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DYNAMIC STABILITY ANALYSIS OF GEAR TRANSMISSIONS

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ABSTRACT

In this paper dynamic stability of spur gear transmissions induced by mesh stiffness variation is numerically investigated. The obtained results were compared with experimental results presented in the references [5],[6] and [7]. It was found a good agreement with the experimental resonances in the sub critical, critical and super critical range. The equation of motion was obtained by reducing the gears system at the line of action and is presented as a Mathieu's equation. The parameter of this equation was considered the fluctuation of mesh stiffness modeled as a harmonic function. The dynamic stability of the system was determined for various values of the ratio: mesh frequency/natural frequency of the system.

Keywords: gears, vibration, dynamic stability, Mathieu's equation

1. INTRODUCTION

Gears transmission plays an important role in modern technology. It transfers power andmotion, and it is used in various types of machines systems. The basic concern in the old studies of gears transmission was the prediction of tooth dynamic loads for designing gears at higher speeds [8]. In later studies of gear dvnamics the target is calculation of dynamic transmission errors to predict gear noise. The predicting the gear noise and dynamic loads acting on the gear tooth is the most important concern in real gear design [8].

The main source of vibrations in a geared transmission system is usually the meshing action of the gears. The vibration models of the gear-pair in mesh have been developed, taking into account the most important dynamic factors such as effects of friction forces at the meshing interface, gear backlash, the time-varying mesh stiffness and the excitation from gear transmission errors [4].

2. PROBLEM FORMULATION

In the present study only spur gears transmission was considered. The mesh action of the gears is schematically represented in the Figure 1.

Periodic variation of the single pair mesh stiffness k(t) [1],[8] and its shape during a period (see the Figure 2 for the steel gears) makes rational to be used a harmonic function to approximate mesh stiffness time-variation as it is proposed by the relation (1):

$$k\left(t\right)=k_m-k_0\cos\Omega t\,, \qquad (1)$$

where k_m is the average of mesh stiffness, k_0 is the amplitude of fluctuations and Ω is the mesh frequency (*gear mesh frequency=number of teeth*shaft speed*). The variation of the mesh stiffness of spur gears teeth was calculated in [1] by taking into account the bending deflection of teeth in respect with rigid gear and the contact deformations using FEM. The ratio k_0/k_m for a pair of teeth in contact of steel gears was equal with 0.169 [1] (see the Figure 2). The relation (1) means parametrical excitation of gear mesh vibrations.

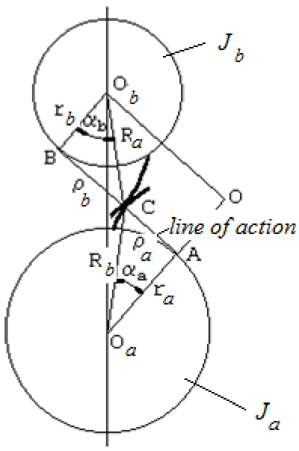

Fig. 1. *Figure 1 Meshing action of the gears; radius of base circles ra (rb), rotary inertia moments about rotation axis Ja (Jb)*

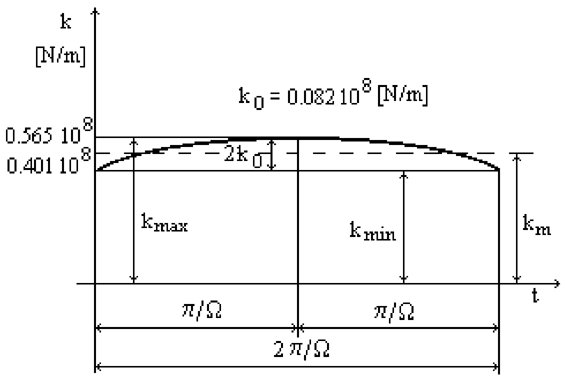

Fig.2. *Variation of mesh stiffness [1]*

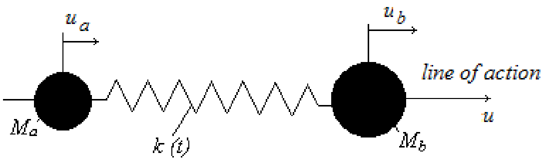

Fig. 3. *Mechanical model of parametrical vibrations induced by mesh stiffness variation [1]*

Mechanical model used to analyze parametric vibrations induced by the variation of mesh stiffness k is given in the Figure 3. In the Figure 3 M_a and M_b are reduced mass in respect with the line of action

$$\left(M_a = \frac{\bar{J}_a}{r_a^2}; M_b = \frac{\bar{J}_b}{r_b^2} \right).$$

The differential equations of motion (without damping) of the two mass are [1]:

$$\begin{aligned} M_a \ddot{u}_a - k(t)(u_b - u_a) &= 0 \\ M_b \ddot{u}_b + k(t)(u_b - u_a) &= 0 \end{aligned} \quad (2)$$

On purpose to eliminate the rigid displacement of the two mass system a new variable $u=u_b-u_a$(relative displacement) is introduced and the two equations are reduced in the form (3):

$$\ddot{u} + \left(\frac{k_m}{M_a} + \frac{k_m}{M_b}\right)\left(1 - \frac{k_0}{k_m}\cos\Omega t\right)u = 0$$

$$\ddot{u} + p^2\left(1 - \frac{k_0}{k_m}\cos\Omega t\right)u = 0 \quad (3)$$

with $$p = \sqrt{\frac{k_m}{M_a} + \frac{k_m}{M_b}} \quad (4)$$

where p is the natural frequency of the system having $k=k_m$=constant.

If notes $\frac{k_0}{k_m} = Eps$ the equation (3) becomes:

$$\ddot{u} + p^2\left(1 - Eps\cos\Omega t\right)u = 0 \quad (5)$$

Equation (5) is a Mathieu's equation [2,3].

If we came across equation (5) in numerical work, we could use the Matlab ODE integrator to investigate the solutions *u(t)* for various different values of parameters *p*, Ω and *Eps*. What would we find is that for some combinations of parameter values the solutions are "well-behaved oscillations with bounded amplitudes"[3], for some combination of parameter values the solutions are uneven oscillations while for other combinations of parameter values the amplitudes of the solutions grow exponentially in time; in this way we have stable oscillations, uneven oscillations and unstable oscillations.

The equation (5) was numerical integrated using Matlab ODE45 function over 20 periods of the slower oscillation (Matlab ODE45 function solve the differential equations using 4th-order Runge-Kutta method). The equation was subjected to random initial conditions [3]. I call the solution unstable if the amplitude exceeds 10x its initial value.

The representation of scattered points of solution is given in the Figure 4 with related notations.

The example of a stable solution (marked by '*' in Figure 4) is given in the Figure 5, an example of a stable solution (marked by '•' in Figure 4) is given in the Figure 6 and an example of an uneven oscillations (marked by 'o' in Figure 4) is given in the Figure 7.

For steel gears the ratio $Eps=k_o/k_m$ is near the value 0.2. In the vicinity of parameter Eps=0.2 (Figure 4) there are some values of Ω/p ratio for which we could have resonances: Ω/p=0.5 (in subcritical range) and Ω/p=1 (in the critical range) and Ω/p=1.5 in the super critical range.

For Ω/p=2 we will always have resonance produced by dynamic instability of the system (super critical range).

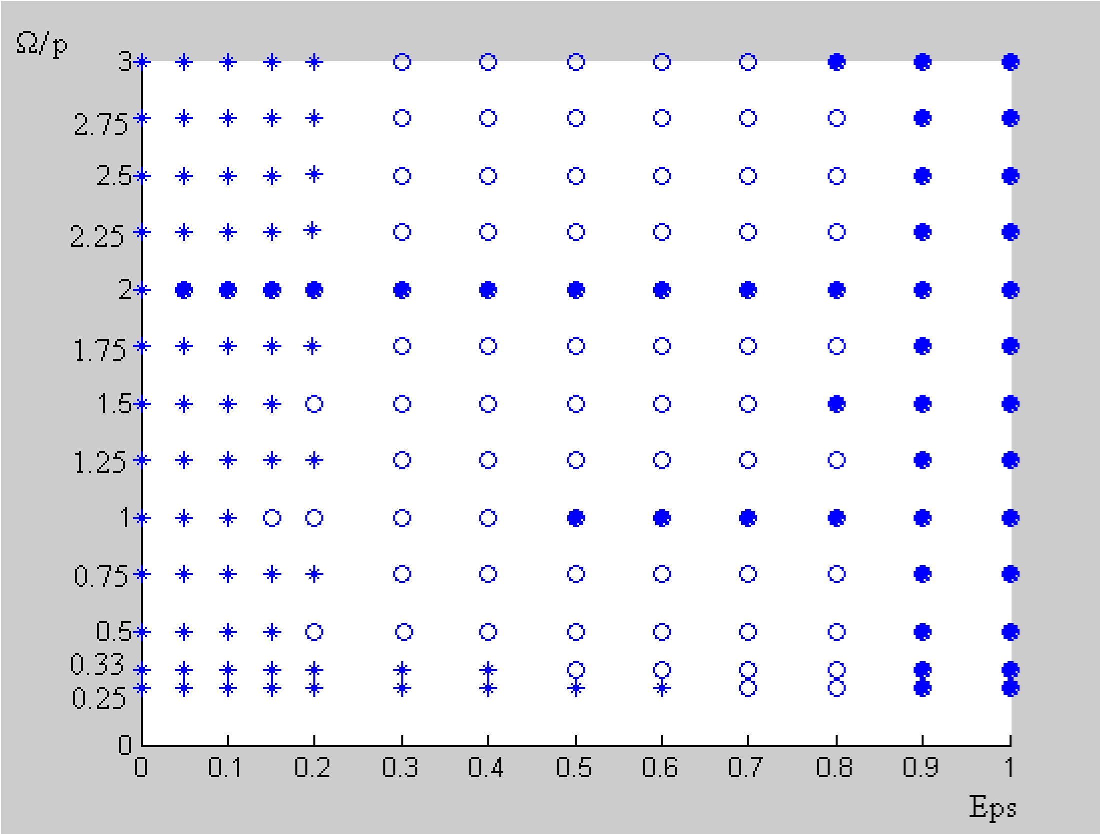

Fig. 4. *The representation of scattered points of solution - Ω/p (mesh frequency/natural frequency) versus Eps=k_o/k_m.*

Points with stable solution oscillations with bounded amplitudes denoted by‘*’, with uneven oscillations denoted by ‘o’ and with unstable solution (the amplitude exceeds 10x its random initial value) denoted by ‘•’.

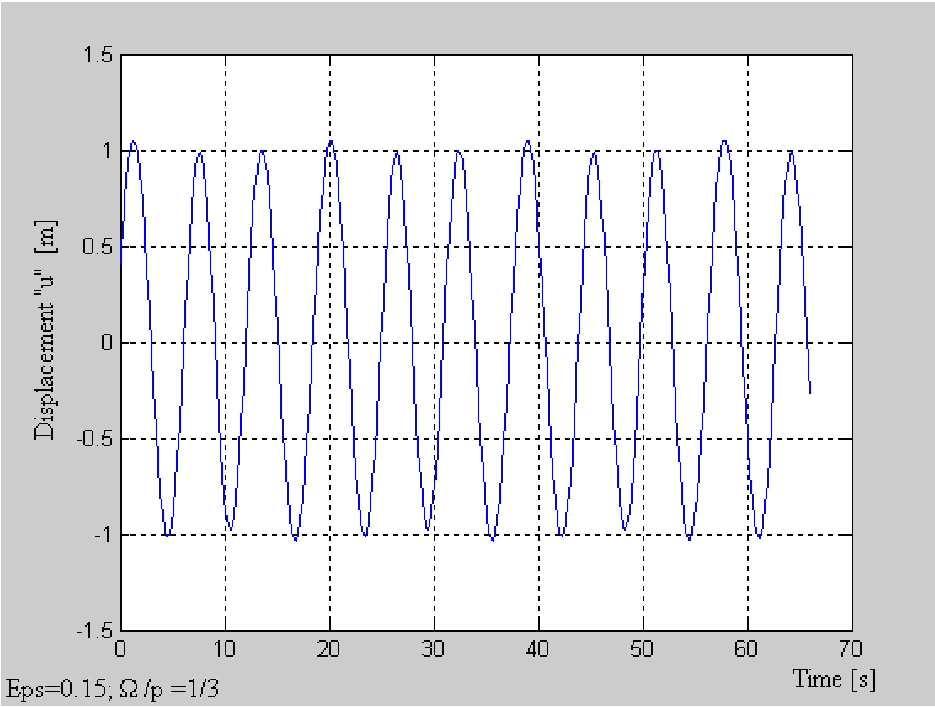

Fig. 5. *Example of stable solution*

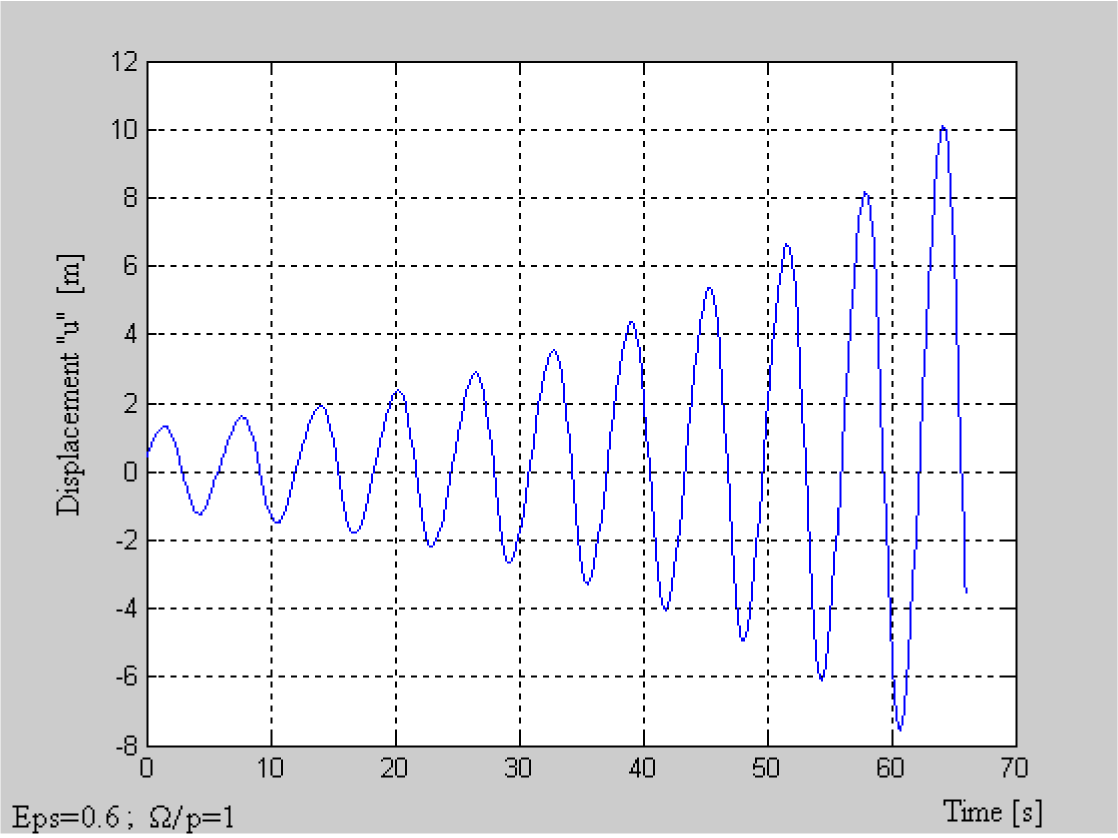

Fig. 6. *Example of unstable solution*

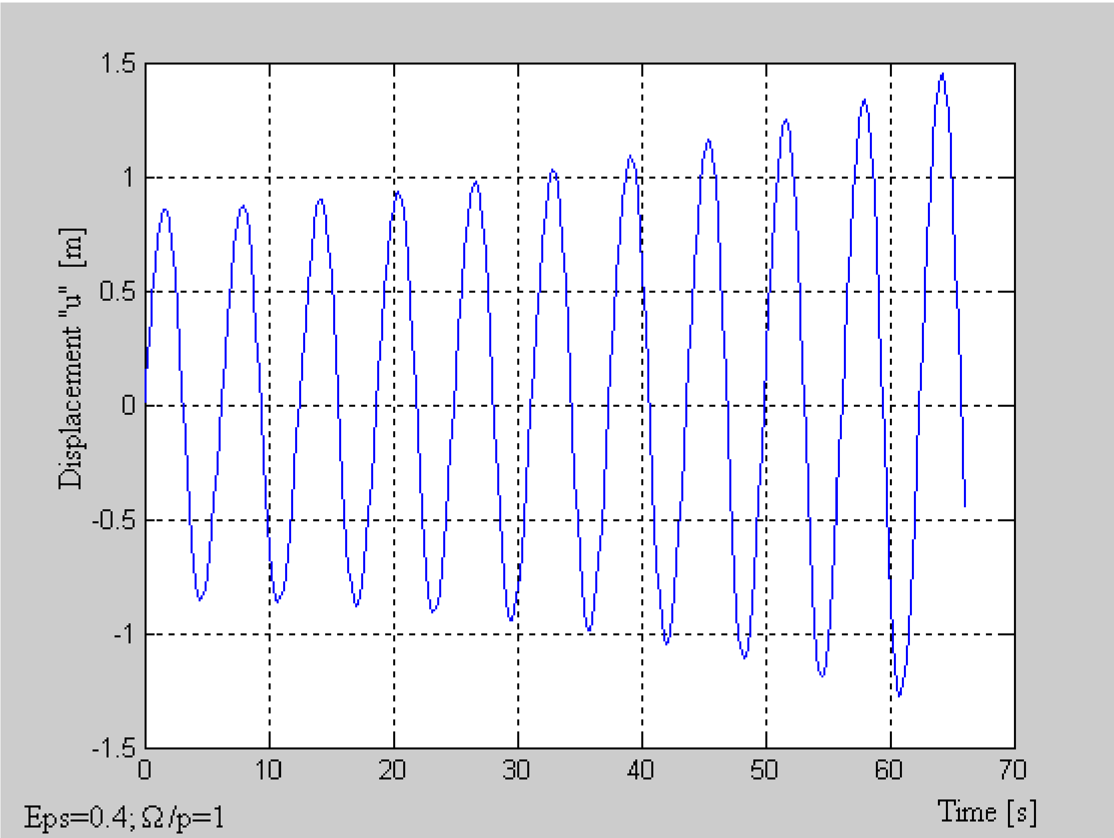

Fig. 7. *Example of uneven oscillation*

In the Figure 8 from reference [5] are shown the resonances of dynamic factor (gear dynamic forces and vibration) in the subcritical, critical and supercritical range. The super critical resonance is highlighted only in the experimental results and not in the calculated values of dynamic factor. The authors of [5] said that a possible explanation of super critical resonance can be found in their model instability. In the present work, in the Figure 4 it is shown that the dynamic instability of system for mesh frequency=2*natural frequency induces that super critical resonance in any conditions.

In the reference [7] for two stage gear system the authors concluded that "the unstable regions of the time-varying stiffness gear system appear at the position of twice its natural frequency".

In the reference [6] was observed that the vibration spectrum calculated by numerical methods and the spectrum of the measured vibration signal (Figure 9) show the same sideband structures.

The Figure 9 shows the resonances in the subcritical, critical and super critical range (for f_z/f_0=0.5, f_z/f_0=1, f_z/f_0=1.5 and f_z/f_0=2 - see values and notations in the Figure 9).

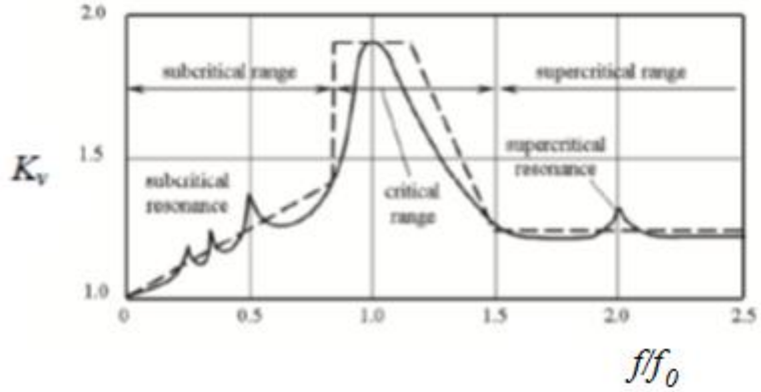

Fig. 8. *Dynamic factor Kv and approximation of gear dynamic forces and vibration (continuous curve represents the experimental results)[5]; f=mesh frequency and f_0=natural frequency*

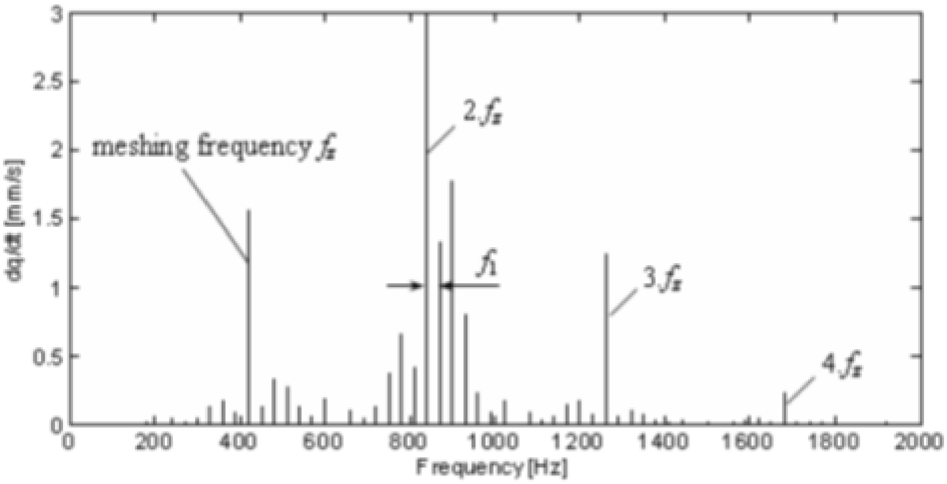

Fig. 9. *Calculated results of frequency spectrum [6]; f_z =420Hz is the mesh frequency; f_0=869 Hz is the natural frequency of the system*

3. CONCLUSIONS

The dynamic stability of the system was investigated for the values of the parameter Eps=k_0/k_m in the range (0, 1] and for various values of the ratio: *mesh frequency/natural frequency* of the system.

It can be concluded that:

- the system could be stable in the subcritical range for the values of parameter Eps<0.15.
- for 0.15<Eps≤0.2the system tends to be unstable (the solution is represented by uneven oscillations) inthe subcritical rangefor *mesh frequency=0.5* natural frequency* of the system, in thecritical rangefor *mesh frequency=natural frequency* and also in super critical rangefor *mesh frequency=1.5*natural frequency*; any

failure of the gear teeth or gear transmission may cause dynamic instability.

- in the super critical domain with *mesh frequency=2*natural frequency*, the system will be unstable for any value of the variable *Eps*.
- the uneven oscillation represents the initial stage manifestation that can lead to instability.

REFERENCES

[1] Boazu D., *Contributions concerning the analysis of gear transmissions, (in Romanian: Contribuţii privind analiza vibraţiilor provocate de angrenaje)*, Doctoral dissertation, "Dunarea de Jos" University of Galati, 1999.

[2] Bolotin V.V., *The dynamic stability of elastic systems, Volume 1*, Translated byV. I. Weingarten, K. N. Trirogoff and K. D. Gallegos, Aerospace Corporation El Segundo, California Contract No. AF 04(695)-169, 1 November 1962.

[3] Homer Reid, *Stability of Mathieu's Equation via Classical and RG Methods*, 18.305 Lecture Notes 2015.

[4] Howard, I., Shengxiang, Jia., & Wang, J. (2001*). The dynamic modelling of a spurgear in mesh including friction and a crack*. Mechanical System and Signal Processing,15(5), 831-853.

[5] MilosavOgnjanovic - FathiAgemi, *Gear Vibrations in Supercritical Mesh-Frequency Range Caused by Teeth Impacts*, Strojniškivestnik - Journal of Mechanical Engineering 56(2010)10, 653-662.

[6] Nguyen Van Khang and Nguyen Phong Dien, *Parametric Vibration Analysis of Transmission Mechanisms Using Numerical Methods*, Advances in Vibration Engineering and Structural Dynamics,Intech 2012, 301-331, http://dx.doi.org/10.5772/51157

[7] Z. Rao , C.Y.Zhou, Z.H.Deng , M.Y.Fu, *Nonlinear torsional instabilities in two-stage gear systems with flexible shafts*, International Journal of Mechanical sciences, 2014, 60-66, http://dx.doi.org/10.1016/j.ijmecsci.2014.02.021.

[8] Yonatan Afework and Tamrat Tesfaye, *Variable mesh stiffness of spur gear teeth using finite element method*, Journal of EEA, VoL23, 2006.