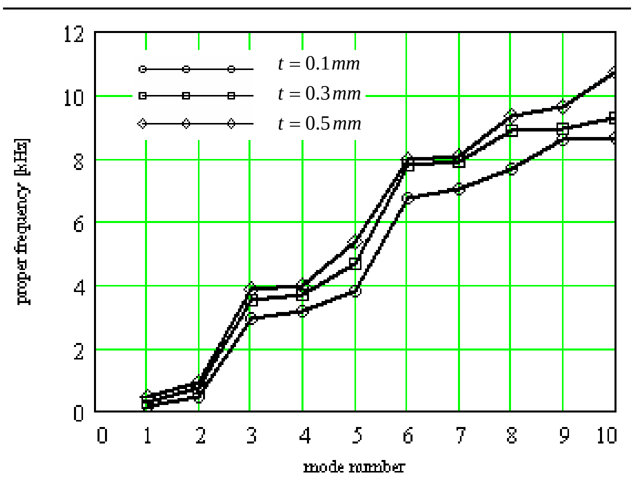

Fig. 10. The natural vibration frequencies corresponding to the first ten vibration modes for three metal sheet thickness, t

4. CONCLUSIONS

The paper presents the analysis of several aspects concerning the shafts coupling using compliant elements. The main advantage of this type of coupling is the simple construction and the existence of the standard coupling elements. The inconveniences met in compliant couplings are of many kinds, that is:

- shaft misalignments must take small values;
- the coupling element and the shaft form a dynamic system that presents an infinity of natural vibration frequencies and for each of these modes, the deformation of the elastic element differs and, from the deformation types, the torsion ones cannot be supported by the bearings; in addition, the torsion is over imposed to the rotation motion to be transmitted and therefore the driven shaft will have a motion different from the driving shaft.

The avoidance of this aspect is practically impossible as the natural frequencies depend both on the compliant element characteristics and on the masses and the geometries of the two shafts. The most convenient way to pass up this aspect is to choose the driving element in a manner that the working rotation speed is lower than the first vibration mode frequency of the system.

As a general final conclusion, one can state that this coupling modality can be used only in the situations when the transmission quality between the two shafts does not require a precise angular velocity ratio. For a strict value of transmission ratio, it is recommended a rigid element coupling, such as linkages or gears.

ACKNOWLEDGMENT

This paper was supported by the project "Progress and development through post-doctoral research and innovation in engineering and applied sciences – PRIDE, contract no. POSDRU/89/1.5/s/57083", project co-funded from European Social Fund through Sectorial Operational Program Human Resources 2007-2013.

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