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ANALYTICAL AND GRAPHICAL MODELLING FOR GENERATION OF A NON CYLINDRICAL HELICAL SURFACE USING A PRE-FORMED TOOL

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ABSTRACT

The issue of generating a non-cylindrical helical surface using pre-formed tools — tools bounded by a revolution cylindrical surface or a conical surface, is the main concern for profiling the active blades of centrifugal rotors or the surface called free form surfaces. In this paper is proposed an analytical approach, based on fundamental theorems of surface generating and, also, a graphical development in CATIA design environment. It is developed the analytic support, based on the principle of surface enwrapping (Olivier theorem), for profiling a helical surface with variable pitch (the blade of a centrifugal rotor) belonging to a revolution body — conical surface with knows generatrix angle. The shape of crossing sections for generated surface is presented for a certain case.

Keywords: centrifugal compressor, graphical method, non-cylindrical helical surface

1. INTRODUCTION

The issue of profiling tools which generate ordered curls of helical surfaces, even for particular cases of compressor rotor constitute special cases which can be approached based on the fundamental theorems of surface enwrapping [1].

Radzevich S. [2] developed the issue of free form surfaces machining, based on fundamental theorems of technical surfaces generating. The analysis is based on surfaces machining process kinematics.

A particular problem is generating compressors and turbines blades surfaces. For this there was developed a suite of solutions, based on the capabilities of CAD/CAM graphical programs. The solutions approach both the blades generating issue and the aspects of roughness for surfaces machined by milling [3].

In case of centrifugal rotor blades, the main problem is the tool path programming in order to avoid vibrations of rotor body and blade. Minimizing the vibrations effect is an important requirement for smooth running of milling process.

Models were created, developing specific modules in C++ programming language, for tool path correction in order to reduce the machining costs [4].

The diversification of tool shape, so that it is no longer a revolution cylinder or tronconic body, rises new problems. Thus, Yao An Lu et al. [5] present the issue of free form generation with tools bounded by a barrel-type surface. By this it is suggested a new approach for generating tool paths used on machine tools with five numerical controlled axes. This allows optimizing the tool position in order to maximize the material detached volume.

Ling Quan et al. [6] propose a new solution for tool path planning when machining a propeller blade with ruled surface. The new solution is based on calculating points where the tool axis vector intersects the free form of the hub. The procedure was simulated in CAM programs and was applied for milling machines with five numerical controlled axes.

2. ROTOR BLADE'S GENERATING PROCESS MODELLING

In the following, it is proposed a modelling of generating process for the active surfaces of conical rotor blade, using an end mill tool, bounded by a primary peripheral surface — revolution cylinder.

In figure 1, is presented the support surface of rotor blades — conical revolution surface with straight line generatrix and the reference systems:

XYZ is the world reference system, joined with the rotor axis, representing the definition system for blade;

$X_0Y_0Z_0$ — mobile reference system, initially with axes overlapped to the axes of world system, and which, during the generation will, has a conical helical movement, with *Z* axis and *p* helical parameter. The system origin is O_0;

$X_0'Y_0'Z_0'$ — auxiliary mobile reference system, with origin in O_0' in plane X_0Y_0, onto the conical support generatrix;

$X_0''Y_0''Z_0''$ — auxiliary mobile reference system, with origin in O_0', having axes revolved around Y_0' with angle ψ_1 constant;

$X_1Y_1Z_1$ — mobile reference system, joined with end mill tool, with X_1 axis representing the axis of revolution body, revolved around the Z_0'' axis of the auxiliary system $X_0''Y_0''Z_0''$ with angle ψ = constant;

The angles ψ and ψ_1 define the disjunctive position of tool’s axis regarding the axis of conical support.

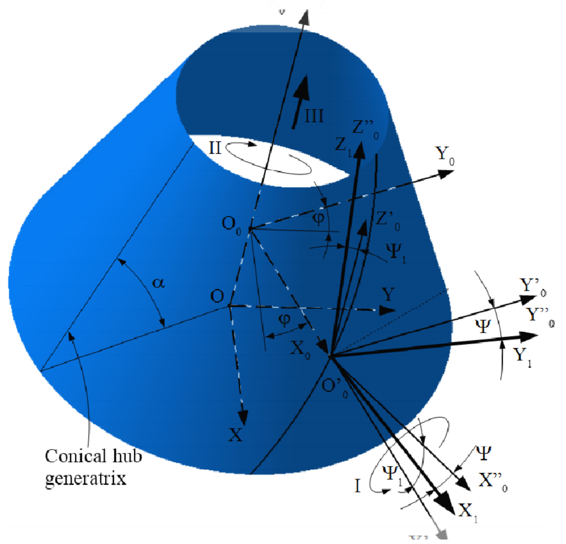

Fig. 1: Reference systems and generating kinematics

The generating kinematics assumes the following motions:

I is the rotation movement of tool around its own axis, X_1. In this motion the tool primary peripheral surface is self-generated;

II — the rotation movement of the rotor around its own axis, *Z*;

III — the translation of contact point between the tool and the rotor, the point O_0'.

The relative position of reference systems $X_1Y_1Z_1$ and $X_0''Y_0''Z_0''$ is defined:

$$X_0'' = \omega_3^T(\psi)\cdot X_1, \tag{1}$$

with $\omega_3^T(\psi)=\begin{pmatrix}\cos\psi & -\sin\psi & 0\\ \sin\psi & \cos\psi & 0\\ 0 & 0 & 1\end{pmatrix}$,

X_1 being the matrix of current point in the space of end mill tool, $X_1Y_1Z_1$.

It is defined the position of reference system $X_0'Y_0'Z_0'$ regarding $X_0''Y_0''Z_0''$:

$$X_0' = \omega_2^T(\psi_1)\cdot X_0'', \tag{2}$$

with $\omega_2^T(\psi_1)=\begin{pmatrix}\cos\psi_1 & 0 & \sin\psi_1\\ 0 & 1 & 0\\ -\sin\psi_1 & 0 & \cos\psi_1\end{pmatrix}$.

The relative position of $X_0Y_0Z_0$ reference system regarding the $X_0'Y_0'Z_0'$ system is given by coordinate transformation:

$$X_0 = X_0' - A,\ \text{with}\ A=\begin{pmatrix}0\\ R-p(Z)\cdot\theta\cdot\tan\alpha\\ 0\end{pmatrix}, \tag{3}$$

where the *p* helical parameter is variable according to the law:

$$p(Z)=c+\frac{b}{Z}+\frac{a}{Z^2}. \tag{4}$$

In (4), *Z* is the coordinate of the $X_0Y_0Z_0$ system origin along the axis of conical body, *R* is the initial radius of the conical helix on the support of surface to be generated and α is the angle of rotor generatrix.

We assume that the helix generated by point O_0' is a conical helix with variable pitch along the axis of conical support.

In this way, the helical motion of the $X_0Y_0Z_0$ system regarding the *XYZ* global reference system, joined with the generated rotor, is:

$$X = \omega_3^T(\theta)\cdot X_0 + B, \tag{5}$$

with $B=\begin{pmatrix}0 & 0 & p(Z)\cdot\theta\end{pmatrix}^T$.

The *B* matrix represents the translation of the $X_0Y_0Z_0$ reference system along the *Z* axis, axis of generated rotor.

From equations (1), (2), (3) and (5) results the helical motion of space associated with the end mill tool, $X_1Y_1Z_1$, regarding the world reference system associated with the generated rotor:

$$X = \omega_3^T(\theta)\cdot\left[\omega_2^T(\psi_1)\cdot\omega_3^T(\psi)\cdot X_1 + A\right]+B. \tag{6}$$

After developments results:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \left[\begin{pmatrix} \cos\psi_1 & 0 & \sin\psi_1 \\ 0 & 1 & 0 \\ -\sin\psi_1 & 0 & \cos\psi_1 \end{pmatrix} \cdot \begin{pmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix} + \begin{pmatrix} 0 \\ R - p(Z)\cdot\theta\cdot\tan\alpha \\ 0 \end{pmatrix} \right] + \tag{7}$$
$$+ \begin{pmatrix} 0 \\ 0 \\ p(Z)\cdot\theta \end{pmatrix}.$$

The matrix equation (7) represents the trajectories family of points from the generating tool’s space regarding the conic hub’s space — the rotor to be generated.

The points belonging to the primary peripheral tool generate, in the XYZ reference system, the blade surface.

As a result, defining only the S surface, the end mill tool primary peripheral surface, from the $X_1Y_1Z_1$ reference system:

$$S\left| X_1 = t; Y_1 = r\cdot\cos v; Z_1 = r\cdot\sin v, \right. \tag{8}$$

with t variable along the X_1 axis, v variable angular parameter and r radius of cylindrical surface S (technological solution).

From (7) and (8) the trajectories family results, regarding the XYZ reference system of the generated blade, belonging to the primary peripheral surface of the generating tool:

$$S(\theta, v, t) = 0\,, \tag{9}$$

or, in vectorial form:

$$\vec{R}_{(\theta)} = X(\theta,v,t)\cdot\vec{i} + Y(\theta,v,t)\cdot\vec{j} + Z(\theta,v,t)\cdot\vec{k}\,, \tag{10}$$

where $\vec{R}_{(\theta)}$ is the position vector of the current point belonging to the surfaces family, in the XYZ reference system. The family enwrapping, defined by the (7) and (8) equations assembly, is determined associating the family equations, the enwrapping condition:

$$\left(\vec{R}_t, \vec{R}_v, \vec{R}_\theta\right) = 0\,, \tag{11}$$

with $\vec{R}_t$, $\vec{R}_v$, $\vec{R}_\theta$ — partial derivatives of the $\vec{R}(t,v,\theta)$ vector, see (10), for variables t, v and θ.

In analytical form, the relation (11) is expressed by determinant:

$$\begin{vmatrix} \dot{X}_t & \dot{Y}_t & \dot{Z}_t \\ \dot{X}_v & \dot{Y}_v & \dot{Z}_v \\ \dot{X}_\theta & \dot{Y}_\theta & \dot{Z}_\theta \end{vmatrix} = 0\,, \tag{12}$$

with X, Y and Z defined by equations (7).

The partial derivatives are calculated for variables t, v and θ from (10).

So, the generated surface characteristic is obtained associating the equations (7) with the enwrapping condition (12), for θ = constant. For various values of the θ parameter, the characteristic curves family on the generated surface is obtained.

The actual form of the enwrapping condition is substantially simplified if, in equations (11)-(12) we accept $\theta = 0$.

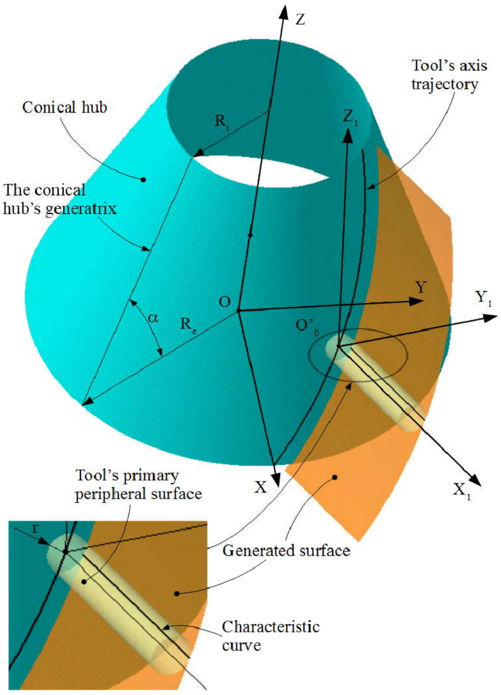

Fig. 2: Relative position of the end mill tool and hub; generated surface and characteristic curve

The (9) and (12) equations assembly, for $\theta = 0$, represents the characteristic curve between the generating cylindrical surface and the future surface of the rotor blade, in the XYZ reference system, in principle, in form:

$$C_{XYZ}\left| X = X(v); Y = Y(v); Z = Z(v). \right. \tag{13}$$

Giving to the characteristic curve (13) the helical motion with $\vec{V}$ axis and p helical parameter, see figure 1, the conic blade surface is generated.

In principle, the helical surface generated by the characteristic curve, is described by equations:

$$X = \omega_3^T(\theta)\cdot X(v) + A. \tag{14}$$

The determining of helical surface with variable pitch shape is possible knowing the crossing sections of the (14) helical surfaces, sections with planes perpendicularly to the conical hub’s axis, $Z = H$, (H arbitrary and variable, see figure 1):

$$Z(v) + p(Z)\cdot\theta = H\,. \tag{15}$$

In this way, the crossing section, in principle, has the form:

$$X = X(\theta); Y = Y(\theta); Z = H. \tag{16}$$

3. GRAPHICAL SOLUTION DEVELOPED IN CATIA

A numerical solution for a rotor is developed. The rotor’s characteristics are: base radius of hub R = 70 mm; external radius of the blade R_e = 120 mm; the angle of rotor’s generatrix α = 20°; generating tool’s radius r = 5 mm; ψ = 0° ψ_1 = 15° and coefficients of the variation law for the helical parameter: a = 10, b = 50, c = 100.

For graphically solving of the problem, crossing planes of conical section were generated. Onto these planes, segments were drawn. The length of each segment corresponds to the radius of intersection circle between the conical surface and the plane. The angle between the X axis and each segment corresponds to the value given by helical parameter and elevation of crossing plane, according to the equation:

$$\theta = \frac{H^3}{a + b \cdot H + c \cdot H^2} \cdot \frac{180^\circ}{\pi} \text{ [deg].} \qquad (17)$$

Between the ends of these segments is drawn a spline curve which will represent the trajectory of a point belongs to the end mill, in its relative motion against the conical surface.

A line inclined with angles ψ and ψ_1 beside Y_0 and Z_1 a drawn. This line represents the end mill tool axis.

Table 1. Coordinates crossing sections points [mm]

H=0 [mm]			H=25 [mm]		
X	**Y**	**Z**	**X**	**Y**	**Z**
68.777	6.030	0.000	57.503	20.316	25.000
71.015	6.113	0.000	72.090	21.611	25.000
73.253	6.193	0.000	86.688	22.762	25.000
75.492	6.268	0.000	101.298	23.768	25.000
77.730	6.340	0.000	115.916	24.627	25.000
H=50 [mm]			**H=75 [mm]**		
X	**Y**	**Z**	**X**	**Y**	**Z**
43.912	29.729	50.000	29.854	33.816	75.000
58.869	31.958	50.000	44.829	37.014	75.000
73.850	34.018	50.000	59.837	40.046	75.000
88.854	35.909	50.000	74.879	42.911	75.000
103.877	37.635	50.000	89.953	45.604	75.000
H=100 [mm]					
X	**Y**	**Z**			
17.116	33.047	100.000			
31.801	37.086	100.000			
46.527	40.974	100.000			
61.293	44.709	100.000			
76.097	48.290	100.000			
17.116	33.047	100.000			

The surface which admits as directrix the previously drawn spline curve and as generatrix the tool axis is the surface where the end mill tool axis moves.

The blade surface, generated by the end mill, is a surface equidistant to the previously determined surface, at distance equal to the tool radius.

In figure 3 and table 1 are presented the crossing sections of the rotor axis, in planes $Z = H$, with H variable

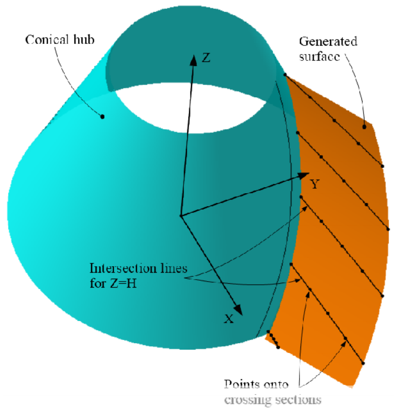

Fig. 3: Crossing sections of rotor blade; points onto crossing sections

4. CONCLUSIONS

In this paper an algorithm based on the fundamental theorem of surface enwrapping is presented. The algorithm is dedicated to analytical modelling of blade surface belonging to a non-cylindrical helical surface. The surface is generated with a pre-formed tool — a cylindrical body, corresponding to the primary peripheral surface of end mill tool.

The surfaces family generated by the cylindrical body are determined, in the helical motion onto a conical helix, with variable pitch, for a tronconic hub. A graphical model in CATIA is presented, for the blade flank generated by characteristic curve. The methodology is simple and easy to use. Similarly, it is possible to analyze the generation with a revolution cylindrical body, with a spherical end.

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