Mechanical Testing and Diagnosis
ISSN 2247 – 9635, 2011 (I), Volume 1, 73-81

OPTIMAL DESIGN OF A ROOF USING BIOMIMETICS

Tina KEGL

II Gymnasium Maribor, Trg Miloša Zidanška 1, 2000 Maribor, SLOVENIA
e-mail: tina.kegl@uni-mb.si

ABSTRACT

In this paper, the attention is focused on the usage of biomimetics in architecture, especially on the sea shell design as an inspiration for a roof structure. Among the modern methods of architecture design, the parametric modelling, the finite element analysis and the optimization are used to design a sea shell-like roof. For this purpose, two research programs are employed: the parametric modelling and the finite element analysis program STAKx and the interactive optimization program iGOx. To get a sea shell-like design, the design variables are related to the thickness and to the geometry of the roof. The objective function, to be minimized, is defined as the total strain energy of the structure. The imposed constraints are related to the maximal displacement and to the maximal volume of the roof. The obtained optimal roof exhibits attractive properties and an aesthetic look.

Keywords: nacre shell, architecture, roof strength analysis, practical implementation

1. INTRODUCTION

The word biomimetics is used to describe the substances, the equipment, the mechanisms and the systems by which humans imitate the natural systems and designs. Today, biomimetics finds applications in all areas, including architecture and building. Biological models may be emulated, copied, learnt or taken as starting points for new technologies. Through studies of biological models, new forms, patterns and building materials arise in architecture. Because of their properties, the biomimetic materials often outperform the conventional materials and constitute future challenges for architecture [1].

If we take a look under the water, more precisely, at various sea shells, we can see that they often resemble wavy hair because of their irregular shapes. This shape allows for a lightweight shell to withstand an enormous pressure. Architects have imitated their structure for designing various roofs and ceilings. For example, the roof of Canada's Royan Market, Figure 1, was designed with the oyster shell in mind.

Fig. 1. A sea shell (left) and Royan Market in Canada (right) [2, 3]

Generally, it seems that regarding the roof design we can learn a lot from the design of sea shells. This is because a sea shell exhibits all the main properties, typically required for a roof structure. These properties are [4]:

- strength and resistance to external loads,
- economic (sparing) use of material,
- aesthetic appearance.

Keeping this in mind, it makes a good idea to include the sea shell design into the design process of a roof structure. Of course, this can be made either in a more or less heuristic manner, or one can go one step further and try to enrich this procedure by modern techniques, such as the computer supported structural analysis and the mathematical optimization.

2. PARAMETRIC MODELLING AND OPTIMIZATION

Generally, in parametric modelling geometric shapes are defined by adequate parameters and the corresponding equations. This has to be done in such a way that a variation of any parameter changes the shape of the structure, but preserves the validity of the design, i.e. it preserves the imposed requirements and constraints [5]. Frequently, the used parameters are related to simple quantities like lengths, widths, thickness, but also to more sophisticated ones like control point positions. An example of a Bezier curve, defined by 5 control points is shown in Figure 2. It should be noted that the shape of this curve can be varied in an elegant manner by simply varying the positions of its control points.

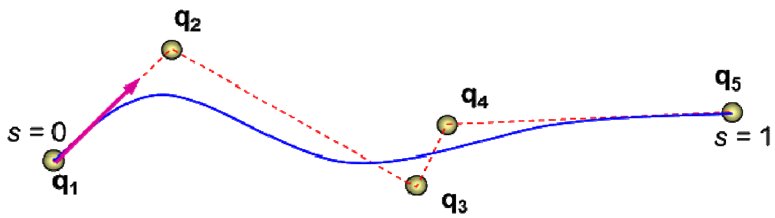

Fig. 2. A Bezier curve, defined by 5 control points

The essence of parametric modelling is to simplify the variation of the design and the shape in such a way that the shape variation can easily be performed by computer-supported procedures. This is of utmost importance especially if we want to use a mathematical optimization in order to support a systematic optimal design procedure.

An optimal design procedure is a systematic computer-supported search for the best solution. In the scope of this paper this means that one has to determine the optimal values of design variables (variable parameters) in order to get the desired properties (measured by the objective function) and fulfill the requirements (measured by the imposed constraints) of the structure. All methods of optimal design search for the minimum of the objective function – the optimum point at which the constraints are fulfilled.

Gradient-based methods, which are commonly used to solve the engineering optimization problems, use the function gradients, evaluated at a current point, to compute a better (optimal) point. More precisely, the solution procedure can be outlined as follows: (i) Choose appropriate initial (starting) values of design variables. (ii) Compute the objective and constraint functions and their gradients. (iii) Submit the computed quantities to the optimizer (optimization algorithm) in order to compute improved (new) values of design variables. (iv) Check some appropriate convergence criteria. If fulfilled, then stop. If not, go back to step 2. Step 2 of the above procedure necessitates a numerical structural analysis. Nowadays this is typically done by using the finite element method, implemented in some engineering software.

In architecture, a lot of commercial programs can be used for parametric modelling and structural analysis, e.g., AutoCAD, Abaqus, Ansys etc. However, these commercial programs are not easily integrated into an optimal design procedure. For this reason, in this work the structural analysis program STAKx has been used, which is easily combined with a gradient-based optimization program iGOx. Both of these research programs were developed at the Faculty of Mechanical Engineering at the University of Maribor. STAKx is actually a finite element program for static analysis of elastic structures. The specialty of this program is its strong orientation into the shape parameterization of structures and the possibility to compute the gradients of response quantities. The employed parameterization is based on the so-called design elements whose shapes are determined by the positions of their control points. The program iGOx is actually a gradient-based optimizer which enables interactive optimization by making use of external response analysis programs like STAKx.

3. THE SEA SHELL-LIKE ROOF STRUCTURE MODEL

In this paper it is assumed that we want to design sea shell-like roof structure to cover a fixed area of an, for example, exhibition pavilion. The program STAKx is used to define the geometrical model of the roof. The top view of the roof is shown in Figure 3, where the thick blue lines mark the supported edges of the structure. To describe the geometry, 3×11=33 control points are used. Some of the control points will be fixed, while the other the control points will be allowed can move in order to change the shape of the roof. By changing the positions of the movable points adequately, the roof may obtain a se shell-like form.

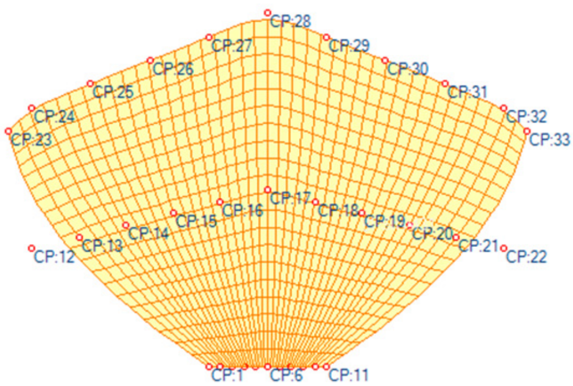

Fig. 3. Top view of the roof and the control points

The span of the roof is 60 m in the x direction and 30 m in the y direction. Its cover area is approximately 1000 m^2. The roof is vertically supported along all edges, except along the front edge, defined by the control points 23-33.The roof is assumed to be loaded in the vertical direction. More precisely, the imposed snow load is 3000 N/m^2. Additionally, the actual weight of the roof was also taken into account. Of course, weight depends on the roof volume and the material chosen. In our case the selected material is concrete. Its modulus of elasticity is $30\cdot10^9$ N/m^2 and its density is taken as 3000 kg/m^3.

4. OPTIMAL DESIGN OF THE ROOF

In order to obtain a roof design that is really optimal with respect to specific criteria, it is necessary to enrich the parametric modelling by one of the optimal design methods. In order to use such a method, one needs to formulate the problem of an optimal design, i.e., to select the design variables and to define the objective function and the constraints.

In the case of our roof, the selected design variables b_i $i = 1,...21$ are related to the roof thickness $d = 1 + b_i$ and x, y or z coordinates of some of the 33 control points as follows:

$$
\begin{array}{lll}
z_{2,4,6,9,10,24,26,29,30,32} = 0.1 + b_2 & y_{17} = 15 + b_8 & x_{16,18} = 4 + 0.4 \cdot b_{14} \\
z_{17} = 1 + 25 \cdot b_3 & y_{16,18} = 14 + b_9 & x_{15,19} = 8 + 0.4 \cdot b_{15} \\
z_{16,18} = 15 \cdot b_4 & y_{15,19} = 13 + b_{10} & x_{14,20} = 12 + 0.4 \cdot b_{16} \\
z_{15,19} = 1 + 20 \cdot b_5 & y_{14,20} = 12 + b_{11} & x_{18,21} = 16 + 0.4 \cdot b_{17} \\
z_{14,20} = 10 \cdot b_6 & y_{13,21} = 11 + b_{12} & x_{27,29} = 5 + 0.4 \cdot b_{18} \\
z_{13,21} = 1 + 12 \cdot b_7 & y_{18,21} = 10 + b_{13} & x_{26,30} = 10 + 0.4 \cdot b_{19} \\
 & & x_{25,31} = 10 + 0.4 \cdot b_{20} \\
 & & x_{24,32} = 10 + 0.4 \cdot b_{21}
\end{array}
\tag{1}
$$

In short, we have 21 design variables that influence the thickness and the shape of the roof.

The objective function is the quantity that has to be minimized during the optimization process. In our case, this could be the strain energy of the roof. The strain energy, here denoted as Π, is the energy that is stored within an elastic body when it is deformed under the influence of the external loads. From practical experience, we know that good designs exhibit low strain energy when subjected to some given external loads. We can use this fact to formulate the problem in the following direction: find such values of the design variables that the strain energy of the roof under the prescribed loads will be minimal.

On the other hand, we also know from the practical experience that the strain energy of the structure can be reduced by keeping the design fixed and just adding material, i.e., in the case of a roof, by making the roof thicker. Because it is not our intention to make the roof too thick, we have to impose a constraint on the volume of the used material. If the structural volume is denoted by V and its maximal allowed value equals V_{max}, the constraint can be written as $V \leq V_{max}$.

In addition to the volume constraint, we impose for practical reasons one further requirement: the vertical displacement Δz of the most exposed point of the roof (CP: 28) should be less than the maximal allowed value Δz_{max}. This constraint can be written as $\Delta z \le \Delta z_{max}$.

If we summarize the above discussion, the optimal design problem can be defined as follows:

- minimize the total strain energy, i.e.,

$$min\ \Pi \tag{2}$$

- subject to constraints

$$V - V_{max} \le 0 \tag{3}$$

$$\Delta z - \Delta z_{max} \le 0$$

where, in our case, $V_{max} = 600\ m^3$ and $\Delta z_{max} = 0.2\ m$. The quantities Π, V and Δz depend on the design variables. They have to be computed by using an adequate software, in our case, by the finite element analysis program STAKx.

The optimization, i.e., the solution process of the above optimal design problem, has been performed by the program iGOx, which can run the program STAKx in order to perform the response and sensitivity analysis of the structure. iGOx is an interactive gradient-based optimization program, which enables a continuous monitoring and an eventual adjustments during the optimization process (Fig. 4).

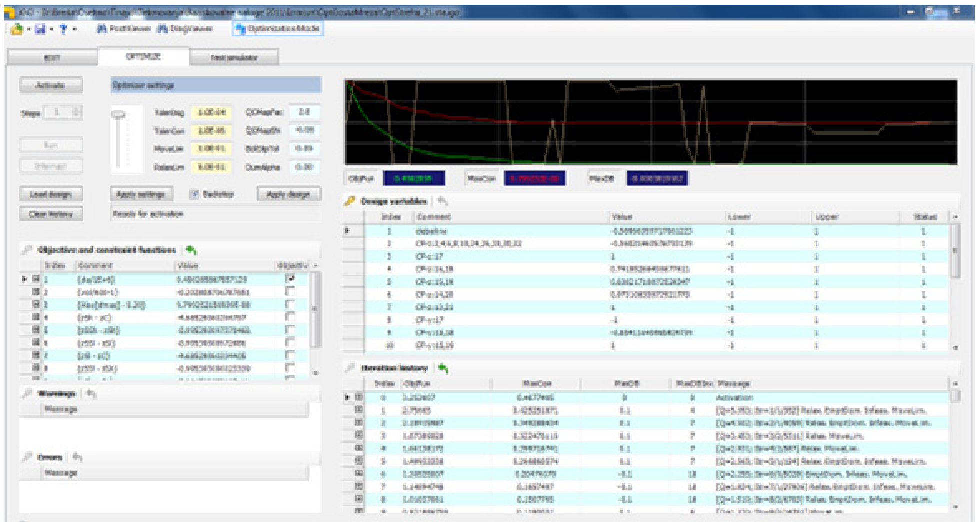

Fig. 4. The interactive optimization program iGOx

The initial values of all design variables were zero: $b_i^{ini} = 0,\ I = 1,...,21$. The corresponding initial thickness and the design dependent coordinates of the control points (in meters) were:

$$\begin{bmatrix} 1.00000 & 0.10000 & 1.00000 & 0.00000 & 1.00000 & 0.00000 & 1.00000 \\ 15.00000 & 14.0000 & 13.0000 & 12.0000 & 11.00000 & 10.00000 & 4.0000 \\ 8.0000 & 12.0000 & 16.0000 & 5.00000 & 10.00000 & 15.00000 & 20.00000 \end{bmatrix}^T$$

The displacements and the distribution of strain energy of the initial roof design are presented in Figure 5.

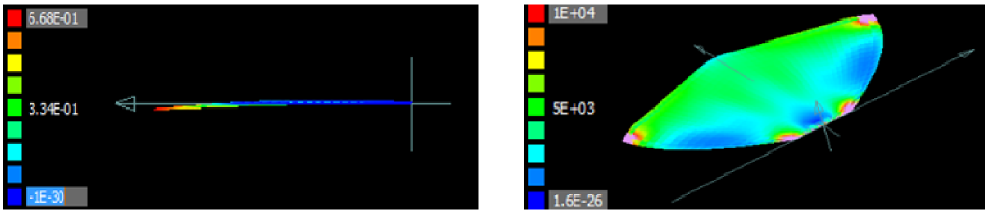

Fig. 5. The displacements and the distribution of strain energy of the initial roof

The optimization process converged nicely after 30 iterations. The final (optimal) values of design variables were as follows:

$$b_i^{opt} = \begin{bmatrix} -0.55015 & 0.04322 & 0.99534 & 0.78061 & 0.640640 & 1.00000 & 1.00000 \\ -1.00000 & 0.568073 & -0.05508 & 0.51398 & 1.00000 & 1.00000 & 0.43664 \\ 0.40026 & 0.69340 & 0.23342 & -0.97349 & -0.24194 & -0.00034 & 1.00000 \end{bmatrix}^T$$

which corresponds the following optimal values of thickness and design dependent coordinates of the control points:

$$\begin{bmatrix} 0.44985 & 0.14322 & 25.88349 & 11.70908 & 13.80794 & 6.00000 & 13.0000 \\ 14.00000 & 14.56807 & 27.56807 & 12.51398 & 12.0000 & 11.00000 & 4.17466 \\ 8.16010 & 0.69340 & 16.09337 & 4.61060 & 9.90322 & 14.99986 & 20.40000 \end{bmatrix}^T$$

The obtained optimal design of the roof, which looks similar to the nacre design, is presented in Figure 6.

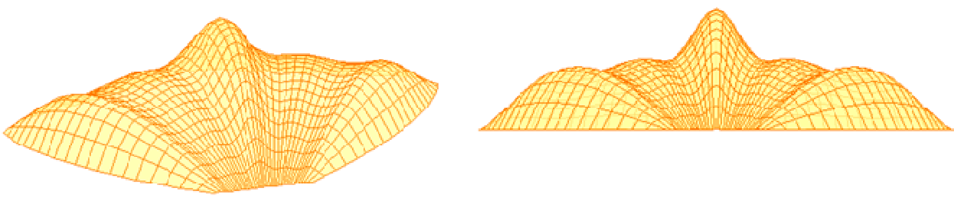

Fig. 6. Optimal design of the roof

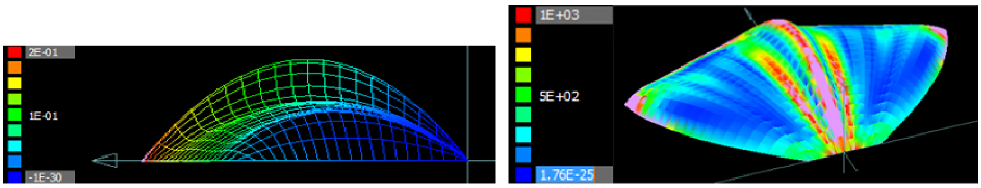

Fig. 7. The displacements and distribution of strain energy of the optimal roof

The response of the optimal roof, i.e., the displacements and the distribution of the strain energy, are presented in Figure 7.

For comparison, the response and some other parameters of initial and optimal design are presented in Table 1. It is evident that by optimization both, the strain energy and the volume of the roof, decreased. Furthermore, all constraints have been fulfilled by the optimization process.

Table 1. Properties of initial and optimal roof design

Parameters of response analysis and optimization	Initial roof design	Optimal roof design
Π (N.m)	3 252 607	496 688
Max. constraint violation	0.46774	< 0
$\lvert \Delta z \rvert$ (m)	0.668	0.1999211
V (m^3)	871.47	531.12
Mass (kg)	2 614 417	1 593 361

The obtained optimal roof design is obviously similar to a sea-shell. On the basis of the properties of the optimal roof design, given in Table 1, it is evident that the roof, which imitates the sea shell design, exhibits low mass and better load carrying capabilities.

5. EXPERIMENTAL VERIFICATION OF OPTIMAL ROOF DESIGN

To verify the correctness of the strength analysis and optimal design procedure, models of initial and optimal roof, reduced by a factor of 200, have been manufactured (Fig. 8). For this purpose rapid prototyping technology on the machine EOSINT P800 has been used. The used material is the composite PA 2200 on the basis of polyamide. Its elastic modulus is 1.7 GPa and its density is about 930 kg/m^3. The surface load of 300 N/m^2 has been used for the numerical simulation and for the experiment. For the experiment, this load was approximated by a water-filled plastic bag (Fig. 9).

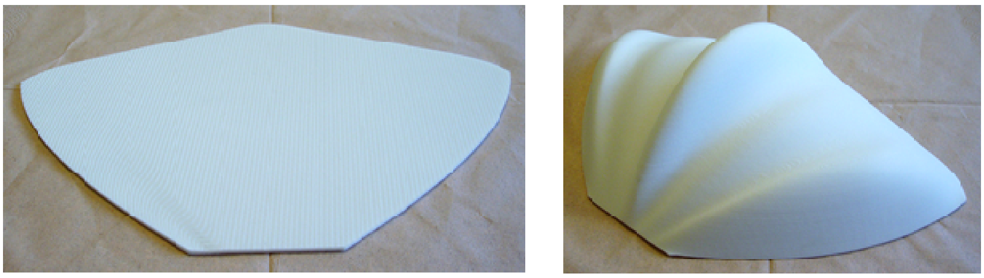

Fig. 8. The manufactured reduced models of the roof initial and the optimal roof

Fig. 9. Measurement of the maximal displacements of the manufactured model

A comparison of the computed and measured displacement for the initial roof design and the optimal one is presented in Table 2. The measurements of the optimal roof model were in the range between 0.3 and 0.4 mm. Therefore, an approximate mean value of 0.35 is listed in Table 2.

Table 2. A comparison of numerically (STAKx) and experimentally obtained displacements

	Manufactured model of initial (flat) roof design		Manufactured model of optimal (nacre) roof design	
	STAKx	experiment	STAKx	experiment
$\lvert \Delta z \rvert$ (mm)	6.0	7.0	0.3	0.35

As one can see from Table 2, the numerically and experimentally obtained displacements agree quite well. This is especially true, if one takes into account that the experimental loading is quite far from an ideal constant distributed load. Furthermore, for practical reasons, the supports in the experiment could not be realized as prescribed in the numerical simulation. Taking only these two quite significant sources of error into account, one may say that the agreement is good enough in order to conclude that the numerical simulation was accurate within reasonable limits.

6. PRACTICAL APPLICABILITY

The roof obtained by the optimization is in some sense similar to the nacre shell. The supporting in the vertical direction of this roof structure is supposed along all three short edges. Therefore, the face side of the roof can be open, offering free access, e.g., for visitors, logistics and so on.

Obviously this nacre shell-like roof could be potentially used, e.g., for:

- exhibitions pavilion (Fig. 10),
- commercial building,
- sport stadium,
- market building and so on.

In the case of an exhibition pavilion (Fig. 10), the front side can be glazed. This would offer various possibilities for light effects, which are very important to attract visitors and potential buyers.

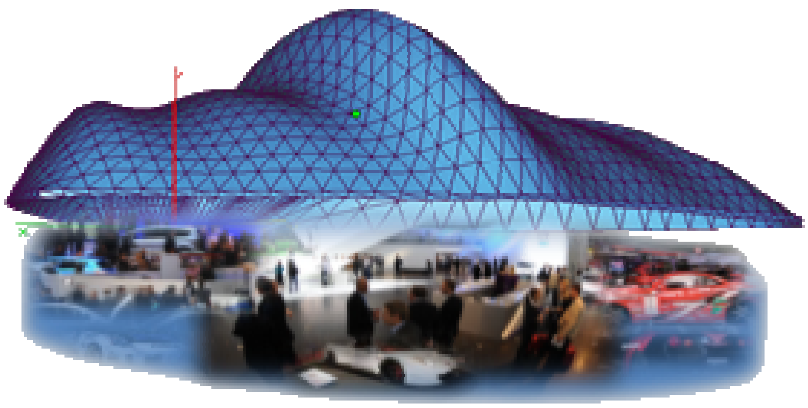

Fig. 10. Usage of nacre shell-like roof for car exhibition pavilion

7. CONCLUSIONS

This paper discusses how to enrich biomimetics by the use of modern methods of architecture in the field of roof design. On the basis of the results obtained in this work, the following conclusions can be done:

- a sea shell surely represents an interesting draft of a roof design,
- some of the most important quantities related to a free form roof design are the strain energy, the displacements, the mass and the volume of the material,
- parametric modelling and optimal design can offer efficient techniques in architecture where statics and aesthetic aspects has to be taken into account,
- the optimization of a free form roof design by minimizing the strain energy can yield a light weight, but strong, roof structure.

ACKNOWLEDGEMENTS

This investigation was supported by the mentors MSc Stane Kodba and Alenka Prapotnik-Zalar at IInd Gymnasium Maribor and by the company CHEMETS, Kranj, d.o.o, Slovenia, where the roof models were manufactured by rapid-prototyping.

REFERENCES

[1] **Zbašnik-Senegačnik, M., Koprivec,** L., 2009, Biomimetics in the architecture of tomorrow. *Architecture Research,* vol. 1, pp. 40-49.

[2] **Yahya, H.,** 2006, Biomimetics, Technology Imitates Nature, Global Publishing, Istanbul.

[3] **Kegl, T.,** 2011, Z biomimetiko do učinkovitestrehenadglavo, Research work. II. Gymnasium Maribor, Maribor, 2011.

[4] **Rizzoa, F., D'Asdiaa, P., Lazzarib, M., Procinoc, L.,** 2011, Wind action evaluation on tension roofs of hyperbolic paraboloid shape, *Engineering Structures*, vol. 33, pp. 445-461.

[5] **Stavric, M., Marina, O.,** 2011, Parametric modeling for advanced architecture. *International Journal of Applied Mathematics and informatics*, vol. 5, pp. 9-16.