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PROPOSED METHOD FOR EVALUATION OF SHEAR STATE IN A PLANE

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ABSTRACT

The shear stress state in a rectangular plate can be practically obtained only on a limited area, because the plate loading is made of concentrated forces and, therefore, in the vicinity of points of load application, the stress field is strongly disturbed. But, at great distance from the force application points, a stress state closed to the ideal pure shear is generated. There are two theoretical ways in obtaining the pure shear stress state. The paper presents a proposed method for qualitative evaluating the degree of reaching pure shear state in a rectangular plate and for predicted loading case. The method considers a loaded rectangular plate with two circular holes. The finite element method analysis of the stress state is made and afterwards compared to the theoretical and experimental (photoelastic) analysis, made by the authors in previous papers. A very good agreement of the results is obtained.

Keywords: Plane elasticity, shear stress, photoelasticity, FEM analysis

1. INTRODUCTION

The pure shear stress state is a theoretical case seldom referred to. In plane elasticity [1], it is stated that the pure shear stress state can be obtained in an elastic plane through two ways, namely by:

- applying at infinity tractions and compression forces, having the same intensity, respectively, on orthogonal directions, (or biaxial loading), as shown in Fig. 1;
- applying tangential forces oriented on orthogonal directions at infinity, as seen in Fig. 2.

In practice, obtaining the pure shear stress state is more difficult due to the finite dimensions of the domain and, regardless of the loading manner, the applied forces are concentrated forces and, therefore, the stress field is rather disturbed.

One of the experimental methods to obtain pure shear stress was proposed by Iosipescu [2, 3]. The method uses a prismatic probe with two V notches, mounted in a special loading device that transforms the traction from the testing machine into shear in the probe section. The pure shear stress state is obtained, as Iosipescu reveals, only in a region close to the vicinity of the two notches.

The present paper presents a qualitative testing methodology for verifying that the pure shear stress state is obtained in a mechanical element under a loading intended to produce pure shear.

The principle proposed in the paper consists in making two circular holes, of small dimensions as compared to the analyzed region. The condition of reduced radii dimension is requested by the requirement that the analysis effect should have a local character. The other condition, the use of two holes, was imposed by the following considerations:

- the compact plane does not have a measure unity and making a circular hole ensures defining a unity of measure;
- the second hole is needed to define a well précised direction. Using a single circular hole, the centre of the hole can be chosen as the origin of coordinate system but, due to circular symmetry, any radius could be chosen as axis;
- a next reason for using the option of making two holes is based on the fact that, for any of the two modalities of loading the elastic plane in order to obtain pure shear stress state, there are presented analytical relations to describe the stress field from the loaded plane with two holes [4-6].

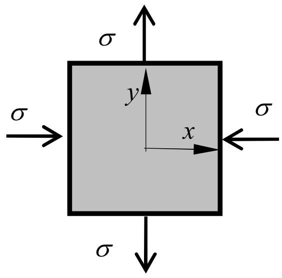

Fig. 1. Pure shear stress state obtained applying normal forces (biaxial loading)

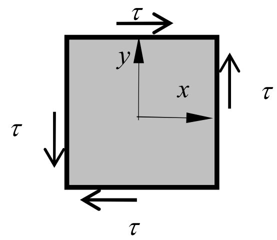

Fig. 2. Pure shear stress state obtained by tangential forces

2. THEORETICAL CONSIDERATIONS

The stress state solution for the problem of the elastic plane with two circular holes is obtained, as principle, by writing in Cartesian coordinates the characteristic potential of the loaded compact plane, next transforming this potential in bipolar coordinates and eventually adding to it an auxiliary potential having the form given by Jeffery [7], such as the boundary conditions on the holes contour can be imposed together with the condition of the stresses regularity at infinity. For the two loading types, the elastic potentials (the Airy functions) characteristic to loadings that produce pure shear stress state are:

$$U(x,y) = \sigma \frac{x^2 - y^2}{2} \tag{1}$$

and:

$$U(x,y) = \tau \cdot xy \tag{2}$$

respectively.

As mentioned, both loading schemes generate pure shear stress states, but they differ as the stress tensors expressing the two states are:

$$\mathbf{T}_1 = \begin{bmatrix} \sigma & 0 \\ 0 & -\sigma \end{bmatrix} \tag{3}$$

and

$$\mathbf{T}_2 = \begin{bmatrix} 0 & \tau \\ \tau & 0 \end{bmatrix} \tag{4}$$

respectively, [1].

3. PROPOSED METHOD FOR SHEAR EVALUATION

As it can be seen, the stress tensor (3) has a diagonal form and the principal axes of the stress state are the coordinate axis *Ox* and *Oy*.

In the case of stress tensor expressed by tensor (4), the principal stresses are found equal and of opposite sign, but rotated at *45°* with respect to the coordinate axes *Ox* and *Oy*. Thus, the necessity of introducing a known direction in the plane is explained and this direction is obtained by the two holes. In previous works [4, 5], the theoretical results were experimentally validated using the photoelastic method, to be more specific, the theoretical isochromatics fields (Fig. 3) were compared to the experimental ones (Fig. 4), obtained in the Laboratory of stress analysis from FIMMM Suceava.

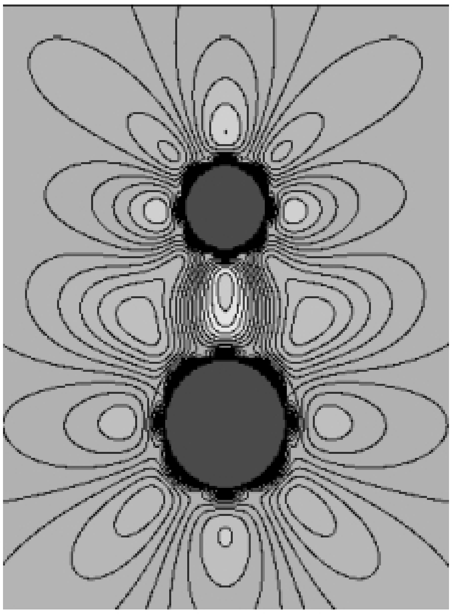

Fig. 3. Analytic share stress field in an elastic plane (isochromatics)

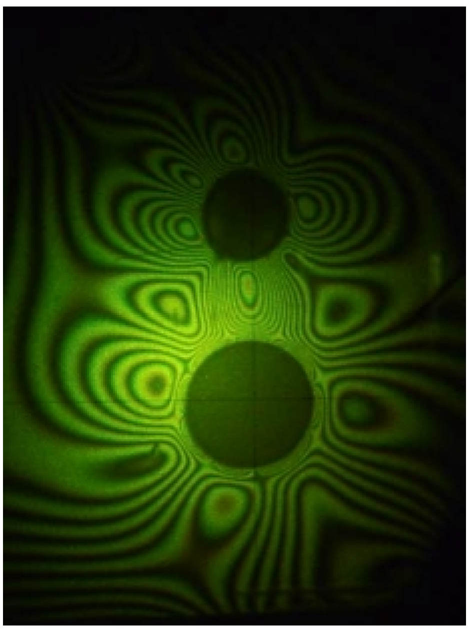

Fig. 4. Experimental shear stress field: photoelastic isochromatics

Since the experimental images were obtained for a probe made of a photoelastic material, loaded by normal forces [4], the coincidence between the theoretical images and the experimental images is obtained only in the vicinity of the holes.

For a supplementary validation of the correctness of the analytical relations, a numerical analysis was performed, using the finite element analysis, for the two loading situations – for holes of equal radii.

Figure 5 presents the shear stress modelled for an elastic plane with two equal holes loaded by tangential forces.

For the theoretical pure shear stress state modelled in Fig. 1, the FEA analysis was performed for an elastic plate with two holes, normally loaded to the sides and the results are plotted in Fig. 6.

The case when the load is normal to the sides, but the direction of the holes center line is rotated with 45°, as presented in Fig. 7, confirms the theoretical results, as the pure shear stress field obtained is the same to that obtained in Fig. 5, but rotated with 45° with respect to the biaxial loading.

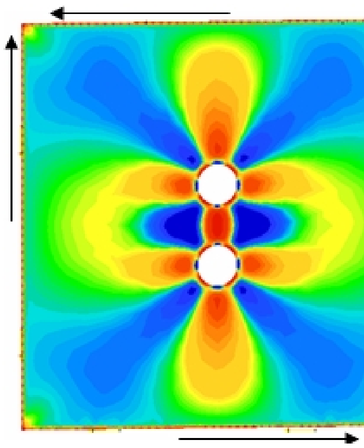

Fig. 5. FEA stress field for tangential loading

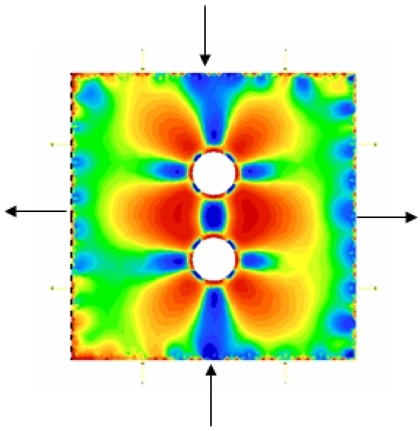

Fig. 6. FEA stress field for biaxial loading

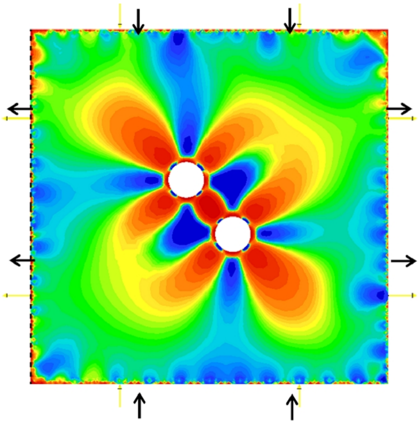

Fig. 7. FEA stress field for pure shear when the direction of the axes line is rotated with 45° with respect to the biaxial loading.

4. CONCLUSIONS

The paper presents some considerations upon laboratory methods for obtaining pure shear stress fields. From the theory of elasticity, the pure shear in an elastic plane can be obtained by tangential (shear) loading of a square element or by biaxial loading.

In practice, the stress field is qualitatively validated by photoelastic methods and compared to analytical results.

The method proposed in the present paper is based on the statement that the loaded plane must be referred to a coordinate system and therefore a direction must be defined in the plane. The mentioned direction is materialized by the centers of two small holes made into the plane. The shear generates a field of stresses in the vicinity of the holes that can be analytically deduced and validated by photoelasticity.

In the present paper, a supplementary validation of the stress state obtained in the two loading modalities was made using numerical models, via the finite element analysis. A very good qualitative agreement between the three methods was found.

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