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DYNAMIC BEHAVIOUR OF THE COMPOSITE DELAMINATED SHIP DECK PLATES

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ABSTRACT

The work presented in this paper analyzes the influence of the material damping properties of the composite plates with delaminations. The linear elastic behaviour of the laminate plates can be predicted from the properties of the individual plies using the laminate theory that can also be used to predict the damping properties of such plates. An orthotropic delamination model, describingthe delaminating mode, using COSMOS/M soft package, is presented in this paper for analyzing the behaviour of the delaminated composite plates during vibrations. So, the damaged part of the structures and the undamaged part have been represented by well-known finite elements (layered shell elements). The influence of the position and the ellipse's diameters ratio of the delaminated zone on the dynamic behaviour of the composite plate is investigated. The plates are made of E-glass polyester, having the dimensions: 320 mm x 320 mm and a thickness of 9.82 mm. The experiments for determining the damping coefficients and the first three natural frequencies for all plates were made with an experimental rig, also presented in the paper.

Keywords: vibrations, delaminations, composite plates

1. INTRODUCTION

The vibration problems involving the orthotropic plates are common, ranging from the vibration control in the composite panels of the structures, in general, and in the ship hull structures, especially. In all these problems, one cannot get very far without values for the essential physical parameters of the material in question, in particular the elastic and damping constants for the relevant frequency range ([1]-[4]). Such values are not readily available in tabulated form, and indeed, for many of the materials which we shall be considering, there is a wide range of variation among samples of the same nominal material, arising from the variations in growth (for natural materials) and in fabrication.

Therefore, it is often necessary to measure the elastic and damping constants for the particular plate under study, if the vibration predictions of any accuracy are needed. In this paper, we describe an approach for making such measurements, based on observing the frequencies and damping factors of the low vibration modes of the plates with clamped boundaries (the most common case in the ship hull structures).

The quantitative determinations of the damping constants require a little more complex apparatus and a much more care ([5]-[7]). Here, the main problem does not lie in measuring the modal damping rates, but in ensuring that the observed damping rate does indeed arise only from the internal damping in the plate and not from the additional losses into the plate's supports or the observer's instrumentation.

Understanding the damping properties of the composite materials is the most important issue of the design and dynamic analysis of the ship structure in general. The offshore and ship structures are loaded by several dynamic forces. The reduction of the structural vibration is one of the key design aspects and the composite materials are used to reduce vibrations. The loss factor and Young's modulus must especially be known in order to develop the finite element codes for a composite structure ([8], [9], [10]).

The damping of an engineering structure is important in many aspects of noise and vibration control, fatigue endurance and so on, since it controls the amplitude of the resonant vibration response ([3]). As a result of the energy dissipation mechanisms in a structure, the inherent material damping significantly contributes to the overall damping, and it is often the primary means of controlling the structure's dynamic behaviour. Thus it is important to be able to control and predict the level of the inherent damping in such materials ([11],[10]).

The linear elastic behaviour of the laminate plates can be predicted from the properties of the individual plies, using the laminate theory, that can also be helpful for evaluating the damping properties of such plates, through the concept of "complex modulus" ([12]-[18]).

An orthotropic delamination model, describing the delaminating mode using COSMOS/M soft package, is developed in this paper for analyzing the behaviour of the delaminated composite plates during vibrations. So, the damaged part of the structures and the undamaged part have been represented by well-known finite elements (layered shell elements). The influence of the position and the ellipse's diameters ratio of the delaminated zone on the dynamic behaviour of the composite plate is investigated ([19]).

2. THE THEORY OF ORTHOTROPIC PLATES VIBRATIONS

In the case of a thin, especially orthotropic plate, the small amplitude vibrational behaviour is governed by four elastic constants, for the simplest case in which shear and rotator inertia are ignored. We choose to use the constants D_1, D_2, D_3, and D_4, introduced in [1].

These are defined through the elastic strain energy of the plate as it vibrates in the *x-y* plane with a centre-plane transverse displacement in the *z*-direction

$$U=\frac{h^3}{2}\int_A \left[D_1\left(\frac{\partial^2 w}{\partial x^2}\right)^2+D_2\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}+D_3\left(\frac{\partial^2 w}{\partial y^2}\right)^2+D_4\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2\right]dA \qquad (1)$$

where h is the plate thickness and the integral is taken over the area of the plate. The constants D_i can be written in terms of the Young's moduli along the two main directions (*x* and *y* axes) of the plate, the two Poisson's ratios between these directions ν_{xy} and ν_{yx}, and the in-plane shear modulus G_{xy}.

$$D_1=E_x/12\mu,\quad D_2=\nu_{xy}E_y/6\mu=\nu_{yx}E_x/6\mu,\quad D_3=E_y/12\mu,\ D_4=G_{xy}/3 \qquad (2)$$

where $\mu=1-\nu_{xy}\,\nu_{yx}$.

Note that, in broad terms, D_1 is associated with bending in the *x* direction, D_3 with bending in the *y* direction, D_4 with twisting motion, and D_2 with Poisson's ratio coupling between the *x* and *y* directions.

The problem of interest is to measure the four elastic constants D_1, D_2, D_3, and D_4, and the four associated damping constants.

Once we know to reasonable accuracy, a particular mode shape and its frequency *f*, it is quite easy to calculate the damping of that mode. It is convenient, and probably at least as accurate as any practical measurement technique, to use a "small damping" approximation, which assumes that the modal *Q* factor is much greater than unity. Before estimating the *Q* factor, we have to introduce the four small quantities η_1 to η_4, defined by

$$\eta_i = Im(D_i)/Re(D_i), \ i = 1,2,3,4 \tag{3}$$

In the upper equation, η_i are the conventional loss factors associated with the individual complex D_i. The reciprocal of the modal *Q* factor of any given mode can be shown from Rayleigh's principle to be simply a weighted sum of the four η_i [3]. The expression is

$$1/Q = \sum_{i=1}^{4} \eta_i J_i \tag{4}$$

where J_i are the real quantities evaluated from the elastic mode shape,

$$J_1 = \frac{D_1 \int_A \left(\frac{\partial^2 w}{\partial x^2}\right)^2 h^3 dA}{f^2 \int_A \rho h w^2 dA}; \quad J_2 = \frac{D_2 \int_A \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} h^3 dA}{f^2 \int_A \rho h w^2 dA};$$

$$J_3 = \frac{D_3 \int_A \left(\frac{\partial^2 w}{\partial y^2}\right)^2 h^3 dA}{f^2 \int_A \rho h w^2 dA}; \quad J_4 = \frac{D_4 \int_A \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 h^3 dA}{f^2 \int_A \rho h w^2 dA}. \tag{5}$$

so that

$$\sum_{i=1}^{4} J_i = 1 \tag{6}$$

The expressions (5) for the J_i simply indicate the partitioning of the potential energy, and thus, the dissipation rate, among the types of the motion associated with each of the D_i in equation (1).

The parameters J_i have another significance purely in terms of mode frequencies, which is also worth noting for a future reference: it follows, again from Rayleigh's principle, that each J_i gives the changing rate of the mode frequency with the corresponding D_i according to

$$J_i = \frac{D_i}{f^2}\frac{\partial f^2}{\partial D_i}, \ i=1,2,3,4. \tag{7}$$

This result proves useful when fitting the trial values of the D_i to the experimental data on the frequency f_i.

Before going on with the experimental method, it is useful to note the relationship between the elastic constants D_1, D_2, D_3, and D_4 and the more familiar elastic constants. One thing we can deduce about them, which applies to any thin flat orthotropic plate, follows from considering a long narrow strip cut from the plate. Such a strip will vibrate as a bending beam. If it is cut along a principal axis of the plate, it is readily shown that the effective Young's moduli for the beam behaviour are, respectively:

$$E_x = 12(D_1 - D_2^2/4D_3); \ E_y = 12(D_3 - D_2^2/4D_1) \tag{8}$$

for a strip cut along the x or y axis. The strip width is taken to be very small as compared to the bending wavelength, but still large as compared to the plate thickness h. For such a strip cut from the plate at an angle θ to one principal axis (the *x*-axis), the equivalent Young's modulus can be shown to be

$$E_\theta = 3D_4(4D_1D_3 - D_2^2)/\Phi(\theta) \tag{9}$$

where

$$\Phi(\theta) = D_4(D_1\sin^4\theta + D_3\cos^4\theta) + (4D_1D_3 - D_2^2 - D_2D_4)\sin^2\theta\cos^2\theta \tag{10}$$

The simplest case occurs when the plate is cut from the solid parallel to one of the principal planes. In that case, the values for the four D_i can be obtained from equations (2).

There is a simple reciprocal property relating the two Young's moduli and the two corresponding Poisson's ratios, so that only three of these are independent, a fact used in exhibiting the symmetrical property of D_i in eq. 2 as:

$$\nu_{yx}/E_y = \nu_{xy}/E_x \tag{11}$$

3. PLATES VIBRATIONS ANALYSIS

Due to the anisotropy of the composite laminates and the non-uniform distribution of the stresses in the lamina under flexural bending, as well as other types of static/dynamic loading, the failure process of laminates is very complex. The large differences in strength and stiffness values of the fiber and the matrix lead to various forms of defect/damage caused during the manufacturing process, as well as under service conditions.

In shipbuilding, many structures made of composite laminates are situated such that they are susceptible to foreign object impacts, which can result in barely visible impact damage. Often, in the form of a complicated array of matrix cracks and interlaminar delaminations, these barely visible impact damages can be quite extensive and can significantly reduce a structure's load bearing capability and behaviour on vibrations.

The delamination or the separation of two adjacent plies in a composite laminate is one of the most common modes of damage. The presence of the delamination may reduce the overall stiffness, as well as the residual strength leading to structural failure. A clear understanding of the influence of delamination on the performance of the laminates is very essential to efficiently use them in structural design applications.

Since such process is generally difficult to detect, the structures must be able to function safely with the present delamination.

Although several studies are available in the literature in the field of delamination prediction and growth, the effect of delamination on the vibration behaviour and the delamination propagation under fatigue loading, the work on the effect of delamination on the first ply failure of the laminate is scarce.

The aim of this study is to present the studies on the influence of the elliptical single and doubled delaminations on the changes in the vibration behaviour of the ship deck plates made of composite materials. An orthotropic delamination model, describing the mixed mode delaminating, using FEM analysis, was applied. Thus, the damaged part of the structures and the undamaged part have been represented by well-known finite elements (layered shell elements). The influence of the position and the ellipse's diameters ratio of the delaminated zone on the natural frequencies was investigated.

If the initial delaminations do exist, these delaminations may close under the applied load. To prevent the two adjacent plies from penetrating, a simple numerical contact model is used.

Taking into account the thickness symmetry of the plates, only the cases of the position of the delamination on one side of the symmetry axis are presented. The variations of the transversal displacement of the point placed in the middle of the plate versus the in-plane applied pressure are plotted for each position of the delamination. The buckling load determination for the general buckling of the plate has been done by a graphical method. The post-buckling calculus has been performed in order to explain the complete behaviour of the plate. Only the cases with one delamination placed between two adjacent laminas are presented here.

There are several ways in which the panel can be modeled for the delamination analysis. For the present study, a 3-D model with 3-node shell composite elements is used. The panel is divided into two sub-laminates by a hypothetical plane containing the delamination. For this reason, the present finite element model would be referred to as the two sub-laminate model. The two sub-laminates are separately modeled with shell composite elements and then joined face to face with the appropriate interfacial constraint conditions for the corresponding nodes on the sub-laminates, depending on whether the nodes lie in the delaminated or the region undelaminated one.

The numerical analysis and the experimental tests were developed on perfect plate and imperfect plates (single and double delaminations).

The material used for plates is E-glass/epoxy, having the mechanical characteristics:

E_x=3.86 GPa, E_y =8.27 GPa, E_z =8.27 GPa, G_{xy}=4.1412 GPa, G_{xz} =4.1412 GPa, G_{yz} =4.1412 GPa, $\nu_{xy}= \nu_{yz}= \nu_{xz}$=0.26.

Table 1. Plate lay-up

Layer	θ	h [mm]
1	45^0	0.195
2	-45^0	0.195
3	0^0	2.36
4	45^0	0.195
5	-45^0	0.195
6	0^0	3.54
7	-45^0	0.195
8	45^0	0.195
9	0^0	2.36
10	-45^0	0.195
11	45^0	0.195

By imposing the diameters ratio of the delamination and from the condition of the same delamination area (400 mm^2), the values of the diameters have resulted. The delaminated plates are presented in Figures 5-9.

4. EXPERIMENTAL ANALYSIS

The experiments for determining the damping coefficients and the first three natural frequencies for the bending vibrations, for all 5 plates with various geometries, were made with the help of an experimental rig, presented in Figure 1. The boundary conditions (clamped on all sides) for the plates were done with a very rigid frame.

The plate was made of E-glass epoxy. The delamination was placed between layers 5 and 6.

The excitation was done with the impact method. For each test, the data are obtained for a time interval of 3 seconds after the plate excitation. This allows for time measuring the damping effects. Then the experimental data are processed to show the effectiveness of the tested control algorithm. A fast Fourier transformation (FFT) is performed by the instrument (vibrometer) in order to provide an acceleration spectral density plot of the plate response, which gives a measure of the signal energy level at different frequencies.

The *Q* of a system is a measure of damping, usually defined from the energy considerations. The *Q* is π times the ratio of peak energy stored to the energy dissipated per cycle and it is equal to π/δ.

There are many methods for measuring the damping of a vibrating system. The logarithmic decrement method and the bandwidth method are the most used in experiments.

The logarithmic decrement method is used to measure the damping in the time domain. In this method, the free vibration displacement amplitude history of a system to an impulse is measured and recorded. A typical free decay curve is shown in Figure 2. The logarithmic decrement is the natural logarithmic value of the ratio of two adjacent peak values of the displacement in the free decay vibration.

The determination of the damping coefficients was made by the vibration equipment, presented in Figure 1. According to the logarithmic decrement, the damping coefficient was determined for all plate specimens.

The damping coefficients determined by the experiments were used in FEM analysis done with the package licenced soft COSMOS/M.

The logarithmic decrement, δ, is used to find the damping ratio of an underdamped system in the time domain. The logarithmic decrement is the natural log of the ratio of the amplitudes of any two successive peaks

$$\delta = \frac{1}{n} \ln \frac{w_1}{w_{n+1}} \tag{12}$$

where w_1 is the greater of the two amplitudes and w_{n+1} is the amplitude of a peak *n* periods away. Then the damping ratio is found from the logarithmic decrement:

$$\varsigma = \frac{1}{\sqrt{1 + (2\pi / \delta)^2}} \tag{13}$$

According to the measurements results, the first three frequencies for the single delaminated plates are presented in Table 2. The identification of the frequencies corresponding to the bending vibrations was done by comparing them to the results obtained from initial FEM calculus.

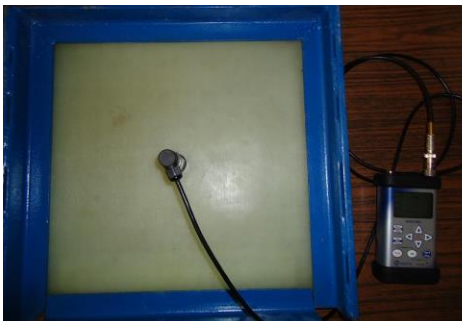

Fig. 1 The experimental rig

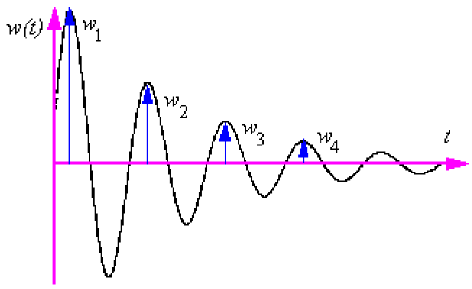

Fig. 2. The motion decay for damped vibrations

Table 2. The first three measured frequencies

Plate case	f_1 [Hz]	f_2 [Hz]	f_3 [Hz]
D_x/D_y=0.5	10.68	21.68	41.78
D_x/D_y=0.75	10.86	21.58	41.83
D_x/D_y=1	10.77	21.68	41.95
D_x/D_y=1.25	10.64	21.62	42.14
D_x/D_y=2	10.74	21.83	42.67
Perfect	10.25	21.86	39.98

5. NUMERICAL ANALYSIS

For the case of a plate with a single delamination placed between layers 5 and 6, the numerical analysis of the damped and nondamped free vibration was done.

The geometry of the delamination placed in the middle of the thickness is considered in the following cases (Figures 5-9):

- Case 1 ($D_x/D_y=0.5$): transversal diameter $D_y=28.28$ *mm*, longitudinal diameter $D_x=14.14$ *mm*;
- Case 2 (D_x/D_y=0.75): transversal diameter $D_y=23.09$ *mm*; longitudinal diameter $D_x=17.32$ mm;
- Case 3 ($D_x/D_y=1$): transversal diameter $D_y=20$ *mm*; longitudinal diameter $D_x=20$ *mm*.
- Case 4 ($D_x/D_y=1.25$): transversal diameter $D_y= 17.32$ *mm*, longitudinal diameter $D_x=23.09$ *mm*;
- Case 5 ($D_x/D_y=2$): transversal diameter D_y=14.14 mm; longitudinal diameter $D_x=28.28$ *mm*.

The plate is considered as clamped on the sides.

The variation of the first three natural frequencies of the bending free vibrations versus the diameters ratio of the delamination area is presented in Figure 3.

The variation of the first three natural frequencies of the bending free damped vibrations versus the diameters ratio of the delamination area, for the case of delamination placed between the layers 5 and 6 is presented in Figure 4.

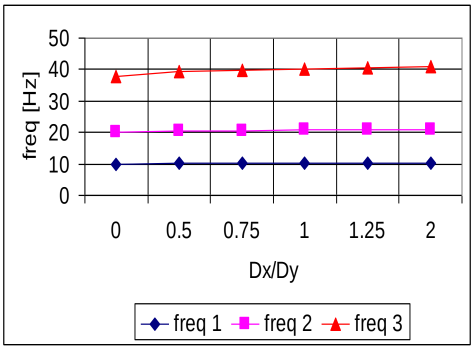

Fig. 3 The variation of the first three natural frequencies (bending vibrations) versus the diameters' ratio of the delamination area

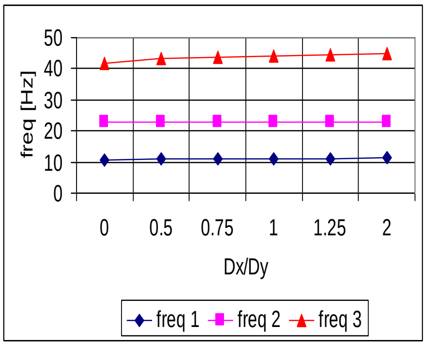

Fig. 4 The variation of the first three natural frequencies (damped bending vibrations) versus the diameters' ratio of the delamination area

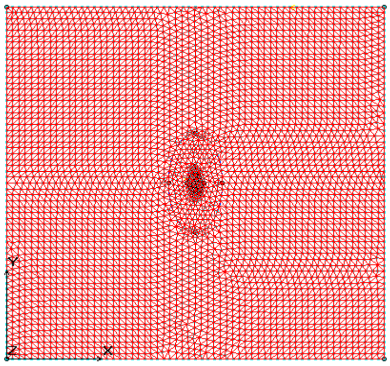

Fig. 5 Plate with single delamination Case 1

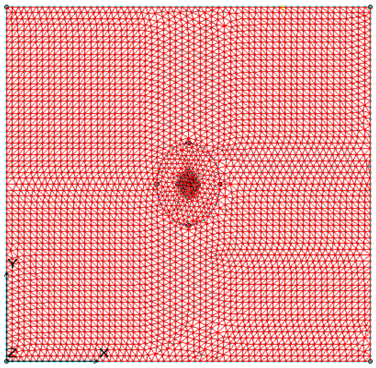

Fig. 6 Plate with single delamination Case 2

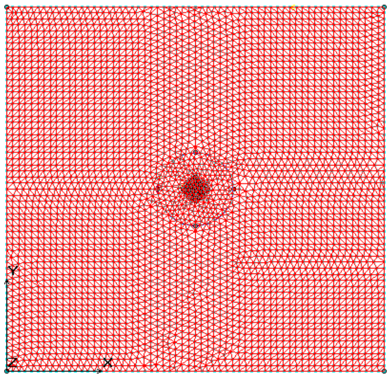

Fig. 7 Plate with single delamination Case 3

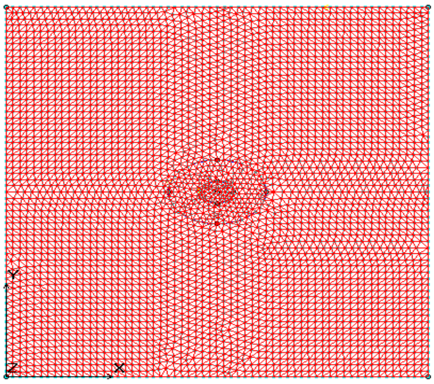

Fig. 8 Plate with single delamination Case 4

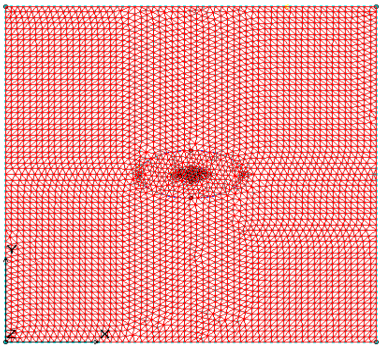

Fig. 9 Plate with single delamination Case 5

6. CONCLUSIONS

The free vibration of a structure is a vibration in which energy is neither added to nor removed from the vibrating system. It will just keep vibrating forever at the same amplitude. Except from some applications there are no free vibrations in nature. They are all damped to some extent.

Damped vibration is one in which there is an energy loss from the vibrating system. The amplitude of a damped vibration will eventually decay to zero.

Basically the natural frequency of a vibrating system increases with the stiffness of the elements and decreases with their mass.

The inertial forces in these systems are large compared to the drag or the friction forces.

It has been verified that thin-plate bending theory can be used with confidence to predict the low-frequency vibration behaviour of the clamped edged, CFRP laminates and that laminate theory is capable of predicting the elastic behaviour with sufficient accuracy to enable the frequency and mode shape predictions. It has also been shown that the laminate theory can be extended to predict the damping properties. The results have been presented in terms of an unfamiliar combination of the elastic constants, introduced in earlier studies and defined in equations (1) and (2). These constants are a convenient set to use when discussing the thin-plate deformation or its vibration. They are also the constants which naturally occur in the expression of the strain energy in terms of the plate centre-plane displacement, so that they lend themselves to use in the Finite-Element computations.

Table 3. The first three frequencies (with and without damping) obtained in numerical analysis

No.	**f_1 [Hz]**		**f_2 [Hz]**		**f_3 [Hz]**	
	free	dump	free	dump	free	dump
1	10.24	11.21	20.65	22.66	39.48	43.32
2	10.21	11.16	20.66	22.65	39.85	43.70
3	10.21	11.21	20.69	22.74	40.14	44.07
4	10.23	11.19	20.75	22.76	40.45	44.35
5	10.31	11.29	20.90	22.94	40.83	44.79
perf.	9.74	10.70	20.71	22.77	37.95	41.68

The elastic and damping properties of the plies of the laminate were deduced from the measurements on the complete laminate, using inversely the laminate theory. This approach might have advantages in some circumstances: it allows for the necessary parameter values to be deduced from a standard “production” laminate, and it obviates the need to make special unidirectional laminates for calibration purposes.

The elastic constants and, particularly, the damping constants are likely to be sensitive to the variations of manufacturing processes. The delaminations, the incomplete bonding among plies are likely to reduce the stiffness and greatly increase the damping.

As it is seen in Table 3, the frequencies in the case of the damping vibrations are greater than the frequencies of the free vibrations (vibrations without damping) for all cases of delaminated plates and for the perfect plate.

The fundamental frequency of the vibration increases as the diameters’ ratio is increasing.

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